A Theory of Partially Directed Search

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This article studies a search model of the labor market in which firms have private information about the quality of their vacancies, they can costlessly communicate with unemployed workers before the beginning of the application process, but the content of the communication does not constitute a contractual obligation. At the end of the application process, wages are determined as the outcome of an alternating offer bargaining game. The model is used to show that vague noncontractual announcements about compensation—such as those one is likely to find in help wanted ads—can be correlated with actual wages and can partially direct the search strategy of workers.

I. Introduction

The search literature has focused on two polar descriptions of the labor market: random and directed search. In the random search model—as exemplified by McCall (1970) and Mortensen (1970)—unemployed workers have no information about the idiosyncratic component of the wage offered to fill different openings. In order to learn about the wage for a particular job, an unemployed worker has to make a relationship-specific investment; that is, he has to spend some time and money applying for that job. In the directed search model—as exemplified by Montgomery (1991), Moen (1997), or Burdett, Shi, and Wright

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(2001)—unemployed workers have perfect information about the different wages offered for different jobs before they decide where to look for work.

In reality, an unemployed worker does not have precise information about wages when he chooses where to apply for a job. Typically, an advertisement for a job opening contains only vague and noncontractual statements about compensation such as "great career opportunities," "competitive pay," or "wage commensurate with experience." Does this mean that random search is the relevant model of the labor market? Or could it be that these vague noncontractual statements are correlated with actual compensation and partially direct the search strategy of workers?

In order to answer this question, I study a search model of the labor market in which firms can communicate with unemployed workers before the application stage, but the content of communication is not contractually binding. More specifically, I consider an economy populated by homogeneous workers—each looking for a job—and heterogeneous firms—each trying to fill one vacancy. First, after having privately observed its productivity, each firm chooses which message to use to advertise its opening. Then, after having observed the entire distribution of help wanted ads, each worker chooses where to apply for a job. Finally, if a firm and an applicant successfully match, the terms of trade are determined as the outcome of an alternating-offer bargaining game.

The main finding of the article is that unless the labor market is either too tight or too slack, there exists an equilibrium in which noncontractual announcements about compensation are correlated with actual wages and direct the search strategy of workers. In this equilibrium, the productivity line is partitioned into a finite number of connected intervals. If the productivity of two jobs falls in the same interval, firms advertise them with the same message. If the productivity of two jobs belongs to different intervals, firms advertise them with different messages. In this equilibrium, workers apply more frequently to those jobs that are advertised with more positive messages about compensation, that is, messages that come from a more productive set of firms. Conversely, firms are more likely to fill their vacancies if they post a more positive message. When a worker and a firm successfully match, they begin to bargain over the terms of trade. At this stage, the worker's beliefs about the gains from trade are affected by the type of message used by the firm to advertise the job. And the more positive the message, the more optimistic the worker is and the higher his wage demands and, ultimately, the wage outcome.

How can noncontractual announcements be informative about wages and play a role in directing the search strategy of workers? The answer provided by this article is based on two insights. First, workers apply more frequently to those jobs that are advertised with more positive messages because they expect to meet more productive firms that, in turn, are more likely to concede to high wage demands. Second, more positive messages are posted by more productive firms because those are the only ones that are willing to face tougher wage demands at the bargaining stage in order to fill their openings with higher probability.

For a partially directed search equilibrium to exist, the labor market must be balanced. Specifically, if there are too many unemployed workers with respect to open vacancies, the incentives to signal higher productivity are not sufficiently strong to make noncontractual advertisement informative about compensation. Conversely, if there are too many job openings with respect to unemployed workers, the incentives to signal high productivity are so strong that noncontractual announcements about compensation cannot be credible. The precise location of these bounds on labor market tightness depends in a simple way on the relationship between the worker's bargaining power and the elasticity of the matching function.

Brief literature review.—In a series of influential papers, Nelson (1970, 1974) argues that inherently uninformative advertisement may be useful to consumers if there happens to be a correlation between a firm's observable advertisement expenditures and the unobservable quality of its good. In turn, such correlation may exist if producers of high-quality goods have more to benefit by attracting new customers than the producers of low-quality goods, for example, because of the possibility of repeat purchases. This idea has been formalized by Milgrom and Roberts (1986). In Bagwell and Ramey (1994), ostensibly uninformative advertisement is useful to consumers because a store's advertisement expenditures happen to be correlated with the unobservable variety of products it carries. In turn, the correlation can be sustained in equilibrium because retail stores that sell more goods have more to benefit from attracting new customers. My article presents an alternative mechanism through which inherently uninformative advertisement may end up being useful to those who are searching the market. In my article, the cost of advertising a high-quality job takes the form of tougher wage demands at the bargaining stage. And the benefit of advertising a high-quality job takes the form of higher probability of filling the opening.

Structure of the article.—In Section II, I describe the environment and specify the equilibrium concept. In Section III, I first characterize the outcome of the bargaining game. Then I identify a simple set of necessary and sufficient conditions for the existence of an equilibrium in which firms use N different messages to advertise their vacancies. In Section IV, I characterize the equilibrium set of the advertisement game in partial equilibrium. Finally, in Section V, I endogenize the value of search and characterize the equilibrium set of the advertisement game in general equilibrium.

II. Model

A. *Environment*

The economy is populated by a continuum of firms with measure 1 and a continuum of workers with measure b. Each firm i has one vacant job that would produce y_i units of output if it was filled by a worker. The productivity of the job is privately observed by the firm that owns it. The density $f(y)$ of job productivities is common knowledge. I assume that the density $f(y)$ is strictly positive over the interval $[\underline{y}, \bar{y}]$, where $0 < \underline{y} < \bar{y} = \infty$, and I denote its cumulative distribution with $F(y)$. Both firms and workers maximize expected consumption.

The two sides of the market come together via search. At the beginning of the period, firm i privately observes the productivity y_i of its job and chooses a message s_i with which to advertise it. The set $\bar{S}$ of messages that the firm can choose from is $\{s_1, s_2, \ldots, s_K\}$, where K is some large number.[1] Next, each worker reads the help wanted advertisements and decides where to apply for a job. That is, worker j seeks jobs advertised with the message s_j, where s_j belongs to the set of messages posted by some firms.

Denote with λ_k the ratio of the workers seeking a job advertised with s_k to the number of firms posting that message. Following Acemoglu and Shimer (1999), I refer to λ_k as the job's queue length. A job with a queue of length λ is filled with probability $\eta(\lambda)$, where $\eta : [0, \infty] \to [0, 1]$ is a twice continuously differentiable function such that $\eta'(\lambda) > 0$, $\eta''(\lambda) < 0$, $\eta(0) = 0$, and $\eta(\infty) = 1$. Symmetrically, if a worker applies for a job that has queue length λ, the worker is hired with probability $\mu(\lambda) = \lambda^{-1}\eta(\lambda)$, where $\mu : [0, \infty] \to [0, 1]$ is a twice continuously differentiable function such that $\mu'(\lambda) < 0$, $\mu(0) = \bar{\mu}$, and $\mu(\infty) = 0$. Denote with $\epsilon(\lambda)$ the elasticity of the job-filling probability $\eta(\lambda)$ with respect to λ. I assume that $\epsilon(\lambda)$ is a strictly decreasing function such that $\epsilon(0) = 1$ and $\epsilon(\infty) = 0$. The reader may find it useful to interpret $\eta(\lambda)$ as $v^{-1}m(\lambda v, v)$, where $m(u, v)$ is a constant return to scale function that tells how many matches are created when u workers seek v jobs. Then, the reader should notice that all the standard matching functions satisfy my assumptions on $\eta(\lambda)$, $\mu(\lambda)$, and $\epsilon(\lambda)$: specifically, the urn-ball func-

[1] Even though the analysis carried out in the article does not directly apply to the case in which $K = \infty$, it is easy to generalize it and show that any equilibrium in which firms use infinitely many messages to advertise their job openings is the limit of a sequence of equilibria in which a finite number N of messages are used, $N = 1, 2, \ldots$. In this sense, the assumption of a finite (but arbitrarily large) message space is made without loss in generality.

tion $v \cdot [1 - \exp(-u/v)]$, the telephone line function $(\alpha u + v)^{-1} \cdot \alpha u$, $\alpha \in (0, 1)$, and the constant elasticity of substitution (CES) function $(\alpha u^{\sigma} + v^{\sigma})^{-1/\sigma} \cdot \alpha^{1/\sigma} uv$, $\alpha \in (0, 1)$ and $\sigma > 0$.[2]

When they meet, a worker and a firm enter a bargaining game to determine the terms of trade. In the first round, the worker advances a wage demand w_1 to the firm. If the firm accepts w_1, the negotiation comes to an end and production takes place. In this case, the worker consumes w_1 units of output and the firm consumes $y - w_1$ units. If the firm rejects w_1, the negotiation breaks down with probability $1 - \exp(-\beta\Delta)$. In this case, neither the worker nor the firm consumes anything. With probability $\exp(-\beta\Delta)$, the negotiation continues, with the firm offering the wage w_2 to the worker. If the worker accepts w_2, production takes place and the two parties consume, respectively, w_2 and $y - w_2$ units of the good. If the worker rejects w_2, the negotiation breaks down with probability $1 - \exp[-(1 - \beta)\Delta]$. With probability $\exp[-(1 - \beta)\Delta]$, the negotiation continues to the next round, where the worker gets to make a wage demand. The process continues without a deadline. I will consider the limit outcome of the bargaining game as $\Delta \to 0$. The reader may find it useful to interpret the parameter Δ as the interval of time between two consecutive wage demands of the worker. Following this interpretation, a fraction β of the interval is spent waiting for the firm to make a counteroffer. And a fraction $1 - \beta$ of the interval is spent waiting for the worker to make his second wage demand. Moreover, while they are waiting, there is an instantaneous breakdown rate equal to one.

B. *Equilibrium*

Let $p_k : [\underline{y}, \bar{y}] \to [0, 1]$ be the probability that a firm with productivity y posts the message s_k, where $\sum_{s_k \in \bar{S}} p_k(y) = 1$. Let S be the set of messages that are posted by a positive measure of firms, that is, $S = \{s_k \in \bar{S} : \int p_k(y) dF(y) > 0\}$. Let $g_k : [\underline{y}, \bar{y}] \to \mathbb{R}_+$ be the density of the worker's beliefs about the productivity of a firm that has advertised its vacancy with the message s_k, where $\int g_k(y) dy = 1$. Let $\pi_{k,w} : [\underline{y}, \bar{y}] \to \mathbb{R}_+$ be the

[2] The urn-ball matching function owes its name to the fact that $1 - \exp(-\lambda)$ is the probability that an urn contains at least one ball if each of $n\lambda$ balls is randomly placed in one of n urns, $n \to \infty$. The telephone line matching function owes its name to the fact that $(\alpha\lambda + 1)^{-1} \cdot \alpha\lambda$ is the probability that a firm contacts a worker by dialing a number randomly selected from a phone book that lists a measure $\alpha\lambda$ of workers and a measure 1 of firms. Notice that the telephone line function satisfies the logical constraint $m(u, v) \leq \min\{u, v\}$ if and only if $\alpha \in (0, 1)$. Finally, the CES matching function owes its name to the fact that it features the constant elasticity of substitution $(1 + \sigma)^{-1}$ between workers and vacancies. Notice that the CES function satisfies the logical constraint $m(u, v) \leq \min\{u, v\}$ if and only if the parameter σ is greater than zero and α is in the interval between zero and one.

worker's expected payoff at the beginning of the bargaining game, given that the firm has posted the message s_k and has a job with productivity y. Similarly, let $\pi_{k,f}: [\underline{y}, \bar{y}] \to \mathbb{R}_+$ be the firm's expected payoff at the beginning of the bargaining game, given that the job has been advertised with s_k and has productivity y.

Definition 1. An equilibrium is a tuple $\{p_k(y), g_k(y), \pi_{k,w}(y), \pi_{k,f}(y), \lambda_k\}_{k=1}^K$ with the following properties:

a. Profit maximization: For all $y \in [\underline{y}, \bar{y}]$, $\sum_{s_k \in S} p_k(y) = 1$, and whenever $p_k(y) > 0$,

$$\eta(\lambda_k)\pi_{k,f}(y) \geq \max_{s_j \in \bar{S}} \eta(\lambda_j)\pi_{j,f}(y).$$

b. Optimal application: For all s_k,

$$\frac{\eta(\lambda_k)}{\lambda_k}\int_{\underline{y}}^{\bar{y}} \pi_{k,w}(y)g_k(y)dy \leq U$$

and $\lambda_k \geq 0$, with complementary slackness, where

$$U = \max_{s_j \in S} \frac{\eta(\lambda_j)}{\lambda_j}\int_{\underline{y}}^{\bar{y}} \pi_{j,w}(y)g_j(y)dy.$$

c. Consistent beliefs: For all $s_k \in S$,

$$g_k(y) = \frac{p_k(y)f(y)}{\int_{\underline{y}}^{\bar{y}} p_k(x)dF(x)}.$$

d. Market clearing:

$$b = \sum_{s_k \in S}\left[\lambda_k \int_{\underline{y}}^{\bar{y}} p_k(y)dF(y)\right].$$

e. Perfection: For all $s_k \in S$, the payoffs $\pi_{k,w}(y)$ and $\pi_{k,f}(y)$ are derived from a sequential equilibrium of the bargaining game between a worker with beliefs $g_k(y)$ and a firm with productivity y. Moreover, the sequential equilibrium satisfies the monotonicity and stationarity conditions described in Section III.*A*.

f. Convexity of posting strategy: If $p_k(y_1)$ and $p_k(y_2)$ are strictly positive, then $p_k(y) > 0$ for all $y \in [y_1, y_2]$.

Condition *a* guarantees that given the queue lengths and bargaining outcomes associated with each message, a firm chooses its advertisement strategy to maximize profits.[3] Similarly, condition *b* guarantees that given

[3] Condition *a* also rules out equilibria in which some messages are posted by a set of firms with measure 0. From the analysis of the necessary and sufficient condition for the

queue lengths and bargaining outcomes, a worker chooses where to apply for a job in order to maximize its expected utility. Moreover, this condition implies that the queue of applicants λ_j that a firm expects to attract by posting an off-equilibrium message s_j makes a worker indifferent between searching that firm and searching elsewhere. Condition *c* guarantees that, whenever possible, the worker's posterior beliefs about the firm's productivity are derived from Bayes' law. Conditions *d* and *e* are self-explanatory. Finally, condition *f* is a technical restriction on the equilibrium strategy that guarantees that the worker's posterior beliefs are well behaved.

Three normalizations can be adopted to eliminate duplications from the equilibrium set. First, I can restrict attention to equilibria in which every message posted by a positive measure of firms is associated with different bargaining outcomes; that is, if s_i and s_j belong to S, then $(\pi_{i,w}, \pi_{i,f}) \neq (\pi_{j,w}, \pi_{j,f})$. In fact, if s_i and s_j were associated with the same outcome, there would be another equilibrium in which only s_i is posted. Second, I can restrict attention to equilibria in which the messages posted in equilibrium are ordered according to the worker's expected bargaining payoff; that is, if s_i and s_j belong to S and i is smaller than j, then $\int \pi_{i,w}(y) g_i(y) dy$ is smaller than or equal to $\int \pi_{j,w}(y) g_j(y) dy$. In fact, one can always take an equilibrium and construct another one in which s_i is relabeled s_j and vice versa. For this same reason, I can also restrict attention to equilibria in which only the first N messages are used, where $N = 1, 2, \ldots, K$.

III. Conditions for an Equilibrium

A. *Bargaining Outcome*

Consider a firm and a worker that have just entered the bargaining stage of the game. The firm has a job with productivity y and has advertised it with the message $s_k \in S$. The worker's beliefs about the productivity of the job are given by the density function $g_k(y)$. Condition *f* in the definition of an equilibrium guarantees that $g_k(y)$ is strictly positive over some connected interval $[\underline{y}_k, \bar{y}_k] \subset [\underline{y}, \bar{y}]$.

As shown by Grossman and Perry (1986), any sequential equilibrium of this bargaining game has the following recursive structure. The worker advances a wage demand w_t. If the job's productivity is sufficiently high, the demand is met and the game ends. If y is sufficiently low, the firm rejects w_t and makes an unacceptable counteroffer (say $w_{t+1} = 0$). If y takes on an intermediate value, the firm rejects w_t and counters with an offer that the worker is willing to accept. Together with a monotonic-

existence of an equilibrium carried out in Sec. III, it follows that this restriction is made without loss in generality.

ity condition on out-of-equilibrium conjectures, this structure implies that a firm can always signal that the job has relatively low productivity by delaying the agreement. In particular, by refusing to trade at the equilibrium wage $w(y)$, a firm $y \in [\underline{y}_k, \bar{y}_k]$ can convince the worker that the productivity of its job lies somewhere between $\underline{y}_k$ and y. Using this observation, one can prove that $w(y)$ cannot be greater than the wage outcome of the perfect information bargaining game between the worker and a firm with productivity y. Also, one can prove that $w(y)$ cannot be smaller than the outcome of the perfect information game between the worker and a firm with productivity $\underline{y}_k$.

A more precise characterization of the bargaining outcomes can be obtained by restricting attention to sequential equilibria in which the firm's strategy is stationary—in the sense that history matters only through its effect on worker's beliefs—and monotonic—in the sense that the possibility of additional high-productivity firms does not lead a low-productivity firm to lower the acceptance wage. Given these conditions, Gul and Sonnenschein (1988) prove that, as the delay between offers converges to zero, all types in the interval $[\underline{y}_k, \bar{y}_k]$ trade instantaneously at the wage w_k. Given the bounds on $w(y)$, it follows that w_k is equal to the wage outcome of the perfect information game between the worker and a firm with the lowest productivity on the support of $g_k(y)$, that is, $w_k = \beta\underline{y}_k$. Also, one can prove that, when a firm with productivity $y \notin [\underline{y}_k, \bar{y}_k]$ enters the negotiation, the outcome of the bargaining game is immediate agreement at the wage w_k if $y > \beta\underline{y}_k$ and no trade if $y < \beta\underline{y}_k$.

From this characterization of the bargaining outcome, I derive the expected bargaining payoffs.

Proposition 1 (Bargaining payoffs). As $\Delta \to 0$, for any sequence of equilibria satisfying the monotonicity and stationary conditions, the limits of the expected payoffs are

$$\int_{\underline{y}}^{\bar{y}} \pi_{k,w}(y) g_k(y) dy = \beta\underline{y}_k \equiv w_k,$$

$$\pi_{k,f}(y) = \max\{y - \beta\underline{y}_k, 0\}.$$

Proof. See Menzio (2005).

B. *Conditions for the Existence of an* N*-Message Equilibrium*

Let $\{p_k, g_k, \pi_{k,w}, \pi_{k,f}, \lambda_k\}_{k=1}^{K}$ be an equilibrium in which the set S has cardinality N. In this equilibrium, the messages posted by a positive measure of firms are $\{s_1, s_2, \ldots, s_N\}$. Each one of these messages is associated with a different bargaining outcome. And the higher the message, the higher the worker's expected payoff at the bargaining stage. Given the characterization result in proposition 1, these conditions imply that $w_1 < w_2 < \cdots < w_N$.

Conjecture that the equilibrium is such that every message $s_k \in S$ attracts a strictly positive queue of applicants. When this is the case, the optimal application condition implies that a worker expects the same utility U from seeking any two jobs. When the worker seeks a job advertised with a low message, he expects to be hired with high probability (because the queue of applicants is short) and to receive a low wage. When the worker seeks a job advertised with a high message, he expects to be hired with low probability (because the queue of applicants is long) and to receive a high wage. Formally, the equilibrium queue lengths $\{\lambda_k\}_{k=1}^{N}$ are such that

$$w_k = w(\lambda_k;\ U) \quad \text{for all } s_k \in S,$$

$$w(\lambda;\ U) = U\frac{\lambda}{\eta(\lambda)}. \tag{1}$$

The function $w(\lambda;\ U)$ returns the wage that a worker must expect to receive in order to be willing to apply for a job whose queue length is λ. I am going to refer to $w(\lambda;\ U)$ as the firm's labor supply curve.[4]

In equilibrium, the productivity distribution of jobs advertised with the message s_k is related to the worker's expectations about the outcome of the bargaining game. In particular, the expected wage w_k is equal to a fraction β of the productivity $\underline{y}_k$ of the lowest type of job that is advertised as s_k, that is,

$$\underline{y}_k = y(\lambda_k;\ U) \quad \text{for all } s_k \in S,$$

$$y(\lambda;\ U) = \beta^{-1} w(\lambda;\ U). \tag{2}$$

The function $y(\lambda;\ U)$ returns the productivity of the lowest type of job advertised with a message that attracts a queue of applicants with length λ.

In equilibrium, a firm with productivity $\underline{y}_k$ advertises its vacancy with the message s_k. From the profit maximization condition, it follows that the benefit from posting s_k rather than s_{k-1}—namely, $[\eta(\lambda_k) - \eta(\lambda_{k-1})] \cdot \underline{y}_k$—is greater than or equal to the cost of doing so—namely, $\eta(\lambda_k) \cdot w_k - \eta(\lambda_{k-1}) \cdot w_{k-1}$. Since the benefit of posting the higher message is increasing in the firm's type and the cost is independent, all firms with productivity above $\underline{y}_k$ strictly prefer s_k to s_{k-1}. However, since $\underline{y}_k < \underline{y}_{k+1} < \cdots < \underline{y}_N$, none of the firms with productivity below $\underline{y}_k$ posts s_k or any higher message. By combining these two observations for $k =$

[4] From the properties of the job-finding probability $\lambda^{-1}\eta(\lambda)$, it follows immediately that the labor supply curve is strictly increasing, $w(0;\ U) = \bar{\mu}^{-1}U$, and $w(\infty;\ U) = \infty$.

2, one can conclude that a firm advertises its vacancy with the message s_1 if and only if the productivity falls in the interval $(\underline{y}_1, \underline{y}_2)$. Then, by iterating the argument, one can prove that a firm posts s_k if and only if its productivity falls in the interval $(\underline{y}_k, \underline{y}_{k+1})$. Obviously, the type $\underline{y}_k$ is indifferent between advertising with the message s_k and s_{k-1}. From equations (1) and (2), this indifference condition can be expressed as

$$\Psi(\lambda_{k+1}, \lambda_k;\ U) = 0 \quad \text{for } k = 1, 2, \ldots, N-1,$$

$$\Psi(\lambda_{k+1}, \lambda_k;\ U) = \eta(\lambda_{k+1})(1-\beta)y(\lambda_{k+1};\ U) - \eta(\lambda_k)[y(\lambda_{k+1};\ U) - w(\lambda_k;\ U)]. \tag{3}$$

In equilibrium, the market for applicants must clear. For $k = 1, 2, \ldots, N-1$, the total number of applications received by the firms posting the message s_k is equal to $\lambda_k \cdot [F(\underline{y}_{k+1}) - F(\underline{y}_k)]$. And the total number of applications received by the firms posting the most optimistic message s_N is equal to $\lambda_N \cdot [1 - F(\underline{y}_N)]$. However, the total number of applicants in the economy is equal to b. Therefore, the market-clearing condition can be written as

$$b = \sum_{k=1}^{N-1} \lambda_k [F(y(\lambda_{k+1};\ U)) - F(y(\lambda_k;\ U))] + \lambda_N [1 - F(y(\lambda_N;\ U))]. \tag{4}$$

Conditions (3) and (4) are necessary for the existence of an N-message equilibrium. It turns out that they are also sufficient. To see why, suppose that $\{\lambda_k\}_{k=1}^N$ is a strictly increasing sequence of queue lengths and U is a value of search such that (3) and (4) are satisfied and $y(\lambda_1;\ U) = \underline{y}$. Then, consider a putative equilibrium in which all firms with productivity in the interval between $y(\lambda_k;\ U)$ and $y(\lambda_{k+1};\ U)$ post the message s_k, attract λ_k applicants, and expect the bargaining payoff $\max\{y - w(\lambda_k;\ U), 0\}$. Workers believe that the message $s_k \in S$ is sent by the firms with productivity in the interval $(y(\lambda_k;\ U), y(\lambda_{k+1};\ U))$ and expect the bargaining payoff $w(\lambda_k;\ U)$. In addition, off-equilibrium messages are believed to originate from the same set of firms that post the message s_1, and they are expected to attract a queue of length λ_1 and to lead to the wage $w(\lambda_1;\ U)$.

Condition (3) implies that a firm with productivity $y(\lambda_{k+1};\ U)$ is indifferent between posting the message s_k and s_{k+1}. Since $\lambda_{k+1} > \lambda_k$, condition (3) also implies that all firms with productivity higher than $y(\lambda_{k+1};\ U)$ prefer s_{k+1} to s_k and all firms with productivity lower than $y(\lambda_{k+1};\ U)$ prefer s_k to s_{k+1}. Combining these observations for $k = 1, 2, \ldots, N-1$, one can conclude that all firms in the interval $(y(\lambda_k;\ U), y(\lambda_{k+1};\ U))$ prefer s_k to any other message. Therefore, the putative equilibrium satisfies the first condition in definition 1. Similarly, one can verify that the putative equilibrium satisfies all the remaining conditions listed in definition 1.[5]

In the following proposition, I vindicate the conjecture that all messages $s_k \in S$ attract a strictly positive queue of applicants. Then, I fill in the details to prove that conditions (3) and (4) are necessary and sufficient.

PROPOSITION 2 (Necessary and sufficient conditions for an equilibrium). (i) In any N-message equilibrium, all messages attract some applicants and all firms make strictly positive profits. (ii) An N-message equilibrium exists if and only if there exists a strictly increasing sequence of queue lengths $\{\lambda_k\}_{k=1}^{N}$ and a value of U such that conditions (3) and (4) are satisfied and $y(\lambda_1;\ U) = \underline{y}$.

Proof. In the Appendix.

IV. Cheap Talk in Partial Equilibrium

In order to characterize the equilibria of the model economy, I proceed in two steps. First, I construct the equilibrium set of the cheap-talk game between firms and workers for an exogenously given value of search U. Mathematically, this means finding the queue lengths $\{\lambda_k\}_{k=1}^{N}$ that satisfy the indifference conditions (3) and the boundary condition $y(\lambda_1;\ U) = \underline{y}$ for a given U, $U \in (0,\ \bar{\mu}^{-1}\beta\underline{y})$.[6] In the second step, I recognize that the amount of information transmitted through the cheap talk affects the value of search, and I endogenize U. Mathematically, this means finding the equilibria $(\{\lambda_k\}_{k=1}^{N},\ U)$ of the cheap-talk game that satisfy the market-clearing condition (4). In this section, I carry out the first part of this procedure.

A. *How to Construct the Equilibrium Set*

For a given value of search U, $U \in (0,\ \bar{\mu}^{-1}\beta\underline{y})$, let $\{\lambda_k\}_{k=1}^{N}$ be an N-message equilibrium of the cheap-talk game between firms and workers; that is, $\{\lambda_k\}_{k=1}^{N}$ is a strictly increasing sequence that satisfies the indifference condition (3) and the boundary condition $y(\lambda_1;\ U) = \underline{y}$. In this equi-

[5] At this point, the reader may have realized that if a tuple $\{p_k,\ g_k,\ \pi_{k,w},\ \pi_{k,f},\ \lambda_k\}_{k=1}^{K}$ satisfies conditions a–d in the definition of an equilibrium and is such that $E[\pi_{w,k}(y)] = w_k$ and $\pi_{k,f}(y) = \max\{y - w_k,\ 0\}$, then this tuple also satisfies condition f. This property does not imply that condition f can be removed from definition 1. In fact, without condition f, the density of the worker's posterior beliefs $g_k(y)$ may not satisfy the regularity conditions required to directly apply proposition 1 and to establish that the outcome of any sequential equilibrium of the bargaining game is such that $E[\pi_{w,k}(y)] = w_k$ and $\pi_{k,f}(y) = \max\{y - w_k,\ 0\}$.

[6] In any equilibrium, a job advertised with the message s_1 attracts a strictly positive queue of applicants λ_1 and pays the wage $\beta\underline{y}$. In any equilibrium, a worker expects utility U from seeking such a job. Combining these two observations, one can conclude that the value of search U is greater than zero and smaller than $\bar{\mu}^{-1}\beta\underline{y}$.

librium, all firms with productivity between $y(\lambda_k;\ U)$ and $y(\lambda_{k+1};\ U)$ post the message s_k, attract a queue of applicants of length λ_k, and pay the wage $w(\lambda_k;\ U)$. Firms with productivity higher than $y(\lambda_N;\ U)$ post the most optimistic message s_N, attract λ_N applicants, and pay the wage $w(\lambda_N;\ U)$.

For this equilibrium, consider the following thought experiment. Of all the firms that advertise with the message s_N, which ones would be willing to publicly disclose the productivity of their vacancies if they were given the opportunity of doing so? If a firm has a vacancy with productivity $y(\lambda;\ U)$ and discloses this information to the public, it attracts a queue of applicants of length λ, it fills the vacancy with probability $\eta(\lambda)$, and it pays the wage $\beta y(\lambda;\ U)$. The firm's expected profits are $\eta(\lambda)\cdot(1-\beta)\cdot y(\lambda;\ U)$. However, if the firm posts the message s_N, its expected profits are $\eta(\lambda_N)\cdot[y(\lambda;\ U)-w(\lambda_N;\ U)]$. The payoff differential between these two alternatives is $\Psi(\lambda;\ \lambda_N,\ U)$, which, with equations (1) and (2), can be written as

$$\Psi(\lambda;\ \lambda_N,\ U) = U\left[\frac{1-\beta}{\beta}\lambda - \frac{\eta(\lambda_N)}{\beta}\frac{\lambda}{\eta(\lambda)} + \lambda_N\right]. \tag{5}$$

Suppose that there is a firm that owns a vacancy with productivity $y(\lambda_{N+1};\ U)$ greater than $y(\lambda_N;\ U)$ and is indifferent between disclosing its private information and posting the messages s_N. Then $\{\lambda_k\}_{k=1}^N \cup \lambda_{N+1}$ is an $(N+1)$–message equilibrium of the cheap talk between firms and workers. In this equilibrium, all firms with productivity lower than $y(\lambda_{N+1};\ U)$ post the same message, attract the same queue, and pay the same wage as in the original N-message equilibrium. Firms with productivity higher than $y(\lambda_{N+1};\ U)$ post the message s_{N+1}, attract λ_{N+1} applicants, and pay the wage $w(\lambda_{N+1};\ U)$.

Next, suppose that all the firms that own a vacancy with productivity $y(\lambda_{N+1};\ U)$ strictly greater than $y(\lambda_N;\ U)$ prefer posting the message s_N rather than disclosing their private information. Then there is no $\lambda_{N+1} > \lambda_N$ such that $\{\lambda_k\}_{k=1}^N \cup \lambda_{N+1}$ is an $(N+1)$–message equilibrium of the cheap talk. Indeed, if workers were to believe that the message s_{N+1} comes from the set of types $(y(\lambda_{N+1};\ U),\ \bar{y})$, the expected wage would be $\beta y(\lambda_{N+1};\ U)$, and there would be some firms with productivity higher than $y(\lambda_{N+1};\ U)$ that strictly prefer s_N to s_{N+1}. Now, suppose that all the firms that own a vacancy with productivity $y(\lambda_{N+1};\ U)$ strictly greater than $y(\lambda_N;\ U)$ prefer disclosing their private information to the public. Also in this case, there is no $\lambda_{N+1} > \lambda_N$ such that $\{\lambda_k\}_{k=1}^N \cup \lambda_{N+1}$ is an $(N+1)$–message equilibrium of the cheap talk. Indeed, if workers were to believe that the message s_{N+1} comes from the set $(y(\lambda_{N+1};\ U),\ \bar{y})$, the expected wage would be $\beta y(\lambda_{N+1};\ U)$, and there would be some firms with productivity lower than $y(\lambda_{N+1};\ U)$ that strictly prefer s_{N+1} to

s_N. Finally, if there are no N-message equilibria for which one can find a type that is indifferent between the two alternatives, then there are no equilibria in which $N + 1$ or more messages are used.

The previous discussion suggests a recursive approach to the characterization of the equilibrium set. Denote with $Z(\lambda)$ the set of zeroes of the function $\Psi(\lambda'; \lambda, U)$ with the property that $\lambda' > \lambda$. The unique one-message equilibrium of the cheap talk is the queue length λ_1 such that $y(\lambda_1; U) = \underline{y}$. The set of $(N + 1)$–message equilibria of the cheap talk is given by all the sequences of queue lengths $\{\lambda_k\}_{k=1}^{N+1}$ such that (i) $\{\lambda_k\}_{k=1}^{N}$ is an N-message equilibrium and (ii) λ_{N+1} belongs to $Z(\lambda_N)$.

B. *Firm's Preference Ordering*

In this subsection, I characterize the firm's preference ordering over revealing its productivity y to the public and advertising its vacancy with the message s_N for all $y \in (y(\lambda_N; U), \bar{y})$. Then I derive the properties of the set $Z(\lambda_N)$.

When the productivity y is close to the boundaries of the interval $(y(\lambda_N; U), \bar{y})$, the firm's preference ordering is determined by the first derivative of the profit differential $\Psi(\lambda; \lambda_N, U)$ with respect to λ, that is,

$$\Psi'(\lambda; \lambda_N, U) = \frac{U}{\beta}\left\{(1-\beta) - \frac{\eta(\lambda_N)}{\eta(\lambda)}[1-\epsilon(\lambda)]\right\}. \tag{6}$$

Since $\Psi(\lambda_N; \lambda_N, U) = 0$ and $\Psi'(\lambda_N; \lambda_N, U) = \epsilon(\lambda_N) - \beta$, the least productive firms in the interval $(y(\lambda_N, U), \bar{y})$ strictly prefer disclosing their private information when $\epsilon(\lambda_N)$ is greater than β. Conversely, when $\epsilon(\lambda_N)$ is smaller than β, the least productive firms strictly prefer advertising their vacancies with s_N. These findings have a clear economic intuition. For a firm with productivity slightly higher than $y(\lambda_N, U)$, the profit-maximizing wage is greater than the wage associated with the message s_N if and only if the elasticity $\epsilon(\lambda_N)$ of the job-filling probability is greater than the worker's bargaining power β. Hence, it is natural that such a firm prefers to reveal its productivity—and, consequently, pay a higher wage—if and only if $\epsilon(\lambda_N)$ is greater than β. It is useful to denote with λ_L the solution to $\epsilon(\lambda_L) = \beta$.

Since $\lim_{\lambda\to\infty} \Psi'(\lambda; \lambda_N, U) = 1 - \beta - \eta(\lambda_N)$, the most productive firms in the interval $(y(\lambda_N; U), \bar{y})$ strictly prefer disclosing their private information to the public when β is smaller than $1 - \eta(\lambda_N)$. Conversely, when β is greater than $1 - \eta(\lambda_N)$, the most productive firms strictly prefer advertising their vacancies with s_N. These findings are easy to understand because for a very productive firm, the increase in revenues obtained from revealing its type is approximately equal to $[1 - \eta(\lambda_N)] \cdot y$, whereas the increase in the wage bill is approximately equal to βy. It is useful to denote with λ_R the solution to the equation $1 - \eta(\lambda_R) = \beta$.

The direction in which the firms' preference ordering varies as the productivity y is raised from $y(\lambda_N;\ U)$ to $\bar{y}$ is determined by the second derivative of the profit differential $\Psi(\lambda;\ \lambda_N,\ U)$ with respect to λ, that is,

$$\Psi''(\lambda;\ \lambda_N,\ U) = -\frac{U}{\beta}\left[\eta(\lambda_N)\frac{d^2(\eta(\lambda)^{-1}\lambda)}{d\lambda^2}\right] = -\frac{\eta(\lambda_N)}{\beta}w''(\lambda;\ U). \quad (7)$$

When the profit differential is concave, more productive firms have a stronger preference for advertising their vacancy with the message s_N. That is, if $\Psi(\lambda;\ \lambda_N,\ U)$ is negative for some $\lambda > \lambda_N$, all firms with productivity higher than $y(\lambda;\ U)$ strictly prefer posting s_N. And if $\Psi(\lambda;\ \lambda_N,\ U)$ is positive for some $\lambda > \lambda_N$, all firms with productivity lower than $y(\lambda;\ U)$ strictly prefer revealing their type.[7] Conversely, when the profit differential is convex, more productive firms have a stronger preference for publishing their private information. In line with basic economic intuition, equation (7) tells us that higher types have a stronger preference for s_N when the labor supply curve is convex, that is, when it takes a higher and higher wage to attract one more applicant to the firm. In addition, equation (7) reminds us that the convexity of the labor supply curve is equivalent to the convexity of the inverse of the job-finding probability.

In light of these findings, I can characterize the set $Z(\lambda_N)$. First, consider the case of a matching function with the property that the inverse of the job-finding probability is convex. If the queue of applicants attracted by the message s_N is greater than λ_L, the set $Z(\lambda_N)$ is empty. In fact, the least productive firms in the interval $(y(\lambda_N;\ U), \bar{y})$ strictly prefer posting s_N than disclosing their private information because $\epsilon(\lambda_N)$ is smaller than β. And all the other firms in the interval $(y(\lambda_N;\ U), \bar{y})$ rank the two alternatives in the same way because, when $\eta(\lambda)^{-1}\lambda$ is convex, more productive firms have a stronger preference for s_N. The set $Z(\lambda_N)$ is empty also when λ_N is smaller than λ_R. In fact, the most productive firms strictly prefer disclosing their private information rather than posting s_N because $1 - \eta(\lambda_N)$ is greater than β. And all the other firms in the interval $(y(\lambda_N;\ U), \bar{y})$ rank the two alternatives in the same way because of the convexity of $\eta(\lambda)^{-1}\lambda$. Finally, if λ_N lies between λ_R and λ_L, the set $Z(\lambda_N)$ contains the point $z(\lambda_N)$. All the firms with productivity higher than $y(z(\lambda_N);\ U)$ strictly prefer s_N. All the firms with

[7] These properties are easily derived. Since the function $\Psi(\lambda;\ \lambda_N,\ U)$ is concave, the set of $\hat{\lambda}$'s is such that $\lambda \geq \lambda_N$ and $\Psi(\lambda;\ \lambda_N,\ U) \geq 0$ is convex. Since $\Psi(\lambda_N;\ \lambda_N,\ U)$ is equal to zero, the set includes the point λ_N. Therefore, the function $\Psi(\lambda;\ \lambda_N,\ U)$ is positive on a connected interval $[\lambda_N, \hat{\lambda}]$.

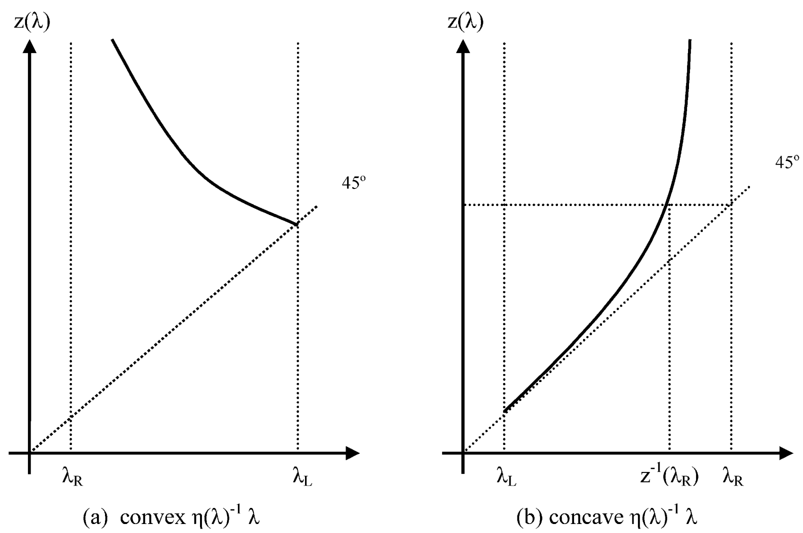

FIG. 1.—Firm's preference ordering

productivity between $y(\lambda_N;\ U)$ and $y(z(\lambda_N);\ U)$ strictly prefer disclosing their type. And the firms that own a vacancy with productivity $y(z(\lambda_N);\ U)$ are indifferent between the two alternatives.

Next, consider the case of a matching function with the property that the inverse of the job-finding probability is concave. If λ_N is smaller than λ_L, the set $Z(\lambda_N)$ is empty because all the firms in the interval $(y(\lambda_N;\ U), \bar{y})$ strictly prefer revealing the productivity of their vacancy to the public. If λ_N is greater than λ_R, $Z(\lambda_N)$ is empty because all these firms strictly prefer to advertise their vacancies with s_N. Finally, if λ_N takes on intermediate values, the firms with productivity $y(z(\lambda_N);\ U)$ are indifferent between the two alternatives. A complete characterization of the zeroes of the profit differential function is contained in the following lemma and illustrated in figure 1.

LEMMA 1 (Firm's preference ordering). Denote with λ_L the solution to $\epsilon(\lambda_L) = \beta$. Denote with λ_R the solution to $1 - \eta(\lambda_R) = \beta$. (i) If $\eta(\lambda)^{-1}\lambda$ is convex, λ_R is smaller than λ_L. For all $\lambda_N \notin (\lambda_R, \lambda_L)$, the correspondence $Z(\lambda_N)$ is empty. For all $\lambda_N \in (\lambda_R, \lambda_L)$, $Z(\lambda_N)$ is a decreasing function $z(\lambda_N)$ such that $\lim_{\lambda_N \to \lambda_R} z(\lambda_N) = \infty$ and $\lim_{\lambda_N \to \lambda_L} z(\lambda_N) = \lambda_N$. (ii) If $\eta(\lambda)^{-1}\lambda$ is concave, λ_L is smaller than λ_R. For all $\lambda_N \notin (\lambda_L, \lambda_R)$, $Z(\lambda_N)$ is empty. For all $\lambda_N \in (\lambda_L, \lambda_R)$, $Z(\lambda_N)$ is a function $z(\lambda_N)$ such that $\lim_{\lambda_N \to \lambda_L} z(\lambda_N) = \lambda_L$ and $\lim_{\lambda_N \to \lambda_R} z(\lambda_N) = \infty$. Moreover, $z(\lambda_N) - \lambda_N$ is strictly increasing.

Proof. In the Appendix.

C. *Equilibrium Set*

Now I am in the position to characterize the equilibrium set of the cheap-talk game between workers and firms for a given value of search U. First, suppose that the matching function is such that the inverse of the job-finding probability is convex. For all $U \in (0, \bar{\mu}^{-1}\beta\underline{y})$, there exists a unique one-message equilibrium. In this equilibrium, every firm's type posts the message s_1 and attracts a queue of applicants with length λ_1, where λ_1 satisfies the boundary condition $y(\lambda_1;\ U) = \underline{y}$. If λ_1 is greater than λ_R and smaller than λ_L, there exists a unique two-message equilibrium as well. In this equilibrium, a firm posts the message s_1 if its productivity is lower than $y(z(\lambda_1);\ U)$ and posts the message s_2 otherwise. The more optimistic announcement attracts a queue of applicants with length $z(\lambda_1)$. Since $z(\lambda_1)$ is greater than λ_L, there are no equilibria of the cheap-talk game involving three or more messages. This impossibility result has a clear economic intuition. When $\eta(\lambda)^{-1}\lambda$ is convex, a three-message equilibrium can exist only if—for a firm with productivity slightly above $y(\lambda_2;\ U)$—the profit-maximizing wage is higher than $\beta y(\lambda_2;\ U)$. However, a firm with productivity slightly below $y(\lambda_2;\ U)$ must prefer to advertise its vacancy with s_1 rather than s_2. For this to be true, the firm's profit-maximizing wage must be lower than $\beta y(\lambda_2;\ U)$. The two conditions are not compatible.

Next, suppose that the matching function is such that the inverse of the job-finding probability is concave. For all $U \in (0, \bar{\mu}^{-1}\beta\underline{y})$, there exists a unique one-message equilibrium of the cheap-talk game. In this equilibrium, all firms post s_1 and attract λ_1 applicants, where $y(\lambda_1;\ U) = \underline{y}$. If λ_1 is greater than λ_L and smaller than λ_R, there exists a unique two-message equilibrium as well. In this equilibrium, firms with productivity lower than $y(z(\lambda_1);\ U)$ post the message s_1 and attract λ_1 applicants to their vacancies. More productive firms post s_2 and attract $z(\lambda_1)$ applicants. This exhausts the equilibrium set if $z(\lambda_1)$ is greater than λ_R or, equivalently, if λ_1 is greater than $z^{-1}(\lambda_R)$. Otherwise, there exists a unique three-message equilibrium as well. More generally, if λ_1 is greater than λ_L and smaller than $(z^{-1})^T(\lambda_R)$, there exists a unique $(T+2)$–message equilibrium. Notice that, as $T \rightarrow \infty$, $(z^{-1})^T(\lambda_R)$ converges monotonically to λ_L.

Proposition 3 (Cheap talk in partial equilibrium). For a given $U \in (0, \bar{\mu}^{-1}\beta\underline{y})$, denote with λ_1 the solution to the boundary condition $y(\lambda_1;\ U) = \underline{y}$. (i) Suppose that $\eta(\lambda)^{-1}\lambda$ is convex. Then there exists a unique one-message equilibrium. In addition, if and only if $\lambda_1 \in (\lambda_R, \lambda_L)$, there exists a unique two-message equilibrium. There are no equilibria with three or more messages. (ii) Suppose that $\eta(\lambda)^{-1}\lambda$ is concave. Then there exists a unique one-message equilibrium. In addition, if and only if $\lambda_1 \in (\lambda_L, (z^{-1})^T(\lambda_R))$, there exists a unique N-message equilibrium for $N = 2, 3, \ldots, T+2$.

V. Cheap Talk in General Equilibrium

In the previous section, I have constructed the set of equilibria of the cheap-talk game between firms and workers for an exogenously given U, $U \in (0, \bar{\mu}^{-1}\beta\underline{y})$. In this section, I use the market-clearing condition (4) to endogenize the value of search.

First, I derive a general expression for the aggregate demand of applicants. Suppose that for a certain value of U in the interval $(0, \bar{\mu}^{-1}\beta\underline{y})$ there exists an N-message equilibrium of the cheap talk. In such an equilibrium, a continuum of firms with measure $F(y(z(\lambda_1);\ U))$ advertise their vacancies with the message s_1 and "demand" a queue of applicants with length λ_1, where λ_1 is such that $y(\lambda_1;\ U) = \underline{y}$. In such an equilibrium, a continuum of firms with measure $F(y(z^{k+1}(\lambda_1);\ U)) - F(y(z^k(\lambda_1);\ U))$ post the message s_{k+1} and demand a queue of applicants with length $z^k(\lambda_1)$, $k = 1, 2, \ldots, N-2$. In addition, a continuum of firms with measure $1 - F(y(z^{N-1}(\lambda_1);\ U))$ post the most optimistic message s_N and demand $z^{N-1}(\lambda_1)$ applicants. Overall, in such an equilibrium, the aggregate demand of applicants is given by

$$b^d(\lambda_1;\ N) = \sum_{k=0}^{N-2} z^k(\lambda_1)[F(y(z^{k+1}(\lambda_1);\ U)) - F(y(z^k(\lambda_1);\ U))] + z^{N-1}(\lambda_1)[1 - F(y(z^{N-1}(\lambda_1);\ U))], \tag{8}$$

where the boundary condition $y(\lambda_1;\ U) = \underline{y}$ implicitly defines U as a function of λ_1.

Now, suppose that the matching function is such that the inverse of the job-finding probability is convex. For all positive values of λ_1, there is a unique one-message equilibrium of the cheap talk between firms and workers. In this equilibrium, the aggregate demand of applicants $b^d(\lambda_1;\ 1)$ is equal to λ_1. For all values of λ_1 in the interval between λ_R and λ_L, there is a unique two-message equilibrium as well. In this equilibrium, every firm with productivity lower than $y(z(\lambda_1);\ U)$ demands the same number of applicants as in the one-message equilibrium. And every firm with productivity higher than $y(z(\lambda_1);\ U)$ demands strictly more applicants. Therefore, the aggregate demand $b^d(\lambda_1;\ 2)$ is strictly greater than $b^d(\lambda_1;\ 1)$. Over the interval (λ_R, λ_L), $b^d(\lambda_1;\ 2)$ is a continuous function that, locally, may be increasing or decreasing depending on the shape of the distribution function $F(y)$. For $\lambda_1 \to \lambda_R$, the fraction of firms posting the message s_2 becomes smaller and smaller and the queue of applicants demanded by each of them grows larger and larger.

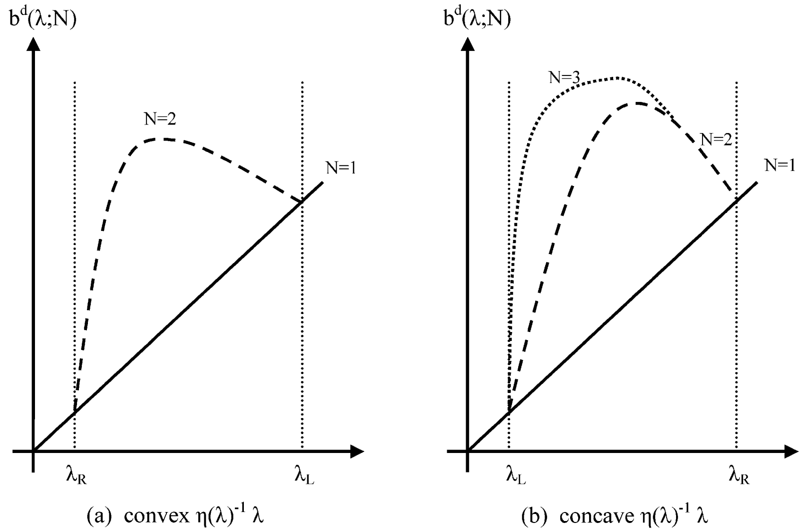

FIG. 2.—Aggregate demand correspondence

As long as the tail of the distribution $F(y)$ is not too thick,[8] the aggregate demand for applicants converges to $b^d(\lambda_1;\ 1)$. For $\lambda_1 \to \lambda_L$, the informative content of the messages s_1 and s_2 becomes more and more similar, and $b^d(\lambda_1;\ 2)$ converges to $b^d(\lambda_1;\ 1)$. Figure 2*a* illustrates the properties of the aggregate demand correspondence when $\eta(\lambda)^{-1}\lambda$ is convex.

Next, suppose that the matching function is such that the inverse of the job-finding probability is concave. For all positive values of λ_1, there is a unique one-message equilibrium of the cheap talk. In this equilibrium, the aggregate demand of applicants $b^d(\lambda_1;\ 1)$ is equal to λ_1. For all values of λ_1 in the interval between λ_L and $(z^{-1})^{N-2}(\lambda_R)$, there is a unique N-message equilibrium as well. In this equilibrium, every firm with productivity lower than $y(z^{N-1}(\lambda_1);\ U)$ demands the same queue of applicants as in the $(N-1)$–message equilibrium associated with λ_1. Every firm with productivity higher than $y(z^{N-1}(\lambda_1);\ U)$ demands strictly more applicants. Therefore, the aggregate demand for applicants $b^d(\lambda_1;\ N)$ is strictly greater than $b^d(\lambda_1;\ N-1)$. For $\lambda_1 \to \lambda_L$, $b^d(\lambda_1;\ N)$ converges to $b^d(\lambda_1;\ 1)$ because the informative content of the messages $s_2,\ s_3,\ \ldots,\ s_N$ becomes more and more similar to s_1. For $\lambda_1 \to (z^{-1})^{N-2}(\lambda_R)$, $b^d(\lambda_1;\ N)$ converges to $b^d(\lambda_1;\ N-1)$ because fewer and

[8] Formally, the condition is

$$\lim_{\lambda_1 \to \lambda_R} z(\lambda_1)\left[1 - F\left(\frac{\eta(\lambda_1)}{\lambda_1}\frac{z(\lambda_1)}{\eta(z(\lambda_1))}\underline{y}\right)\right] = 0.$$

fewer firms post the most optimistic message s_N. Figure 2*b* illustrates the properties of the aggregate demand correspondence when $\eta(\lambda)^{-1}\lambda$ is concave.

Since a general equilibrium in which firms advertise their vacancies with N informationally different messages is a point at which the aggregate supply curve b intersects the aggregate demand of applicants $b^d(\lambda_1;\ N)$, the next theorem follows immediately from the previous characterization of b^d.

Theorem 1 (Partially directed search). Let $\underline{b}$ denote the infimum of $b^d(\lambda_1;\ N)$ for all $\lambda > 0$ and $N \geq 2$. Let $\overline{b}$ denote the supremum of $b^d(\lambda_1;\ N)$ for all $\lambda > 0$ and $N \geq 2$. (i) If $\eta(\lambda)^{-1}\lambda$ is convex, $\underline{b} = \lambda_R$ and $\overline{b} \geq \lambda_L$. For all $b \in (\underline{b}, \overline{b})$, there exists at least one two-message equilibrium. For all $b \notin [\underline{b}, \overline{b}]$, there exists only a one-message equilibrium. (ii) If $\eta(\lambda)^{-1}\lambda$ is concave, then $\underline{b} = \lambda_L$ and $\overline{b} \geq \lambda_R$. For all $b \in (\underline{b}, \overline{b})$, there exists at least one N-message equilibrium, where $N \geq 2$. For all $b \notin [\underline{b}, \overline{b}]$, there exists only a one-message equilibrium.

Theorem 1 is the article's main result. It states that as long as b lies in the interval $(\underline{b}, \overline{b})$, there exists an equilibrium in which the inherently uninformative messages used by firms to advertise their job openings partially direct the search strategy of workers. In such an equilibrium, firms use $N \geq 2$ different messages to advertise jobs with different productivity. Workers apply more frequently for jobs that are advertised with higher messages, that is, messages that are believed to originate from a more productive set of firms. And the outcome of the bargaining game between a firm and a worker is a wage that is higher the higher the message posted by the firm. If, instead, b is greater than $\overline{b}$ or smaller than $\underline{b}$, the unique equilibrium features random search. In this equilibrium, firms use informationally equivalent messages to advertise jobs with different productivity. Workers ignore these messages and apply with the same probability for all vacancies. And the outcome of the bargaining game between a worker and a firm is independent from the message that was posted by the firm. Finally, theorem 1 relates the cutoff $\underline{b}$ and $\overline{b}$ to the fundamentals of the economy. In particular, it states that the interval $(\underline{b}, \overline{b})$ contains the set of worker-to-firm ratios b such that $\lambda_L - b$ and $\lambda_R - b$ have opposite signs or, equivalently, such that $[\epsilon(b) - \beta] \cdot [1 - \eta(b) - \beta] < 0$.

From a qualitative point of view, the main implication of theorem 1 is that even though in reality most help wanted ads contain only vague and noncontractual announcements about compensation, one should not conclude that the search process is random. In fact, these announcements may be informative about the job's quality, they may affect the worker's and firm's bargaining strategies, and, in turn, they may be correlated with actual wages. From a quantitative point of view, the main implication of theorem 1 is suggesting a procedure to test whether noncontractual announcements can partially direct the worker's search strategy in a particular labor market. Specifically, with data on the duration of vacancies, the number of applications, the hiring wages, and labor productivity, it is possible to recover the deep parameters of the model and compute the cutoffs $\underline{b}$ and $\overline{b}$. Then, with data on the unemployment and the vacancy rate, it is possible to verify whether b belongs to the interval $(\underline{b}, \overline{b})$.

Perhaps the reader is concerned about the significance of these findings given that theorem 1 restricts attention to the family of matching functions with the property that the inverse of the job-finding probability is either globally convex or concave. Two remarks are in order. First, most standard matching functions belong to this family. For example, the urn-ball function is such that $\eta(\lambda)^{-1}\lambda$ is globally convex. And the CES function is such that $\eta(\lambda)^{-1}\lambda$ is globally concave when the parameter σ is smaller than one and convex when σ is greater than one. Second, even though this restriction affords a sharper characterization of the equilibrium set, it is not necessary for the existence of a partially directed search equilibrium. For example, the reader can easily verify that as long as $\epsilon(b) - \beta$ and $1 - \eta(b) - \beta$ have opposite signs, there exists a distribution function $F(y)$ that sustains an informative equilibrium of the cheap talk.[9]

VI. Conclusions

This article has shown that ostensibly uninformative statements about compensation—such as those one typically finds in help wanted ads—can be correlated with actual wages and partially direct the search strategy of workers. Intuitively, workers apply more frequently for those jobs that are advertised with more positive messages because they expect to meet more productive firms that, in turn, are more likely to concede to high wage demands. In turn, more positive messages are posted by more productive firms because those are the only ones that are willing to face tougher wage demands at the bargaining stage in order to fill their openings with higher probability.

Appendix

Proof of Proposition 2

Part i.—The optimal application condition implies that higher messages attract longer queues of applicants. The market-clearing condition implies that some

[9] In some cases, this sufficient condition holds even when the matching function does not satisfy the regularity conditions on $\eta(\lambda)$, $\mu(\lambda)$, and $\epsilon(\lambda)$ specified in Sec. II. For example, the condition holds when the matching function $m(u, v)$ is given by the truncated Cobb-Douglas min $\{Au^{\alpha}v^{1-\alpha},\ u,\ v\}$.

of the messages attract a strictly positive queue of applicants. Combining these two observations, we can conclude that in any equilibrium there is a message $s_l \in S$ such that $\lambda_k = 0$ if $k < l$ and $\lambda_k > 0$ if $k \geq l$. Also, the characterization of the bargaining game implies that the lowest type of firm posting the message $s_k \in S$ has productivity $\underline{y}_k = \beta^{-1} w_k$. Since $\{w_k\}_{k=1}^N$ is a strictly increasing sequence, so is $\{\underline{y}_k\}_{k=1}^N$. Hence, in equilibrium, no firm with productivity lower than $\underline{y}_k$ posts s_k or any higher message.

Suppose that l is strictly greater than one. All firms with productivity y in the interval between $\underline{y}_1$ and $\underline{y}_k$ advertise their vacancies with s_{l-1} or a lower message. In equilibrium, all these firms make zero profits. In particular, a firm with productivity $y \in (\beta \underline{y}_k, \underline{y}_k)$ makes zero profits. If, deviating from the equilibrium, this firm was to post the message s_k, its expected profits would be $\eta(\lambda_k) \cdot (y - \beta \underline{y}_k) > 0$, a contradiction. Therefore, it must be the case that l is equal to one, that is, $\lambda_1 > 0$. In turn, this implies that all firms make strictly positive profits in equilibrium.

Part ii.—The necessity of conditions (3) and (4) has been proved in the text. The sufficiency of conditions (3) and (4) is proved by direct verification of the equilibrium conditions *a–f*. QED

Proof of Lemma 1

Part i.—Suppose that $\eta(\lambda)^{-1}\lambda$ is convex. From (5), it follows that $\Psi(\lambda_N; \lambda_N, U)$ is equal to zero. From (7) it follows that $\Psi(\lambda; \lambda_N, U)$ is a strictly concave function of λ.

For all $\lambda_N \geq \lambda_L$, the elasticity of the matching function $\epsilon(\lambda_N)$ is greater than the worker's bargaining power β. This implies that the derivative of the profit differential is negative when evaluated at $\lambda = \lambda_N$, that is, $\Psi'(\lambda_N; \lambda_N, U) \leq 0$. Using the fact that $\Psi(\lambda_N; \lambda_N, U) = 0$ and $\Psi''(\lambda; \lambda_N, U) < 0$, one concludes that $\Psi(\lambda; \lambda_N, U)$ is strictly negative for all $\lambda > \lambda_N$. The correspondence $Z(\lambda_N)$ is empty.

For all $\lambda_N \leq \lambda_R$, the worker's bargaining power β is smaller than $1 - \eta(\lambda_N)$. This implies that the derivative of the profit differential is positive when evaluated at $\lambda = \infty$. Using the fact that $\Psi(\lambda_N; \lambda_N, U) = 0$ and $\Psi''(\lambda; \lambda_N, U) < 0$, one concludes that $\Psi(\lambda; \lambda_N, U)$ is strictly positive for all $\lambda > \lambda_N$. The correspondence $Z(\lambda_N)$ is empty.

Temporarily suppose that λ_L is smaller than λ_R. Then, for all $\lambda_N \in (\lambda_R, \lambda_L)$, the derivative of the profit differential is negative when evaluated at $\lambda = \lambda_N$ and positive when evaluated at $\lambda = \infty$. Because this contradicts the fact that $\Psi(\lambda; \lambda_N, U)$ is a concave function, one concludes that λ_R is smaller than λ_L.

For all $\lambda_N \in (\lambda_R, \lambda_L)$, the derivative of the profit differential Ψ with respect to λ is strictly positive when evaluated at λ_N and strictly negative when evaluated at ∞. Using the fact that $\Psi(\lambda_N; \lambda_N, U) = 0$ and $\Psi''(\lambda; \lambda_N, U) < 0$, one concludes that the profit differential is strictly positive for all $\lambda \in (\lambda_N, z(\lambda_N))$ and is strictly negative for all $\lambda > z(\lambda_N)$. The correspondence $Z(\lambda_N)$ contains $z(\lambda_N)$ only.

By construction, the expression $\Psi(z(\lambda_N); \lambda_N, U)$ is equal to zero for all $\lambda_N \in (\lambda_R, \lambda_L)$. Therefore, its derivative must be equal to zero for all $\lambda_N \in (\lambda_R, \lambda_L)$, that is,

$$\left\{\frac{1-\beta}{\beta} - \frac{\eta(\lambda_N)}{\beta\eta(z(\lambda_N))}[1 - \epsilon(z(\lambda_N))]\right\}\frac{dz(\lambda_N)}{d\lambda_N} + \left[1 - \frac{\eta(\lambda_N)}{\lambda_N}\frac{z(\lambda_N)}{\eta(z(\lambda_N))}\frac{\epsilon(\lambda_N)}{\beta}\right] = 0.$$

The first term in braces is the derivative of the profit differential with respect to λ evaluated at $\lambda = z(\lambda_N)$. Therefore, this term is strictly negative. The second term in braces is strictly negative because $\epsilon(\lambda_N)$ is strictly greater than β and the job-finding probability $\lambda^{-1}\eta(\lambda)$ is decreasing in λ. Therefore, the function $z(\lambda_N)$ is strictly decreasing over its domain (λ_R, λ_L). Moreover, it is easy to verify that $\lim_{\lambda_N \to \lambda_R} z(\lambda_N) = \infty$ and $\lim_{\lambda_N \to \lambda_L} z(\lambda_N) = \lambda_N$.

Part ii.—The analysis of the case in which the labor supply curve is convex is similar and is omitted for the sake of brevity. All details are available on request. QED

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