Research paper

The (Q,S,s) pricing rule: a quantitative analysis

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ABSTRACT

Are nominal prices sticky because menu costs prevent sellers from continuously adjusting their prices to keep up with inflation or because search frictions make sellers indifferent to any real price over some non-degenerate interval? The paper answers the question by developing and calibrating a model in which both search frictions and menu costs may generate price stickiness and sellers are subject to idiosyncratic shocks. The equilibrium of the calibrated model is such that sellers follow a (Q,S,s) pricing rule: each seller lets inflation erode the effective real value of the nominal prices until it reaches some point s and then pays the menu cost and sets a new nominal price with an effective real value drawn from a distribution with support $[S, Q]$, with $s < S < Q$. Idiosyncratic shocks short-circuit the repricing cycle and may lead to negative price changes. The calibrated model reproduces closely the properties of the empirical price and price-change distributions. The calibrated model implies that search frictions are the main source of nominal price stickiness.

1. Introduction

The standard explanation for nominal price stickiness is menu costs (see, e.g., Caplin and Leahy, 1997; Dotsey et al., 1999; Sheshinski and Weiss, 1977, etc...). That is, sellers do not constantly adjust their nominal prices to keep up with inflation because, in order to change their nominal prices, they need to pay some fixed adjustment cost (a menu cost). An alternative explanation for nominal price stickiness is search frictions (see, e.g., Head et al., 2012; Wang et al., 2017). That is, when the outcome of the process by which buyers search for sellers is such that some buyers only come into contact with one seller and others come into contact with multiple sellers, the equilibrium has the property that the profits of a seller are maximized at any price over some non-degenerate interval. Thus, an individual seller is perfectly happy to let inflation erode the real value of its nominal price, as long as this real value remains on the profit-maximizing interval. Whether nominal price stickiness is due to menu costs or search frictions is critical to understand the extent to which a monetary authority can "exploit" the stickiness of nominal prices to affect the real side of the economy. Indeed, if menu costs are the source of stickiness, a monetary authority can exert some control over the real prices. For instance, by increasing the quantity of money, the monetary authority can lower real prices as some sellers will not be willing to pay the menu cost to adjust their prices. If, on the other hand, search frictions are the source of nominal price stickiness, a monetary authority cannot exploit such stickiness. If the monetary authority increases the quantity of money, the equilibrium distribution of prices will respond to fully neutralize the monetary injection leaving real prices unaffected. Some individual sellers may not

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change their prices as a matter of indifference, but the overall nominal price distribution must immediately reach its new equilibrium.

The first attempt at answering the question of whether price stickiness is due to menu costs or search frictions is made by Burdett and Menzio (2017). Burdett and Menzio (2017) develop a model which allows for both the menu cost explanation of price stickiness–by positing that sellers may need to pay a menu cost to adjust their nominal price–and the search frictions explanation of price stickiness–by positing that the outcome of the search process is such that some buyers may come into contact with only one seller while others may come into contact with multiple sellers. Burdett and Menzio (2017) find that, as long as the fractions of buyers with one and multiple contacts are both positive and menu costs are not too large, the equilibrium is such that sellers follow a (Q,S,s) pricing rule. According to this rule, each seller lets inflation erode the real value of its nominal price until it reaches some point *s*. Then the seller pays the menu cost, changes the nominal price so that the real value of the nominal price is randomly drawn from some distribution with support [*S, Q*], with *Q* > *S* > *s*. In a (Q,S,s) equilibrium, both search frictions and menu costs contribute to the stickiness of nominal prices. As inflation drives the real value of the price from *Q* to *S*, the seller would not want to change its nominal price even if it could do it for free because, due to search frictions, the seller's profit is maximized everywhere over the interval [*S, Q*]. As inflation drives the real value of the price from *S* to *s*, the seller would like to change its nominal price but chooses not to so as to avoid paying the menu cost. Burdett and Menzio (2017) then calibrate the model to match the average duration of prices and the average dispersion of prices in the data and find that, indeed, its equilibrium is of the (Q,S,s) variety. They then use the calibrated model to measure how much of the price stickiness that we observe in the data is due to menu costs and how much to search frictions. Their finding is very stark: almost all of price stickiness is due to search frictions and almost none of price stickiness is due to menu costs.

The main flaw of Burdett and Menzio (2017) is that their model predicts a grossly counterfactual distribution of prices and a grossly counterfactual distribution of price changes. Their model predicts a distribution of prices with a decreasing density, while empirically the price distribution is hump-shaped (Kaplan and Menzio, 2015). Their model predicts a distribution of price changes featuring only positive changes, while empirically the price change distribution features both positive and negative changes (Nakamura and Steinsson, 2008). This flaw of Burdett and Menzio (2017) is not only problematic on its own, but it also casts some doubt on their measurement of the contribution of menu costs and search frictions to price stickiness. In fact, the observation that as many as 30% of price changes are negative suggests that sellers have additional motives for changing their prices besides keeping up with inflation. If a model abstracts from these additional motives–as is the case in Burdett and Menzio (2017)–then such a model will likely underestimate the magnitude of menu costs and, in turn, the contribution of menu costs to nominal price stickiness.

In this paper, we generalize the model of Burdett and Menzio (2017) to allow for seller-specific shocks. In our model, sellers are subject to shocks to the value of the amenities that they offer to their customers (e.g., quality of the salesmen, lighting in the store, etc...). The value of the amenities offered by a seller increases proportionally the customers' utility from purchasing the good, the seller's cost of producing the good, and the seller's menu cost. Shocks to the amenity value give sellers a second motive for changing their nominal prices besides keeping up with inflation and can induce sellers to actually lower their nominal prices. Shocks to the amenity value introduce an additional source of price dispersion which we empirically identify with the dispersion of the store-component of prices.

In the first part of the paper, we define and characterize the properties of a (Q,S,s) equilibrium for our model. We show that the structure of the (Q,S,s) equilibrium is as simple as in Burdett and Menzio (2017), even though sellers are heterogeneous. In fact, a buyer's decision on where to purchase is based on the ranking of sellers' effective real prices (which are defined as real prices divided by amenity values). Similarly, a seller's pricing strategies is only based on the effective real value of its nominal price, not on the real value of the price and the value of the amenities separately. As a result, in a (Q,S,s) equilibrium, every seller follows the same pricing rule irrespective of the value of its amenities. The seller lets inflation erode the effective real value of its price until it reaches some point *s*, then pays the menu cost and chooses a new nominal price so that the effective real value of the price is a random draw from some distribution with support [*S, Q*]. The cyclical repricing process gets short-circuited when the seller is hit by the amenity shock. In a (Q,S,s) equilibrium, both menu costs and search frictions contribute to nominal price stickiness. As inflation drives the effective real value of the nominal price from *Q* to *S*, the seller has no desire to change its price as its profit remains maximized. Only as inflation drives the effective real value of the nominal price from *S* to *s*, the seller would like to change its price but does not to avoid paying the menu cost.

In the second part of the paper, we calibrate our model. We choose the parameter values so as to match the extent of price dispersion observed in the data, the extent of price stickiness observed in the data and the fraction of price changes that, in the data, are negative. We find that the calibrated model closely reproduces the properties of the empirical distribution of prices and of the empirical distribution of price changes, even though it has only a handful of parameters. In particular, the model generates a price distribution that, as in the data, is hump-shaped and leptokurtic. Further, the model generates a distribution of price changes that, as in the data, features both positive and negative price changes, both small and large price changes and is leptokurtic. We then use the calibrated model to break down the observed stickiness of nominal prices into a component due to menu costs and a component due to search frictions. We find that, in agreement with Burdett and Menzio (2017), search frictions are the main source of price stickiness. However, we find that menu costs contribute to a larger fraction of price stickiness than in Burdett and Menzio (2017).

Our model adds seller-specific shocks to the (Q,S,s) model of Burdett and Menzio (2017) in order to account for the observation that a large fraction of price changes are negative and, hence, cannot be driven by the sellers' desire to keep up with inflation. The (Q,S,s) model of Burdett and Menzio (2017) is not the only model that cannot rationalize negative price changes absent idiosyncratic shocks. Indeed, the same is true of all (S,s) models. The (S,s) literature typically rationalizes negative price changes by introducing shocks to the cost that a particular seller has to bear to produce a given good (see, e.g., Golosov and Lucas, 2007 or Midrigan, 2011). Our approach is different because we introduce shocks to the value of the services offered by a seller to its customers together with all the sellers' products. Thus, instead of introducing shocks that are specific to a seller/good combination, we introduce shocks that are specific to a seller but common to all of the seller's goods.

We believe that our approach is preferable. Seller-specific shocks can be identified from the data without using information on the distribution of price changes. In fact, seller-specific shocks can be identified from the seller component of prices (which is observed in the data). The price change distribution can then be used as a test of the model. Here, we do not follow this approach fully, as we calibrate the stochastic process of seller-specific shocks to match the cross-sectional dispersion of the store component of prices and the fraction of negative price changes (rather than another moment of the store component of prices). However, we believe that we would obtain similar results if we targeted the autocorrelation of the store component of prices instead of the fraction of negative price changes, as both in our calibrated model and in the data the autocorrelation of the store component of prices is very high. In contrast, shocks that are specific to a seller-good combination do not have an immediate counterpart in the data and have to be inferred directly from the price change distribution. Moreover, we find it hard to believe that retailers operating in the same market and often purchasing from the same wholesaler would face very different costs for the same item.

2. Environment

We generalize the model of Burdett and Menzio (2017)–which is a dynamic and monetary version of a model of imperfect competition in the spirit of Butters (1977); Varian (1980) and Burdett and Judd (1983)–by introducing seller-specific shocks.

More specifically, we consider the market for an indivisible good. On one side of the market, there is a population of long-lived sellers with measure 1. Each seller chooses the nominal price d at which it sells the good so as to maximize the present value of real profits discounted at the rate $r > 0$. Sellers are heterogeneous with respect to the value $x \in \mathbb{R}_+$ of amenities that they provide to their customers. A seller of type x produces the good at the constant marginal cost kx, with $k > 0$. A seller of type x can change its nominal price by paying the real menu cost cx, with $c > 0$. Sellers are subject to two types of idiosyncratic shocks: amenity shocks and death shocks. In particular, a seller of type x is hit by an amenity shock at the rate $\lambda \geq 0$. Upon being hit by the amenity shock, the value of the seller's amenities moves to $x(1 + \epsilon)$ with probability $1/2$ and to $x(1 - \epsilon)$ with probability $1/2$, where $\epsilon \in (0, 1)$. Furthermore, upon being hit by the amenity shock, the seller resets the nominal price of its good for free. A seller of type x is hit by a death shock at the rate δ, with $\delta \geq 0$. Upon being hit by the death shock, the seller permanently exits the market. In every interval of time of length dt, a measure $\delta \cdot dt$ of new sellers enters the market, thus maintaining the overall measure of sellers in the market at 1. All new sellers start with an amenity value of $x_0 = 1$.

The other side of the market is populated by a continuum of short-lived buyers. In every interval of time of length dt, a measure $b \cdot dt$ of buyers enters the market. Each buyers searches for sellers. The outcome of the search process is such that the buyer comes into contact with one randomly-selected seller with probability α, and he comes into contact with two randomly-selected sellers with probability $1 - \alpha$, where $\alpha \in (0, 1)$. If the buyer comes into contact with one seller, we say that he is *captive*. If the buyer comes into contact with two sellers, we say that he is *non-captive*. The buyer observes the prices posted and the amenities offered by each of the contacted sellers and decides whether and where to purchase the good. If the buyer purchases the good at a nominal price of d from a seller with an amenity level of x, he attains a utility of $Qx - \mu(t)d$, where Qx is the buyer's valuation of the good together with amenities x and $\mu(t)d$ is the buyer's valuation of d dollars in period t. If the buyer does not purchase the good, he attains a utility of zero. Whether the buyer purchases the good from the contacted sellers or not, he exits the market.

The utility value of a dollar declines at the constant rate π, with $\pi > 0$. Therefore, if a nominal price remains unchanged during an interval of time of length dt, the real value of the price falls by $\exp(-\pi \cdot dt)$. In this paper, we do not describe the demand and supply of dollars. It would, however, be straightforward to embed our model into either a standard cash-in-advance framework (see, e.g., Lucas and Stokey, 1987) or in a standard money-search framework (see, e.g., Lagos and Wright., 2005) and show that, in a stationary equilibrium, the depreciation rate π is equal to the growth rate of the money supply.

The environment described above generalizes the one studied by Burdett and Menzio (2017) along two dimensions.[1] The first generalization is to allow for idiosyncratic shocks to the value of the amenities offered by a seller to its customers, i.e. $\lambda > 0$. The amenity shock gives the seller a second reason for changing its nominal price besides keeping up with inflation.

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[1] The environment generalizes several models. Burdett and Judd (1983) is a static version of our model without inflation or menu costs ($\pi = c = 0$). Bénabou (1988) is essentially a version of our model in which all buyers are captive ($\alpha = 1$) and sellers do not face idiosyncratic shocks ($\lambda = \delta = 0$). Head et al. (2012) is a version of our model in which menu costs are set to zero ($c = 0$) and sellers do not face idiosyncratic shocks ($\lambda = \delta = 0$).

More importantly, the amenity shock gives the seller a reason for lowering its nominal price, an event that would never occur if the seller only changed its price to keep up with inflation. We assume that the level of amenities x scales up the seller's cost of producing the good, the seller's cost of resetting prices and the buyers' valuation of the good. Also, we assume that, upon being hit by an amenity shock, the new value of the seller's amenities is, in expectation, equal to the old value of the seller's amenities. These assumption guarantee that the present value of the seller's profits is homogeneous of degree 1 in x. We interpret amenities as the services provided by the seller to its customers, such as the quality of the personnel tending to the customers, the quality of the refrigeration system, etc... It is then natural to think of amenities as being specific to the seller, and not to a particular seller/good combination. Given our interpretation of amenities, we will identify the stochastic process for x from the store component of prices. Given our interpretation of amenities it is also natural to assume that, upon being hit by the amenity shock, the seller can readjust the price of all of its goods for free.[2] The second generalization of Burdett and Menzio (2017) is to allow for the entry and exit of sellers, i.e. to allow for $\delta > 0$. The assumption guarantees that, even though the level of amenities of a particular seller follows a martingale, the cross-sectional distribution of sellers' amenities has an ergodic distribution.

3. Definition and properties of equilibrium

In this section, we define a (Q,S,s) equilibrium and an (S,s) equilibrium for our generalized version of Burdett and Menzio (2017) described in Section 2. We then solve for the (Q,S,s) equilibrium and show that it features the same properties as in Burdett and Menzio (2017). In particular, the (Q,S,s) equilibrium is such that both menu costs and search frictions contribute to nominal price stickiness.

3.1. Definition of equilibrium

We begin by defining a stationary (Q,S,s) equilibrium. In a stationary (Q,S,s) equilibrium, each seller lets inflation erode the effective real value z of its nominal price d–effective real price which is defined as the real value of the nominal price $\mu(t)d$ divided by the value x of the amenities offered by the seller–until it reaches some point $s \in (0, Q)$. Then, the seller pays the menu cost and changes its nominal price. The effective real value of the seller's new nominal price is randomly drawn from a distribution G with support $[S, Q]$, where $s < S < Q$. This cyclical repricing process is short-circuited if the seller is hit by the death shock δ, in which case the seller exits the market, or if the seller is hit by the amenity shock λ, in which case the seller resets the effective real value of its nominal price by drawing from the distribution G. We denote as F the cumulative distribution function of real effective prices across sellers.[3] We find it useful to define $z(t)$ as $Q \exp(-\pi dt)$. In words, $z(t)$ is the current real value of a nominal price that had a real value of Q t units of time ago. We also find it useful to define T_1 as $\log(Q/S)/\pi$ and T_2 as $\log(S/s)/\pi$. In words, T_1 is the time it takes for inflation to drive the real value of a nominal price down from Q to S, and T_2 is the time it takes for inflation to drive the real value of a nominal price down from S to s.

Consider the flow profit $R(z(t), x)$ enjoyed by a seller that has an effective real price of $z(t) \in [0, Q]$ and an amenity value of $x > 0$. The seller's flow profit is given by

$$R(z(t), x) = b[\alpha + 2(1 - \alpha)(1 - F(z(t)))](z(t)x - kx). \tag{1}$$

The above expression is easy to understand. The arrival rate of captive buyers at the seller's location is $b\alpha$. Each one of these buyers purchases the good from the seller with probability 1. For each sale made to a captive buyer, the seller enjoys a real profit of $z(t)x - kx$. The arrival rate of non-captive buyers at the seller's location is $2b(1 - \alpha)$. Each one of these buyers purchases the good from the seller if and only if his second contact is a supplier with an effective real price greater than $z(t)$, an event which occurs with probability $1 - F(z(t))$. For each sale made to a non-captive buyer, the seller enjoys a real profit of $z(t)x - kx$. Note that the seller's flow profit $R(z(t), x)$ is homogeneous of degree 1 in x. That is, $R(z(t), x) = xR(z(t), 1)$.

Now, consider the present value of profits $V(t, x)$ for a seller that has effective real price of $z(t) \in [0, Q]$ and an amenity value of $x > 0$. The seller's present value of profits is given by

$$V(t, x) = \max_T \int_t^T e^{-(r+\lambda+\delta)(\tau-t)}\big[R(z(\tau), x) + \lambda\mathbb{E}V^*(\hat{x})\big]d\tau + e^{-(r+\lambda+\delta)(T-t)}[V^*(x) - cx]. \tag{2}$$

The above expression is also easy to understand. After $\tau - t$ units of time, the seller has not yet been hit by the death or amenity shock with probability $\exp(-(\delta + \lambda)(\tau - t))$. In this case, the seller's nominal price has an effective real value of $z(\tau)$ and the seller enjoys a flow profit of $R(z(\tau), x)$. Conditional on not having been yet hit by the amenity or death shocks, the seller faces the hazard δ of receiving the death shock. If the seller is hit by the death shock, it exits the market and enjoys a continuation present value of profits of zero. The seller also faces the hazard λ of receiving the amenity shock. If the seller is hit by the amenity shock, the new value $\hat{x}$ of the seller's amenities is $x(1 + \epsilon)$ with probability 1/2 and $x(1 - \epsilon)$

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[2] The assumption is meant to capture the idea that, when a seller is hit by an amenity shock, it incurs some fixed cost of reorganizing the store and that, upon paying such fixed cost, the marginal cost of changing the price of a particular good is zero.

[3] Note that F must be a continuous distribution function. In fact, there cannot be a positive measure of sellers with the same effective real price z in a stationary equilibrium in which each seller follows the same (Q,S,s) rule.

with probability 1/2. Then, the seller resets its nominal price for free and enjoys the maximized continuation present value of profits of $V^*(\hat{x})$. If after $T - t$ units of time it has not yet been hit by a shock, an event which occurs with probability $\exp(-(\delta + \lambda)(T - t))$, the seller pays the menu cost cx, changes its nominal price, and enjoys the maximized continuation present value of profits $V^*(x)$.

The present value of profits $V(t, x)$ for a seller with an effective real price of $z(t)$ and an amenity value of x, as well as the maximized present value of profits $V^*(x)$ for a seller with an amenity value of x are functions that are homogeneous of degree 1 in x. To see why, note that $V^*(x)$ is given by

$$V^*(x) = \max_{t,T} \int_t^T e^{-(r+\lambda+\delta)(\tau-t)} \big[R(z(\tau), x) + \lambda \mathbb{E}V^*(\hat{x}) \big] d\tau + e^{-(r+\lambda+\delta)(T-t)} [V^*(x) - cx]. \tag{3}$$

It is immediate to verify that the value function V^* is the fixed-point of a contraction map. It is also immediate to see that the contraction maps value functions that are homogeneous of degree 1 in x into value functions that are also homogeneous of degree 1 in x. From these observations, it follows that $V^*(x)$ is homogeneous of degree 1 in x. It then follows from (2) that $V(t, x)$ is homogeneous of degree 1 in x.

In a (Q,S,s) equilibrium, a seller of type x chooses to pay the menu cost when the effective real value of its nominal price is equal to $s = z(T_1 + T_2)$. The seller finds it optimal to pay the menu cost when the effective real value of its price is s only if

$$x[R(s, 1) + \lambda V^*(1)] = (r + \delta + \lambda)x[V^*(1) - c]. \tag{4}$$

The expression in (4) is the first-order condition with respect to T of the seller's maximization problem in (2), evaluated at $T = T_1 + T_2$. The expression is independent of x because x multiplies both the left and the right hand sides of (4). The expression is easy to understand. The left-hand side of (4) is the marginal benefit of delaying a nominal price adjustment by an instant, which is given by the flow profit $R(s, 1)$ plus the arrival rate of the amenity shock times the expected continuation value conditional on being hit by the shock $\lambda V^*(1)$. The right-hand side of (4) is the marginal cost of delaying a nominal price adjustment by an instant, which is given by the annuitized value of paying the menu cost and attaining the maximized present value of profits, i.e. $(r + \delta + \lambda)[V^*(1) - c]$. The expression in (4) then states that the seller finds it optimal to pay the menu cost at s only if the marginal benefit of delaying a price adjustment equals the marginal cost. Condition (4) is also sufficient if[4]

$$R(z(t), 1) + \lambda V^*(1) \geq (r + \delta + \lambda)[V^*(1) - c], \ \forall t \in [0, T_1 + T_2]. \tag{5}$$

In a (Q,S,s) equilibrium, a seller of type x resets its nominal price so that the effective real value of the new price is drawn from the distribution G with support $[S, Q]$. The seller finds it optimal to follow this strategy if and only if the present value of profits attains its maximum for all effective real prices $z \in [S, Q]$ and it is non-greater than the maximum for all effective real prices $z \in [s, S]$. That is[5],

$$xV(t, 1) = xV^*(1), \ \forall t \in [0, T_1], \tag{6}$$

$$xV(t, 1) \leq xV^*(1), \ \forall t \in [T_1, T_1 + T_2]. \tag{7}$$

Note that the conditions (6) and (7) are independent of x because x multiplies both the left and the right hand sides. It is convenient to restate condition (6) as

$$R(z(t), 1) = (r + \delta)V^*(1), \ \forall t \in [0, T_1], \tag{8}$$

and

$$V^*(1) = \int_{T_1}^{T_1+T_2} e^{-(r+\lambda+\delta)(\tau-T_1)} [R(z(\tau), 1) + \lambda V^*(1)] d\tau + e^{-(r+\lambda+\delta)T_2} [V^*(1) - c]. \tag{9}$$

Let us explain the two conditions above. Suppose that (6) holds. In this case, the present value of profits for a seller with an effective real price of $S = z(T_1)$ and amenity value of 1 is equal to $V^*(1)$. This is condition (9). Moreover, the derivative of the present value of profits with respect to the age t of the price is equal to zero for all $t \in [0, T_1]$. Since $\partial V(t, 1)/\partial t = (r + \delta + \lambda)V(t, 1) - R(z(t), 1) - \lambda V^*(1)$ and $V(t, 1) = V^*(1)$, we obtain condition (8). Conversely, if (8) and (9) hold, $V(t, 1) = V^*(1)$ for all $t \in [0, T_1]$.

The cross-sectional distribution F of effective real prices is stationary if and only if, during an arbitrarily small interval of time of length dt, the measure of sellers whose effective real price enters the interval $[s, z]$ is equal to the measure of sellers whose effective real price exits the interval $[s, z]$ for any $z \in [s, Q]$. For $z \in (s, S)$, the inflow-outflow equation is given by

$$\big[F(ze^{\pi dt}) - F(z)\big]e^{-(\delta+\lambda)dt} = \big[F(se^{\pi dt}) - F(s)\big]e^{-(\delta+\lambda)dt} + F(z)\big[1 - e^{-(\lambda+\delta)dt}\big]. \tag{10}$$

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[4] Formally, one would also have to check $R(z(t), 1) + \lambda V^*(1) \leq (r + \delta + \lambda)[V^*(1) - c]$ for $t > T_1 + T_2$. It is straightforward to verify that this inequality is always satisfied.

[5] Formally, one would also have to check $V(t, 1) \leq V^*(1)$ for all $t < 0$. It is straightforward to verify that this inequality is always satisfied.

The term on the left-hand side of (10) is the flow of sellers into the interval [s, z], which is given by the measure of sellers that currently have an effective real price between z and $z\exp(\pi dt)$ and that, in the next dt units of time, are not hit by the death or amenity shocks. In fact, every one of these sellers is pushed by inflation into the [s, z] interval. The term on the right-hand side of (10) is the flow of sellers out of the interval [s, z], which is made up of two groups. The first group is composed by the sellers that currently have an effective real price between s and $s\exp(\pi dt)$ and that, in the next dt units of time, are not hit by any shock. In fact, every one of these sellers reaches the point s, pays the menu cost and resets the effective real value of its nominal price to some point above z. The second group is composed by the sellers that currently have an affective real price below z and, in the next dt units of time, are hit by the amenity or death shock. In fact, the sellers that are hit by the amenity shock reset the effective real value of their nominal price somewhere above z. The sellers that are hit by the death shock exit the market and are replaced by new sellers with an effective real price above z.

For $z \in (S, Q)$, the inflow-outflow equation is given by

$$\left[F(ze^{\pi dt}) - F(z)\right]e^{-(\delta+\lambda)dt} + [1 - F(z)]\left[1 - e^{-(\lambda+\delta)dt}\right]G(z)$$
$$= \left\{\left[F(se^{\pi dt}) - F(s)\right]e^{-(\delta+\lambda)dt} + F(z)\left[1 - e^{-(\lambda+\delta)dt}\right]\right\}(1 - G(z)). \tag{11}$$

The term on the left-hand side of (11) is the flow of sellers into the interval [s, z], which is made up of two groups. The first group is composed by the sellers that currently have an effective real price between z and $z\exp(\pi dt)$ and that, in the next dt units of time, are not hit by any shock. In fact, each of these sellers is pushed by inflation in the [s, z] interval. The second group is composed by a fraction $G(z)$ of the sellers that currently have an effective real price above z and are hit by a shock. In fact, if a seller with a real price above z is hit by the amenity shock, there is a probability $G(z)$ that it will set a new effective real price below z. Similarly, if a seller with a real price above z is hit by the death shock, there is a probability $G(z)$ that the seller that replaces it will set an effective real price below z. The term on the right-hand side of (11) is the flow of sellers out of the interval [s, z], which is also made up of two groups. The first group is composed by a fraction $1 - G(z)$ of the sellers that currently have an effective real price between s and $s\exp(\pi dt)$ and that, in the next dt units of time, are not hit by the amenity or death shock. The second group is composed by a fraction $1 - G(z)$ of the sellers that currently have an effective real price smaller than z and that, in the next dt units of time, are hit by either the amenity or the death shock.

Dividing by dt and taking the limit for $dt \to 0$, the inflow-outflow condition (10) becomes

$$F'(z)z = F'(s)s + \theta F(z), \forall z \in (s, S), \tag{12}$$

where θ is defined as $(\delta + \lambda)/\pi$. Similarly, the inflow-outflow condition (11) becomes

$$F'(z)z + \theta(1 - F(z))G(z) = \left\{F'(s)s + \theta F(z)\right\}(1 - G(z)), \forall z \in (S, Q). \tag{13}$$

Condition (12) is a differential equation for F over the interval (s, S). Condition (13) is a differential equation for F over the interval (S, Q). The boundary conditions associated with these differential equations are

$$F(s) = 0, F(Q) = 1, F(S-) = F(S+). \tag{14}$$

Intuitively, $F(s) = 0$ as there are no sellers who let their price fall below s and the distribution F is continuous. Similarly, $F(Q) = 1$ as there are no sellers who reset the effective real value of their nominal price above Q. Finally, $F(S-) = F(S+)$ as the cross-sectional distribution of effective real prices is continuous.

We are now in the position to define a (Q,S,s) equilibrium.

Definition 1. A stationary (Q,S,s) equilibrium is a cumulative distribution of effective real prices $F : [s, Q] \to [0, 1]$, a cumulative distribution of effective real new prices $G : [S, Q] \to [0, 1]$, a pair of prices (s, S) with $0 < s < S < Q$, and a maximized present value of seller's profits $V^*(1)$ that satisfy the optimality conditions (4), (5), (7)–(9) and the stationarity conditions (12)–(14).

For the sake of completeness, it is also useful to define a stationary (S,s) equilibrium. In an (S,s) equilibrium, each seller lets inflation erode the effective real value z of its nominal price d until it reaches some value $s \in (0, Q)$. Then, the seller pays the menu cost and changes its nominal price so that its effective real value is Q. This cyclical repricing process is interrupted if the seller is hit by either the death or the amenity shocks. Basically, one can think of an (S,s) equilibrium as a (Q,S,s) equilibrium in which $S = Q$.

Formally, the definition an (S,s) equilibrium is as follows.

Definition 2. A stationary (S,s) equilibrium is a cumulative distribution of effective real prices $F : [s, Q] \to [0, 1]$, a pair of prices (s, S) with $0 < s < S = Q$, and a maximized present value of seller's profits $V^*(1)$ that satisfy the optimality conditions (4), (5), (7), (9) and the stationarity conditions (12) and (14).

3.2. Solution of equilibrium

In this section, we solve for the (Q,S,s) equilibrium. Consider the optimality condition (8), which guarantees that the derivative of the seller's present value of profits with respect to the age t of the price is zero for all $t \in [0, T_1]$. For $t = 0$, (8) states that the flow profit of a seller with an effective real price of Q is equal to the annuitized maximum present value

of seller's profits, i.e. $R(Q, 1) = (r + \delta)V^*(1)$. Since a seller with an effective real price of Q only trades with captive buyers, it enjoys a flow profit of $R(Q, 1) = b\alpha(Q - k)$. Using this observation, we can solve $R(Q, 1) = (r + \delta)V^*(1)$ with respect to $V^*(1)$ and find

$$V^*(1) = b\alpha \frac{Q - k}{r + \delta}. \tag{15}$$

Consider again the optimality condition (8). For $t \in [0, T_1]$, (8) states that the flow profit of a seller with an effective real price of $z \in [S, Q]$ is equal to the annuitized maximum present value of seller's profits, i.e. $R(z, 1) = (r + \delta)V^*(1)$. Since $R(z, 1)$ is given by $b[\alpha + 2(1 - \alpha)(1 - F(z))](z - k)$ and $V^*(1)$ is given by (15), we can solve $R(z, 1) = (r + \delta)V^*(1)$ with respect to the price distribution F. We then find

$$F(z) = 1 - \frac{\alpha}{2(1 - \alpha)} \frac{Q - z}{z - k}, \ \forall z \in (S, Q). \tag{16}$$

Note that this is the same price distribution that emerges in Burdett and Judd (1983). This finding is easy to understand. In Burdett and Judd (1983), the price distribution is such that the seller's flow profit is the same everywhere on the support of the distribution. Here, the price distribution fulfills the same function over the interval $[S, Q]$ and, hence, it has the same shape.

Now, consider the stationarity condition (12), which is a differential equation for the price distribution F over the interval (s, S). Solving the differential Eq. (12) using the boundary conditions $F(s) = 0$ and $F(S-) = F(S+)$, we find

$$F(z) = \left[1 - \frac{\alpha}{2(1 - \alpha)} \frac{Q - S}{S - k}\right] \frac{z^\theta - s^\theta}{S^\theta - s^\theta}, \ \forall z \in (s, S), \tag{17}$$

where $\theta = (\delta + \lambda)/\pi$. Note that this is the price distribution that would emerge in an (S, s) model in which sellers die at a rate of $\delta + \lambda$. In fact, here as in such an (S, s) model, sellers enter the interval $[s, S]$ from S, they exit the interval from s, they travel through the interval at the speed of π and, while travelling through the interval, they exit with an hazard of $\delta + \lambda$.

Next, consider the stationarity condition (13), which is a differential equation for the price distribution F over the interval (S, Q). Using the fact that F is given by (16) and (17), we can solve (13) with respect to the new price distribution G and find

$$G(z) = \frac{\theta F(z) + F'(s)s - F'(z)z}{\theta + F'(s)s}, \ \forall z \in (S, Q). \tag{18}$$

As it is clear from its construction, the role of G is to generate the price distribution F that keeps the seller's flow profit constant (and, consequently, the seller's present value of profits) constant for all effective real prices z in the interval $[S, Q]$. Note that, in order to fulfill its role, G has mass points at both S and Q. These mass points are respectively given by

$$\begin{aligned} \chi(S) &= \frac{\theta F(S) + F'(s)s - F'(S)S}{\theta + F'(s)s}, \\ \chi(Q) &= 1 - \frac{\theta + F'(s)s - F'(Q)Q}{\theta + F'(s)s}. \end{aligned} \tag{19}$$

Finally, we need to solve for the cutoff prices s and S. The optimality condition (4) states that, when the seller's effective real price is s, the benefit of delaying the price adjustment by an instant is equal to the cost of delaying the price adjustment by an instant. Formally, (4) states that $R(s, 1) + \lambda V^*(1)$ equals $(r + \delta + \lambda)[V^*(1) - c]$. Since a seller with an effective real price of s sells to all of the customers it meets, $R(s, 1) = b[\alpha + 2(1 - \alpha)](s - k)$. Using this observation and the fact that $V^*(1)$ is given by (15), we can solve condition (4) with respect to s and find

$$s = k + \frac{\alpha}{2 - \alpha}(Q - k) - \frac{r + \delta + \lambda}{2 - \alpha} \frac{c}{b}. \tag{20}$$

The optimality condition (9) states that a seller with an effective real price of S must attain the maximized present value of profits $V^*(1)$. After substituting F, $V^*(1)$ and s, we can write the optimality condition (9) as an equation of the form $\varphi(S) = 0$. The only candidate (Q,S,s) equilibria are such that S is a solution of the equation $\varphi(S) = 0$ such that $S \in (s, Q)$, $V^*(1)$ is given by (15), F is given by (16) and (17), G is given by (18) and s is given by (20). A candidate (Q,S,s) equilibrium is indeed an equilibrium if it also satisfies the optimality conditions (5) and (7) and G is a proper cumulative distribution function. These additional conditions can be checked numerically once a candidate (Q,S,s) equilibrium is found.

We have thus established the following proposition.

Proposition 1. *Any stationary (Q,S,s) equilibrium is a tuple $\{F, G, S, s, V^*(1)\}$ such that: S is a solution to the equation $\varphi(S) = 0$ in the interval (s, Q), F is given by (16) and (17), G is given by (18) and is a proper cumulative distribution function, s is given by (20). If the tuple $\{F, G, S, s, V^*(1)\}$ also satisfies the optimality conditions (5) and (7), then it is a (Q,S,s) equilibrium.*

Burdett and Menzio (2017) show that a (Q,S,s) equilibrium of their basic model exists and there is no other type of equilibrium if the menu cost c is small enough and/or the inflation rate π is small enough. They also show that, if the menu cost c is large enough and/or the inflation rate π is high enough, any equilibrium is an (S,s) equilibrium. For intermediate

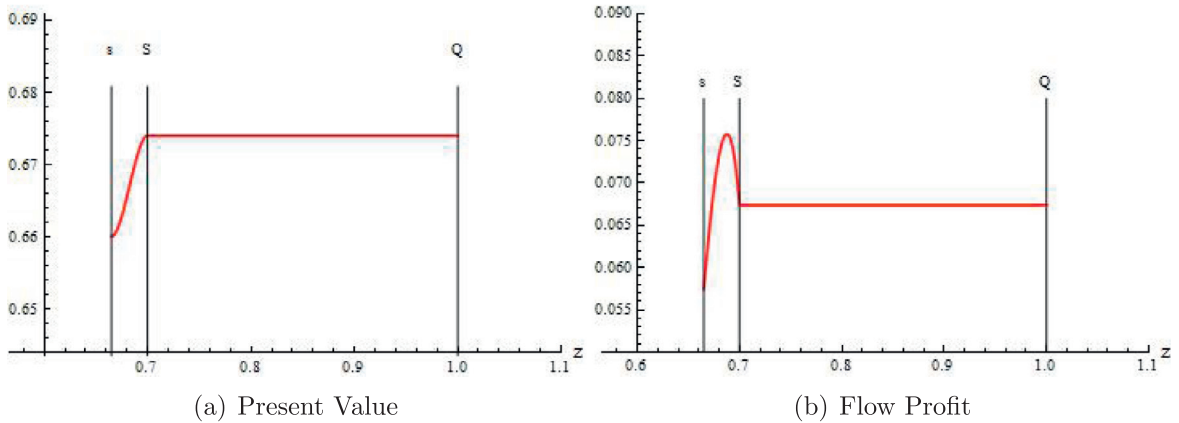(a) Present Value   (b) Flow Profit

Fig. 1. Present value of profits and flow profit. *Notes:* The present value of profits V and the flow profit as a function of the effective real price z given the parameter values $r = 0.05$, $\pi = 0.03$, $b = 1$, $Q = 1$, $k = 0.63$, $\lambda = 0.61$, $\epsilon = 0.021$, $\alpha = 0.183$, $c = 0.014$.

values of c and π, they show that a (Q,S,s) equilibrium may coexist with an (S,s) equilibrium. We do not attempt to extend these theoretical results to our general model. However, our numerical analysis makes us believe that these results do indeed apply to our model as well.

3.3. Illustration of equilibrium

Before embarking on the quantitative analysis, it is useful to illustrate the properties of the (Q,S,s) equilibrium with a numerical example. The illustration is carried out using the parameter values associated with our benchmark calibration and reported in Section 4. Panel (a) of Fig. 1 illustrates the present value of profits for a seller with an effective real price of z and an amenity value of 1. Inflation drives down the effective real value of the seller's nominal price, so we shall read the figure from right to left. As inflation pushes the value of the price from Q to S, the present value of the seller's profits remains constant at its maximum $V^*(1)$. As inflation pushes the value of the price from S to s, the present value of the seller's profits declines monotonically. When inflation drives the price to s, the present value of the seller's profits is $V^*(1) - c$. Then, the seller pays the menu cost and adjusts its nominal price so that the effective real value of the new nominal price is drawn from the distribution G with support $[S, Q]$. This cyclical repricing process is short-circuited when the seller is either hit by the death shock (in which case, it is replaced by a new seller starting with an effective real price drawn from G) or hit by the amenity shock (in which case, the seller automatically resets its nominal price).

Panel (b) of Fig. 1 illustrates the flow profit for a seller with an effective real price of z and an amenity value of 1. Again, we read the figure from right to left. As inflation pushes the value of the price from Q to S, the seller's flow profit remains constant at $(r + \delta)V^*(1)$. As inflation pushes the value of the price from S to s, the seller's flow profit increases at first, peaks and then declines monotonically. As explained in Burdett and Menzio (2017), the fact that the seller's flow profit reaches its maximum at a price smaller than S is what makes the seller indifferent between resetting the effective real value of its nominal price to any point in the interval $[S, Q]$. Indeed, if the seller resets the effective real value of the price to S, it will pay the menu cost sooner but it will also enjoy the highest flow profit sooner. If the seller resets the effective real value of the price to Q, it will pay the menu cost later but it will also enjoy the highest flow profit later.

Panel (a) of Fig. 2 illustrates the density of the cross-sectional distribution F of effective real prices. Over the interval $[S, Q]$, the density of F is such that the seller's flow profit remains constant. Over the interval $[s, S]$, the density of F is the same as in a standard (S,s) model with an exogenous exit hazard of $\lambda + \delta$. There is a discontinuity in the density of F at S. Specifically, the density of F to the right of S is strictly smaller than the density of F to the left of S, i.e. $F'(S+) < F'(S-)$. As explained in Burdett and Menzio (2017), the discontinuity in the density of F implies that the derivative of seller's flow profit jumps up as the real value of the price falls below S. In turn, this implies that the seller's flow profit starts increasing as the real value of the price falls below S.

Panel (b) of Fig. 2 illustrates the density of the distribution G of effective real new prices, together with the two mass points at S and Q. As explained in Burdett and Menzio (2017), the mass point at S is what generates a discontinuity in the density of F, which is necessary to guarantee that the seller's flow profit increases when the effective real price falls below S. The mass point at Q is what guarantees that the density of the cross-sectional price distribution F is strictly positive at Q, which is necessary in order to guarantee that the seller's flow profit is constant for effective real prices in a neighborhood of Q.

The numerical illustration reveals that the (Q,S,s) equilibrium of the generalized model studied in this paper has the same qualitative properties as the (Q,S,s) equilibrium of the basic model analyzed in Burdett and Menzio (2017). Most importantly, as in Burdett and Menzio (2017), the (Q,S,s) equilibrium is such that both menu costs and search frictions contribute to price
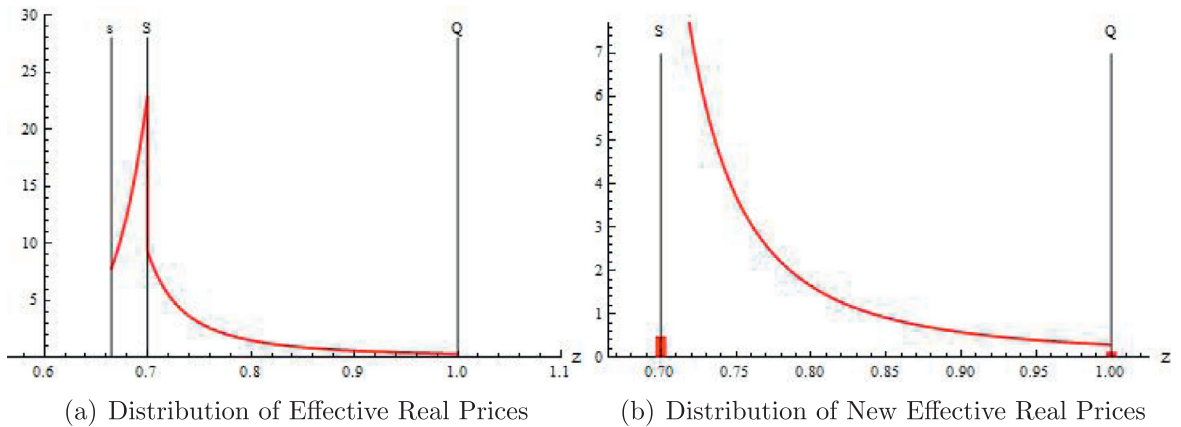
(a) Distribution of Effective Real Prices   (b) Distribution of New Effective Real Prices

Fig. 2. Price distributions. *Notes:* The densities of the equilibrium distribution of effective real prices and of the equilibrium distribution of new effective real prices given the parameter values $r = 0.05$, $\pi = 0.03$, $b = 1$, $Q = 1$, $k = 0.63$, $\lambda = 0.61$, $\epsilon = 0.021$, $\alpha = 0.183$, $c = 0.014$.
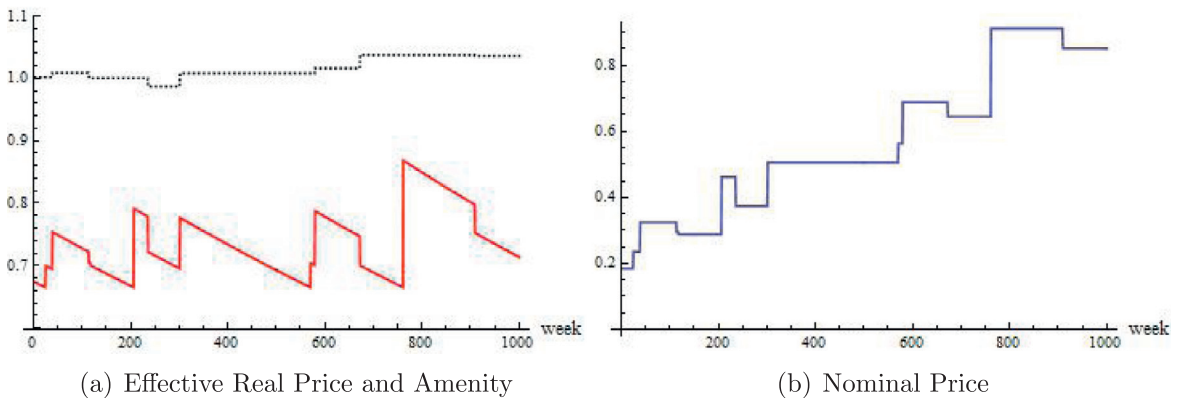

(a) Effective Real Price and Amenity   (b) Nominal Price

Fig. 3. Sample path of prices and amenities. *Notes:* Simulation of prices and amenities for a seller in economy with $r = 0.05$, $\pi = 0.03$, $b = 1$, $Q = 1$, $k = 0.63$, $\lambda = 0.61$, $\epsilon = 0.021$, $\alpha = 0.183$, $c = 0.014$.

stickiness. Indeed, consider a seller who just reset its nominal price to an effective real value of Q. Conditional on not being hit by a shock, the seller keeps the nominal price unchanged until inflation drives the value of the price down to s. As inflation drives the value of the price from Q to S, the seller would not want to change its nominal price even if it could do it for free. Indeed, as inflation pushes the value of the price from Q to S, the seller's present value of profits remains maximized. Only as inflation drives the value of the price from S to s, the seller would like to change its nominal price but does not do so to avoid paying the menu cost. The overall duration of the seller's nominal price is $T_1 + T_2 = \log(Q/s)/\pi$. Only the last T_2 units of time in the life of the price are due to menu costs. The first T_1 units of time in the life of the price are due to search frictions–and, specifically, to the coexistence of captive and non-captive buyers in the market–which create, in equilibrium, an entire interval of prices over which the seller's profits are maximized.

There are two critical differences between the (Q,S,s) equilibrium of the generalized model studied in this paper and the (Q,S,s) equilibrium of the basic model of Burdett and Menzio (2017). The first difference is that the properties of equilibrium in the generalized model apply to effective real prices, while they apply to real prices in the basic model. Importantly, in the generalized model, the price distribution F is a distribution of effective real prices, while F is a distribution of real prices in the basic model. In the generalized model, the distribution of real prices–which is what we observe in the data–depends on the distribution of effective real prices and on the distribution of amenities across sellers with different effective real prices. The second difference between our generalized model and the basic model of Burdett and Menzio (2017) is that, in our model, sellers change their nominal price in response to idiosyncratic amenity shocks. Fig. 3 illustrates a sample path for the effective real price of an individual seller, the value of the seller's amenities, and the seller's nominal price. The seller changes the nominal price to keep up with inflation (it does so when the effective real price reaches s) and to respond to idiosyncratic shocks to the value of its amenities (it does so automatically whenever the amenity value changes). While the price changes carried out to keep up with inflation are always positive, the price changes carried out to respond to amenity

shocks may also be negative. Indeed, the reader can see in Fig. 3 that, when the value of the amenities falls, the seller sometimes lowers the nominal price.

4. Quantitative analysis

In this section, we calibrate our model to match the extent of price dispersion observed in the data, the extent of price stickiness observed in the data, and the fraction of price changes that, in the data, are negative. We show that–in contrast to the basic model of Burdett and Menzio (2017)–the calibrated version of our model reproduces quite closely the key properties of the empirical distribution of prices and the key properties of the empirical distribution of price changes, even though it has very few parameters. We then show that–similarly to the basic model of Burdett and Menzio (2017)–the calibrated version of our model implies that search frictions and not menu costs are the main cause of nominal price stickiness.

4.1. Calibration

The parameters of the model are the following: the real interest rate, r, the inflation rate, π, the arrival rate of buyers, b, the fraction of captive buyers, α, the buyer's valuation of the good, Q, the seller's menu cost, c, and the seller's cost of producing the good, k, the arrival rate of the amenity shock, λ, the magnitude of the amenity shock, ϵ, and the arrival rate of the death shock, δ.

We set the real interest rate r to 5% and the inflation rate π to 3%. We set the death rate δ to 1/20, so that the average life of a seller is 20 years.[6] We note that the equilibrium objects F, G, S and s increase by a factor ρ whenever the parameters Q, k and c increase by the same factor ρ.[7] For this reason, Q is simply a choice of units of measure and we can normalize it to 1. Next, we note that the equilibrium objects F, G, S and s depend on c and b only though their ratio c/b.[8] For this reason, we can normalize the inflow of buyers b to 1 and interpret the seller's menu cost as a fraction of b. We calibrate k so that the average mark-up in the model is 15%.[9] We calibrate the remaining parameters α, c, λ and ϵ to match empirical measures of price stickiness and price dispersion, as well as to match the fraction of negative price changes observed in the data.

Nakamura and Steinsson (2008) measure the extent of price stickiness for consumer goods using the Bureau of Labor Statistics microdata underlying the Consumer Price Index. During the 1998–2005 period, they find that the average duration of nominal prices is 7.7 months if sales and product substitutions are included in the data, and 13 months if sales and product substitutions are excluded. Nakamura and Steinsson (2008) also measure the fraction of nominal price changes that are negative. During the 1998–2005 period, Nakamura and Steinsson (2008) find that the fraction of negative price changes is around 40% if sales are included in the data, and around 30% if sales are excluded.

Kaplan et al. (2016) measure the extent of price dispersion for consumer goods using the Kielts–Nielsen Homescan Dataset. They measure the standard deviation of prices (measured in logs) for the same item (defined by its Unique Product Code) in the same market (defined as a Scantrack Metro Area) in the same week. They find that the average standard deviation of prices is 15%. Then they decompose the price of each good at each store into a store component and a store-good component, and estimate an ARMA process for each of the two components. The estimation reveals that the variance of the store component accounts for 15.5% and the variance of the store-good component accounts for 84.5% of the overall variance of the price for the same good in the same market and in the same week. The estimation also reveals that 36% of the variance of the store-good component is due to persistent differences in the store-good component of prices, while 64% is due to temporary differences. The temporary differences in the store-good component of prices presumably reflect temporary sales.

Our model abstracts from temporary sales and product substitutions. For this reason, we target an average duration of nominal prices of 13 months and a fraction of negative price changes of 30%, which are measures constructed by Nakamura and Steinsson (2008) excluding sales and substitutions. We interpret the seller's amenity x as a bundle of services that are provided with each one of the seller's goods. Under this interpretation, the seller's amenity x is the component of the seller's prices $p_i = z_i x$ that is common to each of the seller's goods i and, hence, it is the model-equivalent of the store-component of the price in the data. For this reason, we target a standard deviation of x across sellers of 5.9%, which is the

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[6] We choose 5% for r, as this is a common choice for the real interest rate. We choose 3% for π, as this is close to the aggregate rate of inflation over the period 1998–2014, from which the data on the duration of prices and on the dispersion of prices is collected. The choice of δ is meant to capture the idea that sellers are long-lived relative to prices. However, the particular choice of δ is somewhat arbitrary. Using alternative values for δ does not significantly affect our findings.

[7] Formally, the following homogeneity property holds: Let $(F, G, s, S, V^*(1))$ be a (Q,S,s) equilibrium given the parameters Q, k and c. Then, for all $\rho > 0$, $(\hat{F}, \hat{G}, \hat{s}, \hat{S}, \hat{V}^*(1))$ is a (Q,S,s) equilibrium given the parameters ρQ, ρk and ρc, where $\hat{F}(\rho z) = F(z)$, $\hat{G}(\rho z) = G(z)$, $\hat{s} = \rho s$, $\hat{S} = \rho S$, and $\hat{V}^*(1) = \rho V^*(1)$. An analogous property holds for an (S,s) equilibrium.

[8] Formally, the following homogeneity property holds: Let $(F, G, s, S, V^*(1))$ be a (Q,S,s) equilibrium given the parameters b and c. Then, $(F, G, s, S, V^*(1))$ is a (Q,S,s) equilibrium given the parameters ρb and ρc, for all $\rho > 0$. An analogous property holds for an (S,s) equilibrium.

[9] The empirical literature on markups has not reached a consensus on the magnitude of the ratio between price and marginal cost. Basu and Fernald (1997) find gross markups between 0.66 and 1.32 depending on the sector and use of instrumental variables. They conclude that the typical industry has small markups over marginal cost. Klette (1999) reaches a similar conclusion using a different estimation technique. De Locker and Warzynski (2012) find gross markups between 1.03 and 1.22 depending on the estimation strategy. In our benchmark calibration, the markup is 15%. When we target a higher (lower) markup, the calibrated value of the menu cost and its contribution to price stickiness falls (increases).

Table 1
Calibration targets and outcomes, full model.

| Calibration targets	(1)	(2)	(3)	(4)	(5)
Std store/good component of prices	8.2%	8.2%	8.2%	5.5%	11.0%
Std store component of prices	5.9%	5.9%	5.9%	3.9%	7.8%
Average duration of prices (months)	13	10	16	13	13
Fraction negative price changes	30%	30%	30%	30%	30%
Average markup	15%	15%	15%	15%	15%
Calibration Outcomes	(1)	(2)	(3)	(4)	(5)
Captive buyers α	18.3%	16.4%	20.3%	10.4%	10.4%
Menu cost c	1.4%	0.03%	6.5%	0.80%	0.80%
Production cost k	0.63	0.62	0.64	0.75	0.52
Arrival rate idiosyncratic shock λ	0.61	0.72	0.61	0.80	0.57
Magnitude idiosyncratic shock ϵ	2.1%	2.1%	1.3%	1.3%	2.9%
Contribution of c to stickiness	(1)	(2)	(3)	(4)	(5)
Average time spent in $[s, S]$	51%	18%	67%	46%	46%
Counterfactual with $c = 0$	14%	1%	29%	28%	11%
Counterfactual with $\alpha = 1$	6%	7%	5%	6%	6%
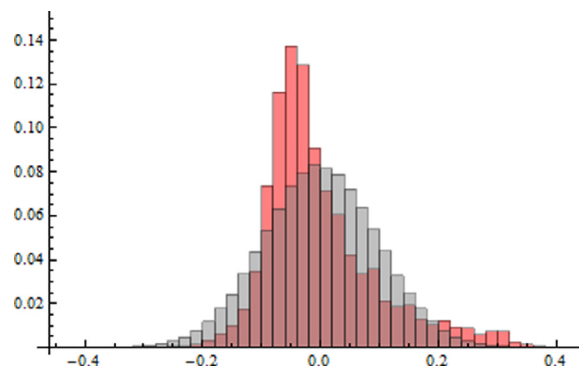
Fig. 4. Price distribution. *Notes:* Dark histogram: Distribution of log prices measured in differences from the average log price. Light histogram: Normal distribution with same mean and variance as the distribution of log prices generated by the model.

standard deviation of the store-component of prices in the data (the square root of 15.5% of the overall variance of prices of 0.15^2). Similarly, under our interpretation of amenities, the effective real price z of a seller for a particular good is the model equivalent of the store-good component of the price. Thus, we target a standard deviation of z across sellers of 8.2%, which is the standard deviation of the persistent part of the store-good component of prices in the data (the square root of 36% of 84.5% of the overall variance of prices of 0.15^2). We target the standard deviation of only the persistent part of the store-good component of prices, as the temporary part is presumably due to temporary sales.

The above targets define our benchmark calibration. However, we also consider some alternative targets, as there is a lot of heterogeneity across different types of goods with respect to the extent of price stickiness and the extent of price dispersion. For example, Nakamura and Steinsson (2008) find that, excluding sales and product substitutions, the duration of nominal prices is 3.5 months for unprocessed food and 27.3 months for apparel. Similarly, Kaplan and Menzio (2015) find that the standard deviation of prices for Health&Beauty products is roughly 3 times larger than for Alcoholic Beverages.

4.2. Properties of the calibrated model

The first column of Table 1 reports the outcomes of our benchmark calibration. Under this calibration, we find that the fraction of captive buyers α is 18%, the menu cost c is 0.014, the production cost k is 0.63, the arrival rate of seller's idiosyncratic shocks λ is 0.61, and ϵ is 0.021. Given these parameter values, the equilibrium is such that sellers follow (Q,S,s) pricing strategies with $s = 0.66$, $S = 0.70$ and $Q = 1$.

The dark histogram in Fig. 4 is the cross-sectional distribution of normalized real prices generated by the calibrated model, where the normalized price is constructed as the difference between the log of the real price and the average log real price. The model-generated price distribution matches quite well the key features of the typical price distribution in the data as documented by Kaplan and Menzio (2015). First, the model-generated price distribution is hump-shaped. Kaplan and Menzio (2015) find that this is also the shape of the typical price distribution in the data. Second, the model-generated price distribution is leptokurtic. That is, compared to a Gaussian distribution with the same mean and variance (Gaussian distribution which is plotted as the light histogram in Fig. 4), the cross-sectional price distribution has more mass around
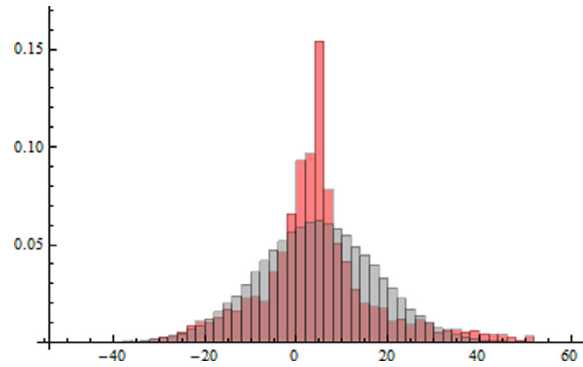
Fig. 5. Price change distribution. *Notes:* Dark histogram: Distribution of price changes measured in percentage deviations for the benchmark calibration. Light histogram: Normal distribution with same mean and variance as the distribution of price changes generated by the model.

the mode as well as more mass in the tails. Kaplan and Menzio (2015) find that the typical price distribution in the data is also leptokurtic.

The dark histogram in Fig. 5 is the price-change distribution generated by the calibrated model, where price changes are measured in percentage deviations. The model-generated price-change distribution fits quite well the key features of the empirical price change distribution. First, the model-generated price change distribution matches the fraction of price changes that are positive (70%) and negative (30%) in the data. This is not surprising, as the fraction of negative price changes was used as a target for the calibration. Second, the model-generated price-change distribution features both small and large price changes. Indeed, the model generates several price changes as large as 20% and a large number of price changes smaller than 5%. The co-existence of small and large price changes is also a feature of the empirical price-change distribution, as documented by Klenow and Kryvtsov (2008). Third, the model-generated price-change distribution is leptokurtic, with a kurtosis of approximately 4. Again, this means that, compared with the Gaussian distribution with the same mean and variance (Gaussian distribution that is plotted as the light histogram in Fig. 5), the model-generated price-change distribution has more mass around the model and in the tails. As documented in Alvarez et al. (2016) the empirical price-change distribution is also leptokurtic.

The fact that our calibrated model matches so well the features of the empirical price and price-change distributions is surprising. After all, our model is a simple generalization of the model in Burdett and Menzio (2017), and their model cannot match these features. First, the model of Burdett and Menzio (2017) generates a price distribution with a strictly decreasing density. Our model generates a hump-shaped distribution of prices because the introduction of seller-specific shocks generates a hump-shaped distribution of effective real prices and then adds hump-shaped noise to such distribution. Without seller-specific shocks, the distribution of effective real prices would have a decreasing density. The density would be decreasing over the interval [S, Q], as this feature is necessary to keep the seller's flow profit constant. The density would be decreasing over the interval [s, S], as the distribution associated with sellers entering the interval from S, exiting from s and travelling from S to s at a constant speed of π is log-uniform and a log-uniform distribution has decreasing density. With seller-specific shocks, the density of the effective real price distribution remains decreasing over the interval [S, Q]. However, the density of the distribution becomes upward sloping over the interval [s, S], leading to an overall hump-shape as seen in Fig. 2(a). In fact, seller-specific shocks cause sellers to exit the interval [s, S] as they travel from S to s and, when this exit rate is high enough, they lead to a density than is increasing over [s, S]. Moreover, the introduction of seller-specific shocks adds some hump-shaped noise to the distribution of effective prices. Indeed, the distribution of prices p is basically given by the distribution of effective prices z plus a hump-shaped disturbance term due to the distribution of amenity values x across sellers.

Second, the model of Burdett and Menzio (2017) generates a price-change distribution with only positive price changes. Our model generates a price-change distribution with both negative and positive price changes by introducing amenity shocks. Intuitively, in Burdett and Menzio (2017), the only reason for changing prices is keeping up with inflation. Thus, all price changes are positive. In our model, sellers also change price in response to amenity shocks. Since sellers may find it optimal to lower their price when the value of their amenities falls, our model can generate negative price changes alongside positive price changes.

We now use our calibrated model–which does much better than the model of Burdett and Menzio (2017) in fitting the distributions of prices and price changes–to revisit their key quantitative exercise. That is, we break down the average duration of nominal prices in a component due to menu costs and one due to search frictions. We carry out this break-down using three alternative measures of the contribution of menu costs to price stickiness. The first measure is the fraction of time that, on average, a nominal price spends in the [s, S] interval. This is a measure of the contribution of menu costs to price stickiness because, only in the [s, S] interval, the seller does not change its nominal price to avoid the menu cost. The second measure is obtained by setting the menu cost c to zero. Specifically, we compute the complement to 1 of the ratio between the average duration of prices in the counterfactual model with $c = 0$ and the average duration of prices in the

properly calibrated model (which, by construction, is equal to the average duration of prices in the data). The third measure is obtained by setting the fraction α of captive buyers to 1. Specifically, we compute the ratio of the average duration of prices in the counterfactual model with $\alpha = 1$ to the average duration of prices in the properly calibrated model. This is a measure of the contribution of menu costs to price stickiness because, when $\alpha = 1$, all buyers are captive, all sellers are monopolists and, just like in Sheshinski and Weiss (1977) or Bénabou (1988), search frictions do not cause any price stickiness. The contribution of search frictions can be recovered as the fraction of price stickiness not accounted for by menu costs.

The last three rows in the first column of Table 1 report the contribution of menu costs to price stickiness for the benchmark calibration of the model. We find that menu costs contribute to approximately half of the empirical duration of nominal prices, if we measure their contribution as the time spent by prices in the [s, S] interval. In words, we find that half of the time a seller does not change its nominal price because it does not want to incur the menu cost. We find that menu costs contribute to 14% of the empirical duration of nominal prices, if we measure their contribution using the counterfactual where $c = 0$. In words, we find that, in a counterfactual world where menu costs are zero, the average duration of prices is only 14% lower than in the data. Finally, we find that menu costs contribute to only 6% of empirical duration of nominal prices, if we measure their contribution using the counterfactual where $\alpha = 1$. In words, we find that, in a counterfactual world where search frictions do not play any role in generating price stickiness, the average duration of prices is 6% of what it actually is in the data.

Columns (2) through (5) of Table 1 report the parameter values and the contribution of menu costs to price stickiness using alternative calibration targets. Column (2) reports the results of a calibration in which we consider some product with a duration of prices of 10 months, which is lower than the average duration of 13 months. Under this calibration, menu costs are smaller and their contribution to the duration of nominal prices tends to be lower than in the benchmark calibration. Column (3) reports the results of a calibration in which we consider some product with an above-average price duration of 16 months. Under this calibration, menu costs are larger and their contribution to price stickiness tends to be bigger than in the benchmark calibration. These findings are very intuitive. As explained in Head et al. (2012), our model generates some price stickiness simply because of search frictions. The lower (higher) is the targeted duration of nominal prices, the smaller (greater) is the menu cost that needs to be added on top of the search frictions in order to match the target, and the smaller (greater) is the contribution of menu costs to price stickiness.

Column (4) reports the results of a calibration in which we consider some product with a standard deviation of prices of 10%, which is lower than the average standard deviation of prices in the data (15%). Using the fact that the store component of prices accounts for 15.5% and the (persistent) store-good component of prices accounts for 30% of the overall variance of prices, we target a standard deviation of x of 3.9% and a standard deviation of z of 8.2%. Under this calibration, menu costs are larger relative to the benchmark calibration and so is their contribution to price stickiness. Column (5) reports the results of a calibration in which we consider some product with an above-average standard deviation of prices of 20% (which implies that we target a standard deviation of x of 7.8% and a standard deviation of z of 11%). Under this calibration, menu costs are smaller and so is their contribution to price stickiness. Also these findings are intuitive. The extent of price stickiness generated by search frictions is greater the larger is the targeted dispersion of prices. In turn, this implies that the larger is the targeted dispersion of prices, the smaller are the menu costs required to match the observed duration of prices, and the smaller is the contribution of menu costs to price stickiness.

The take-away of Table 1 is that both menu costs and search frictions contribute to the observed stickiness of nominal prices, although search frictions are the relatively more important factor. Qualitatively, this is the same finding as in Burdett and Menzio (2017). Quantitatively, though, the contribution of menu costs to price stickiness is larger than what found by Burdett and Menzio (2017). It is easy to understand why this is the case. In Burdett and Menzio (2017), sellers only change their nominal price to keep up with inflation. In our model, sellers also change their nominal price to respond to idiosyncratic shocks. For this reason, in order to match the same empirical duration of nominal prices, our model requires a larger menu cost than Burdett and Menzio (2017). In turn, this implies that the contribution of menu costs to price stickiness is larger in our model than in Burdett and Menzio (2017).

5. Conclusions

In this paper, we developed a generalized version of the model by Burdett and Menzio (2017) in order to answer the question of whether nominal price stickiness is due to menu costs (which prevent sellers from freely adjusting their nominal prices to keep up with inflation) or to search frictions (which make sellers indifferent between every real price over an entire interval). Our model generalizes Burdett and Menzio (2017) by introducing idiosyncratic shocks to the value of the amenities offered by a seller to its customers. These shocks give sellers a second reason to change their nominal prices in addition to keeping up with inflation and lead to the possibility of negative price changes. We calibrated the model to match the extent of price dispersion, price stickiness and the fraction of negative price changes observed in the data. We found that, in the equilibrium of the calibrated model, sellers follow a (Q,S,s) pricing rule as in Burdett and Menzio (2017). This implies that both menu costs and search frictions contribute to the stickiness of nominal prices. We found that, in contrast to Burdett and Menzio (2017), our model reproduces quite closely the properties of the empirical price distribution (hump-shape and excess kurtosis) as well as the properties of the empirical price change distribution (both negative and positive price changes, both small and large price changes, and excess kurtosis). We also found that, in agreement with Burdett and

Menzio (2017), our model implies that search frictions are the main cause of price stickiness. However, our model implies that menu costs account for a larger fraction of price stickiness than what found by Burdett and Menzio (2017).

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