High-Frequency Wage Rigidity

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Abstract

In the context of a frictional model of the labor market with off and on the job search, I advance a novel model of wage determination where contracts are non-binding and firms have private information about the productivity of labor. The characterization of the intra-firm bargaining game leads to a reduced-form model where the firm chooses the wage subject to a non-discrimination and consistency constraints. The fundamental property of the optimal firm-wage policy is high-frequency wage rigidity. While the firm does not respond to productivity shocks whose persistence falls below a critical threshold, the wage is a non-degenerate function of the long-term component of labor productivity. A calibrated version of the model shows that the cyclical behavior of the model is quantitatively consistent with the empirical regularities of the labor market at the business cycle frequency. Among other things, wages are nearly acyclical, the semi-elasticity of the average labor productivity to unemployment is smaller than one, and vacancies are almost perfectly correlated with unemployment.

1 Introduction

Recent research has shown that the standard job-search model cannot account for some important empirical regularities of the labor market at the business cycle frequency. Shimer (2005) makes the most compelling case. When aggregate fluctuations are driven by job-destruction shocks, the model counterfactually predicts a positive correlation between unemployment and vacancies. When shocks to productivity drive the business cycle, the model predicts large responses in the wage and small responses in the rate of unemployment. Quantitatively, the simulated semi-elasticity of average labor productivity to unemployment is an order of magnitude greater than its empirical counterpart.

In the standard model, unemployed workers and firms search for trading opportunities. Once a match is created, the two trading partners are locked in a bilateral monopoly and the wage cannot be identified uniquely by the intersection of their individual rationality constraints. This typical terms-of-trade indeterminacy is resolved by appealing to the Axiomatic Nash Bargaining Solution, using the outside options as disagreement points. While any allocation within the core is Pareto efficient, the specific selection has dramatic effects on the ex-ante incentives to create job vacancies and, in turn, on the macroeconomic behavior of the model. Since the selection of the bargaining outcome is neither theoretically nor empirically a compelling assumption of the model, we can interpret Shimer's result as a call for research on wage setting protocols in frictional labor markets. This paper advances an alternative model of wage determination based on the notion of "firm-wage policy," held by personnel economics and shows how this modification leads to a vast improvement in the business cycle predictions of the search model.

Doeringer and Piore (1971) offer a classic description of what the "firm-wage policy" is. According to their view, labor markets are imperfect and firms offering different wages to similar workers coexist in the same marketplace. The wage is considered a policy because the firm strategically chooses whether to be a low or a high paying employer. In choosing its wage policy, the firm faces both external and internal constraints. The external constraint is a trade-off between turnover and payroll costs. Since workers receive job offers even when already employed, high-wage firms are able to attract more workers and retain them for longer periods than low-paying employers. Internally, the wage policy is constrained by a non-discrimination principle. The narratives in personnel economics stress how underpaid workers are likely to divert time from production into rent-seeking activities (haggling), making worker-specific wages an impracticable option. In this sense, the wage policy is firm-specific.

The wage setting process described above contains elements of wage-posting and intra-firm bargaining that at first sight seem contradictory. I formalize the "firm-wage policy" as the combination of a signaling and bargaining game. More specifically, I assume that workers search for employment opportunities both off and *on the job*. When a contact between a firm and a worker is created, the firm advances a wage offer to the potential employee. The wage offer is *not a binding contractual obligation*, but it is an informative signal of the firm's idiosyncratic productivity, a variable that is *privately observed* by the firm. Based on a comparison between the expected value of the current and prospective jobs, the worker chooses a trading partner for the current period. The wage is a determined as the outcome of extensive-form asymmetric information bargaining between the firm and each individual employee, subject to a stability condition. Specifically, an outcome profile at the firm is said to be stable when no worker has an incentive to re-open the bargaining table after having observed her co-workers' wage. When the size of the workforce is sufficiently large, the characterization of the stable wage profile is extremely simple. First, it is possible to rule out equilibria where two employees are paid differently. Intuitively, if a worker observes that her office-mate receives a higher wage, she updates her beliefs about the firm's productivity and starts a renegotiation. When a stable equilibrium is reached, every worker has the same posterior beliefs and receives the same wage. The second characterization result is that the information spillover allows the firm to credibly mimic the lowest type compatible with the intersection of workers' beliefs. Therefore, the common wage equals the outcome of the perfect-information bargaining game between a worker and a firm whose productivity is at the lower bound of the support of the distribution of workers' beliefs (conditional on the initial wage offer).

If technically the firm-wage is a bargaining outcome, economically is a policy. Through the choice of the wage offer, the firm voluntarily discloses information about its productivity, affects the lower bound on workers' beliefs and, in turn, the wage. In fact, under certain technical conditions, the signaling-and-bargaining game has a reduced-form where the firm directly chooses its wage subject to a couple of constraints. First, the non-discrimination constraint requires the same wage being paid to every employee, independently of her particular employment history. Secondly, the consistency constraint introduces a link between firm-wage and labor productivity. More specifically, the firm-wage has to coincide with a realization of productivity that is possible conditional on the information disclosed in the past by the firm.

The characterization of the optimal firm-wage policy leads to the main finding of the paper: *high-frequency wage rigidity*. To illustrate this result, consider an economy where the firm's labor productivity is the sum of an acyclical component and a transitory shock. Suppose that a firm is considering increasing the wage in response to a positive realization of the shock. The cost of a marginal wage raise is proportional to the measure of employees that are currently trading with the firm, because of the non-discrimination property. The benefit depends on the increase in the acceptance rate of contacted workers and on the reduction in the separation rate of current employees. In turn, the size of these effects depends on how the wage raise is affecting the workers' expectations about the firm-wage policy in the future. Under rational expectation, the marginal benefit is proportional to the expected duration of the productivity shock. If the persistence of the transitory shocks falls below a critical threshold, the optimal firm-wage policy becomes stationary. The common wage is the lowest that workers are willing to accept, given the information that the firm has revealed them about the acyclical component of productivity. By applying the same argument, it is possible to show that the firm-wage policy is strictly increasing in the long-term productivity of the firm. In terms of the signaling game that lies behind the reduced-form model, the firm's announcements about the persistent component of productivity are informative; those referred to the realization of transitory shocks are just noise.

High-frequency wage rigidity translates into acyclical wages, given a couple of assumptions about the nature of the driving force of the business cycle. First, if aggregate shocks affect each firm in an idiosyncratic way, workers cannot use the public information about the state of the aggregate economy to successfully renegotiate their wage. Secondly, if the persistence of the aggregate shock is sufficiently low, the firm has no incentives to adjust its wage in response to cyclical fluctuations. When these two conditions are met, wages are rigid and the cyclical fluctuations in labor productivity are fully bore by the firm. This contrasts with the standard Nash Bargaining matching model where wages are strongly procyclical and the return from filling a vacancy is nearly acyclical. As discussed by Hall (2005), wage rigidity magnifies the effect that productivity fluctuations have on vacancy creation and, in turn, unemployment. This amplification effect is stronger the smaller are the stationary rents of the firm, which in the model are determined by the efficiency of the job-search process. Therefore, when the persistence of aggregate shocks is sufficiently low and the average job-finding rate is high, the model predicts large unemployment fluctuations in response of small productivity shocks. When the model is calibrated to the U.S. data, the aggregate statistics of the simulated business cycle match closely with their empirical counterpart and significantly improve upon the standard matching model with Nash Bargaining. A one percent increase in productivity is sufficient to make the unemployment rate fall from 6 to 5 percent, wages remain constant over the cycle and vacancies are strongly procyclical.

Related Literature

Technically, the model of wage determination advanced in this paper is similar to Stole and Zwiebel's (1996 a, b). They describe an environment where the firm bargains separately with every individual employee, and each worker can ask for a renegotiation of her wage after having observed the bargaining outcome of her co-workers. The focus of their analysis is the effect of intra-firm competition between workers on the wage outcome, in an environment where the productivity of labor is diminishing. My model differs from Stole and Zwiebel's because the productivity of labor is constant and privately observed by the firm. In this context, the interaction between co-workers is relevant because it creates an informational spillover that affects the equilibrium bargaining strategy of the firm. Because of the mechanism of voluntary information disclosure, my model is conceptually closer to a wage posting game à la Burdett-Mortensen (1998). In their model, workers search off and on the job and the firm chooses its wage to optimally trade-off payroll and turnover costs. Mutatis mutandis, the same economic logic applies to this paper On the contrary, in the standard matching model there is no on-the-job search and no trade-off between prices and quantities. The wage represents a non-allocative division of the surplus determined by non-market forces (specifically the bargaining power of the two trading parties).

The main contribution of the paper is the theory of high-frequency wage rigidity. The result is a property of the optimal wage strategy of a firm, subject to the non-discrimination and the consistency constraints. Both restrictions play a role in the derivation of the result. By linking the wage of successive cohorts of workers, the non-discrimination principle makes the cost of a wage increase proportional to the size of the firm rather than to the inflow of workers. On the other hand, the marginal benefit depends on the increase in hiring induced by the wage raise. As the expected duration of transitory shocks falls, only the marginal benefit becomes smaller. The consistency constraint allows the firm to commit to a baseline wage above the monopsony level, by disclosing information about the permanent component of productivity. Kennan (2004) obtains acyclical wages in the context of a bilateral matching model and through a different mechanism. Formally, he studies a protocol where a randomly selected party makes a take-it-or leave-it offer to the other. Because the firm's productivity is privately observed by the firm, the worker faces a higher probability of rejection if he demands for a higher wage. Depending on the cyclical properties of the match-specific productivity distribution, the wage demand of the worker can be pro or countercyclical. In my paper, wage rigidity is an optimal *wage-posting* strategy: the allocative benefit is not worth the cost of increasing the wage to the entire workforce. In Kennan, acyclical wages are the consequence of an optimal *bargaining* strategy.

Recent empirical work on matched employer-employee data offers some support to the theory of high-frequency wage rigidity. Guiso, Pistaferri and Schiavardi (2003) are able to identify transitory and persistent shocks to value added in a sample of Italian firms. Their main finding is that wages are independent from the short-term component of labor productivity, and yet significantly responsive to persistent shocks. While it is important to verify that this result holds in countries where the labor market is less regulated than in Italy, the finding is certainly encouraging for the theory advanced in this paper.

Structure of the Paper

The first part of Section [2] describes the model economy and discusses the empirical content of the assumptions. In the second half of the section, I immediately introduce the reduced-form wage posting model and provide an intuition for its derivation from first principles. The interested reader can find all the formal steps in the Appendix and a robustness analysis in the companion paper. In Section [3], I study a simple version of the model with exogenous search intensity on both sides of the labor market. First, I characterize the optimal firm-wage policy in partial equilibrium. Then, the general equilibrium analysis is carried over in order to derive the equilibrium wage distribution and the sufficient conditions for acyclical wages. Section [4] extends the model to allow for endogenous search intensity. A calibrated version of the model is studied and the simulated moments are compared to their empirical counterpart. Section [5] briefly concludes.

2 Signal Posting: The Model and its Reduced Form

The first part of the section describes the extensive-form of the signaling-and-bargaining game. The physical environment is identical to Burdett and Mortensen (1998), with workers searching for trading partners off and on the job. On the other hand, the contractual environment is similar to Stole and Zwiebel's (1996 a, b), where the wage is the outcome of an intra-firm bargaining game. The presence of asymmetric information between the firm and its employee makes the present environment different from both BM98 and SZ96, and leads to a novel *Signal Posting* model of wage determination. The firm advances non-binding wage offers that are informative about its productivity and affect the bargaining strategy of the workers. In the second half of the section, I present the reduced-form model of wage posting and informally discuss its derivation from first principles. The interested reader can find all of the technical details in Appendix [A].

2.1 The Model

Players In the model economy, there are three kinds of players.

Workers. The economy is populated by a unit measure of ex-ante identical workers, indexed by $j \in [0,1]$. The preferences of the worker are described by a utility function $u_w : \mathbb{R}^\infty \mapsto \mathbb{R}$, which is specialized to the take on the additively-separable form

$$u_w(\tilde{c}) = \sum_{t=0}^{\infty} \delta^t \tilde{c}_t \tag{1}$$

The symbol $\tilde{c}$ in [1] denotes a sequence of consumption levels $\{\tilde{c}_t\}_{t=0}^{\infty}$ and $\delta \in (0,1)$ denotes the discount factor. The linearity of the period-utility with respect to consumption is justified in terms of an environment where the capital markets are perfect and the (constant) interest rate is $1/\delta$. In every period, the worker is endowed with: (i) a unit of labor, which she can trade with a firm, (ii) an unemployment-benefit/ home-production output w_0, $w_0 \geq 0$, conditional on not having accepted any job-offer.

Firms. There is a unit measure of heterogeneous firms, indexed by $i \in [0,1]$. Define a firm type with a two-dimensional vector $\left(b_i, \rho_{i,t}\right)$, where $b_i \in [\underline{b}, \overline{b}] \subset \mathbb{R}^2_{++}$ denotes the persistent component of labor productivity and $\rho_{it} \in [1, \overline{\rho}]$ is a time-t shock to productivity. The permanent-type distribution of firms is given by a twice continuously differentiable CDF $\Gamma(b)$ with full support (i.e. $\Gamma'(b) \equiv \gamma(b) > 0$ for all $b \in [\underline{b}, \overline{b}]$). The transitory productivity component of firm i is distributed according to a time varying CDF $\Phi_t(\rho)$. For every t, $\Phi_t(\rho)$ is twice continuously differentiable and has full support on the interval $[1, \overline{\rho}]$. The boundedness of the supports of b and ρ is a technical assumption that allows for a simple characterization of the bargaining game outcome. Once that the vector (b_i, ρ_{it}) is determined, the average and marginal productivity of labor is given by $p_{it} = b_i \rho_{i,t}$. Finally, the objective of a firm is to maximize the (expected) discounted sum of profits, i.e. the firm's preferences are described by

$$u_f(\tilde{\pi}) = \sum_{t=0}^{\infty} \delta^t \tilde{\pi}_t \tag{2}$$

where $\tilde{\pi} = \{\tilde{\pi}_t\}_{t=0}^{\infty}$ is a sequence of profits.

Nature. At the beginning of each period, Nature draws a realization of the economy-wide state $z_t \sim Z$. Conditional on z_t, every firm i draws a scalar $\rho_{i,t}$ from the distribution $\Phi_t(\rho) \equiv \Phi(\rho|z_t)$.

Markets In the model economy, there are only two markets.

Labor. The worker's employment decisions are constrained by a time-consuming search process. At each date t, worker j contacts a randomly selected firm with probability $\lambda \in (0,1)$. When the two parties are in contact, and before the worker makes a final employment decision, firm i makes a wage offer $m_{i,j,t} \in M$. If worker j decides to trade with firm i, a match is created. The employment relationship continues until either the worker is forced into unemployment (*displacement*) or when he accepts an offer from a firm i_2 (*endogenous separation*). The first event occurs with probability $\sigma \in (0,1)$. The second event is determined by the probability λ of finding a job and by the employment strategy of the worker. For the sake of simplicity, job-finding and displacement are assumed to be mutually exclusive events (within the same date t). From the firm's point of view, the parameter λ is the measure of workers contacted over one period and σ is the fraction of its workforce that is exogenously displaced.

Consumption Good. At date t, the output of the match between firm i and worker j is a quantity $p_{i,j,t} = p_{i,t}$ of a consumption good. The price of the consumption good is constant and normalized to one.

Information All aggregate variables are public. This category includes the fundamentals of the model $\{\Gamma, \Phi(|z), \sigma, \lambda, w_0, \delta, \xi\}$, the economy-wide shock z_t, the unemployment rate u_t and the wage distribution $F(.)$. On the other hand, the productivity type $(b_i, \rho_{i,t})$ is privately observed by firm i. Finally, there is a signal $I_{i,t,.}$ about the wage policy of firm i that is observed exclusively by those workers employed by i at date t.

Contractual Environment In this paper, I model a contractual environment based on the legal doctrine of "employment at will," which typically applies to non-union labor relationships in the U.S. market. According to this view, both trading parties retain the option of leaving the relationship at any time and at no cost. When employment is at will, at any time before production, a worker can approach its employer and start a renegotiation of a previous agreement. Similalrly, the firm can re-open the bargaining table with any of its employees.

When worker j contacts firm i at date t, the firm advances a wage offer $m_{i,j,t}$. Because employment is at will, the offer $m_{i,j,t}$ does not constitute a contractual obligation. Nevertheless, the cheap-talk might be informative about the productivity $\left(b_i, \rho_{i,t}\right)$ of the firm. Therefore, the message space (wage-offer space) M is restricted to be the type space, i.e. $M = \left[\underline{b}, \overline{b}\right] \times [1, \overline{\rho}]$.

If worker j chooses firm i as her trading partner, she physically moves to i's productive location. Once worker j has committed to firm i, she cannot join an alternative employer during date-t. Similarly, at this stage of the game firm i cannot replace its employees during date-t. The two parties are in a bilateral monopoly situation. Therefore, either the firm or the worker can start a renegotiation of the initial wage offer $m_{i,j,t}$. The renegotiation game takes the form of an extensive-form bargaining game under asymmetric information.

Even though each firm trades with a continuum $n \in \mathbb{R}_+$ of workers, it is convenient to describe the wage determination game between firm i and a finite subdivision of employees $n_\Delta \in \mathbb{N}$ of equal size n/n_Δ. The case of interest is obtained by taking the limit for $n_\Delta \to \infty$.

Once the allocation of labor is final, every worker $j = 1, 2, ...n_\Delta$ observes a signal $I_{t,0}$ about the wage that the firm has paid to the cohort of old workers and a signal $h_{t,0}$ about the current wage offers of the firm. More specifically, $h_{t,0}$ is defined as

$$h_{t,0} = \left\{ \max_{j=1,..n_\Delta} m^1_{j,t}, \max_{j=1,..n_\Delta} m^1_{j,t} \cdot m^2_{j,t} \right\} \tag{3}$$

Having observed $I_{t,1} = I_{t,0} \cup h_{t,0}$, the firm or the worker can decide to enter a pair-wise bargaining game. The object of a pair-wise negotiation is a two-dimensional wage (w, υ), where the first component is a baseline salary and the second component is a transitory bonus.[1] The protocol of the pairwise bargaining game is such that the worker (the uninformed party) makes all the wage demands and the firm either accepts or rejects the proposition.[2] At the $\tau-$th round of the $k-$th bargaining session between i and j, the worker demands a wage $(w_{j,t,k,\tau}, \upsilon_{j,t,k,\tau})$. If the firm accepts this demand, the $k-$th bargaining session is concluded.

[1] The representation of the wage as a two dimensional object allows the firm to communicate to the worker which part of the wage he should expect to receive in the future. As discussed in Section [3], this information plays a critical role in allocating labor across firms.

[2] The selected bargaining protocol is identical to Fudenberg, Levine and Tirole (1983). Allowing the firm to advance wage offers complicates the solution of the bargaining game, but does not undermine the qualitative results obtained in the paper.

If the firm declines the offer, the worker demands $(w_{j,t,k,\tau+1}, \upsilon_{j,t,j,\tau+1})$. If an agreement is reached, the outcome of the $k-th$ bargaining session between i and j is denoted by $\Omega_{j,t,k}$, where

$$\Omega_{j,t,k} = (w_{j,t,k,\tau}, \upsilon_{j,t,k,\tau}, \tau_{j,k}) \tag{4}$$

While the first two entries of $\Omega_{j,t,k}$ are obvious, the last denotes the number of times that the firm has rejected a wage demand by worker j. If an agreement is never reached, the bargaining outcome is denoted by

$$\Omega_{j,t,k} = (0, 0, \infty) \tag{5}$$

An outcome profile $\{\Omega_{j,t,k}\}_{j=1}^{n_\Delta}$ becomes final and production takes place, if a renegotiation-proofness test is passed. More specifically, it is required that no party wants to start a renegotiation after having observed the signal $I_{t,k+1}$

$$\begin{aligned} h_{t,k} &= H(\{\Omega_{j,t,k}\}_{j=1}^{n_\Delta}) = \left\{ \max_{j=1,..n_\Delta} \Omega^1_{j,t,k}, \max_{j=1,..n_\Delta} \Omega^1_{j,t,k} \cdot \Omega^2_{j,t,k} \right\} \\ I_{t,k+1} &= I_{t,k} \cup h_{t,k} \end{aligned} \tag{6}$$

where $\Omega^1_{j,t,k}$ and $\Omega^2_{j,t,k}$ denote the first and second component of the bargaining outcome $\Omega_{j,t,k}$.[3] In other words, a wage profile is final when there is no worker j who wants to begin a renegotiation after having observed the highest wage that the firm has accepted or offered to an employee $-j$. If the wage profile is final then the payoffs for workers $j = 1, 2, ...n_\Delta$ and the firm are respectively

$$\begin{aligned} c_{j,t} &= (w_{j,t,k,\tau} \cdot \upsilon_{j,t,k,\tau})\, \beta^{\tau_{j,k}} \qquad \text{for } j = 1, 2, ...n_\Delta \\ \pi_{i,t} &= \tfrac{n}{n_\Delta} \textstyle\sum_{j=1}^{n_\Delta} \beta^{\tau_{j,k}} \left(b_i \rho_{i,t} - w_{j,t,k,\tau} \cdot \upsilon_{j,t,k,\tau} \right) \end{aligned} \tag{7}$$

where $\beta \in (0, 1)$ measures the productivity loss induced by a delayed agreement. Before, the first bargaining session at date $t+1$, every worker employed by firm i observes the signal $I_{i,t+1,0}$, which contains information about the history of firm-wages

$$I_{i,t+1,0} = I_{i,t,k+1} \tag{8}$$

On the other hand, if some agreements are renegotiated, the $(k+1)$-th bargaining session is started. The process continues until the stability condition is met.[4]

Timing of Events It is useful to summarize the structure of the model with a time-line of events (see also Figure [1]).

[3] Any alternative specifications of H that allow the worker to identify $\left(\max_j \Omega^1_j, \max_j \Omega^1_j \cdot \Omega^2_j\right)$ can be adopted to derive the reduced-form model derived in Appendix [A].

[4] The extensive-form wage determination game closely resembles the model of intra-firm bargaining devel-

- *Shocks.* Nature draws a realization of the aggregate shock z_t from the distribution Z. Then every firm observes the realization of the idiosyncratic productivity shock $\rho_{i,t} \sim \Phi(\rho|z_t)$.

- *Contacts and Separations.* Every employed worker loses her job with probability σ and has to spend date-t in the unemployment pool. Every worker contacts one randomly selected firm with probability λ.

- *Signal Posting.* Firm i advances a wage offer $m_{i,j,t} \in M$ to each worker j that enters in contact with the firm. This category includes those workers who were employed by firm i at date $t-1$ and were not displaced and a measure λ of new contacts[5]. The wage offer $m_{i,j,t}$ is not a binding agreement, rather a cheap-talk signal about the productivity $(b_i, \rho_{i,t})$ of the firm.

- *Final Allocation.* At date t, a worker j might be in contact with zero, one or two firms. After having observed the wage offers from the contacted firms, the worker chooses a trading partner and physically moves to the selected productive location. At this point, the worker cannot trade with anyone else during date t.

- *Bargaining.* Firm i enters a pair-wise bargaining game with each of its employees. An outcome profile $\{\Omega_{i,j,t,k}\}_{j=1}^{n_\Delta}$ is final when no one has an incentive to ask for a renegotiation, given the signal $I_{i,t,k+1}$.

oped by Stole and Zwiebel (henceforth SZ96), and it shares the same basic view on non-binding contracts. Economically, the main difference between the two models regards the questions addressed. The focus of SZ96 is the effect on bargaining outcome of intra-firm worker's competition when the production process uses both labor and (fixed) capital. In this paper, the focus is on the effect on the bargaining outcome of informational externalities across co-workers'. Formally, the bargaining protocol I have just described differs from SZ96 for two technical reasons and in one more substantive way. First, I assume that the worker makes all the wage demands, while SZ96 consider a more general process that includes alternating offers. This is just a simplifying assumption and does not affect the qualitative results derived in Section [3]. Secondly, I do not allow for the possibility of an exogenous risk of breakdown à la Binmore-Rubinstein-Wolinsky (1986). Again, I believe it would be simple to extend the results in this direction. The substantive difference with respect to SZ96 is to assume that the wage is a two-dimensional vector as the underlying productivity of the firm. By splitting the current terms-of-trade into a baseline salary and a bonus, I allow firm to signal which part of the wage is related to the long-term productivity component.

[5] Whether a firm observes the employment alternatives of a newly contacted worker is not relevant (as shown in Appendix [A]).

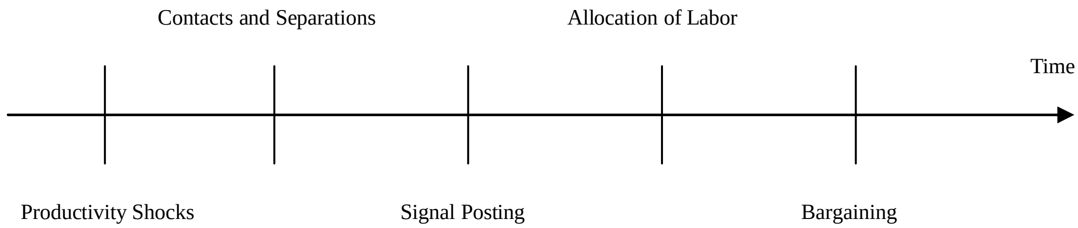


Figure 1: TimeLine

2.2 The Reduced-Form Model

In the technical appendix, I show that the signaling-and-bargaining model has a simple "wage posting" reduced-form. More specifically, the firm announces a two-part wage $(w_{i,j,t}, \upsilon_{i,j,t})$, where the first component $w_{i.j,t}$ is a baseline salary and the second dimension $\upsilon_{i,j,t}$ is a temporary bonus. The wage policy of the firm is subject to two constraints. First, a *Non-Discrimination Rule* requires that every worker j who is employed by i at date t has to receive the same wage: $(w_{i,j,t}, \upsilon_{i,j,t}) = (w_{i,t}, \upsilon_{i,t})$. Secondly, the firm-wage $(w_{i,t}, \upsilon_{i,t})$ has to represent a possible realization of the productivity vector $\left(b_i, \rho_{i,t}\right)$, conditional on the firm's wage history (the *Consistency* constraint). In particular, the firm-wage $(w_{i,t}, \upsilon_{i,t})$ has to belong to the set $D(x_{i,t})$, where

$$D(x_{i,t}) = \{(w, \upsilon) \in \left[\underline{b}, \overline{b}\right] \times \left[1, \underline{\rho}\right] : w_{i,t} \geq x_{i,t}\} \tag{9a}$$

The state variable $x_{i,t}$ in [9a] represents the baseline salary that, in the past, the firm has announced it would pay independently of the realization of the transitory productivity shock $\rho_{i,t}$. Therefore, the law of motion for $x_{i,t}$ is given by the dynamic relationship

$$x_{i,t+1} = w_{i,t} \tag{10}$$

In equilibrium, a worker j who accepts an offer $(w_{i,t}, \upsilon_{i,t})$ is paid accordingly and her expected payoff is

$$c_{j,t} = w_{i,t} \cdot \upsilon_{i,t} \tag{11}$$

On the other hand, the payoff of a firm $\left(b_i, \rho_{i,t}\right)$ that announces the wage $(w_{i,t}, \upsilon_{i,t})$ and has n employees is given by

$$\pi_{i,t} = n \left(\max\left\{b_i \rho_{i,t} - w_{i,t} \upsilon_{i,t}, 0\right\}\right) \tag{12}$$

The arguments that lead to the reduced-form model from the first principles of the signaling-and-bargaining model are informally developed in the next few paragraphs. First, I want to argue that there are no renegotiation-proofness equilibria of the bargaining game where two co-workers are paid differently. By contradiction, suppose that $(w_{j_1,t,k}, \upsilon_{j_1,t,k}) \neq (w_{j_2,t,k}, \upsilon_{j_2,t,k})$ and that $w_{j_1,t,k} \cdot \upsilon_{j_1,t,k} > w_{j_2,t,k} \cdot \upsilon_{j_2,t,k}$. At the end of the $k-th$ bargaining session, worker j_2 observes that the firm is willing to pay a perfectly identical employee a wage above $w_{j_2,t,k} \cdot \upsilon_{j_2,t,k}$. Since the firm always retains the otpion not to trade with j_1 at date t, worker j_2 concludes that $b_i \rho_{i,t} \geq w_{j_1,t,k} \cdot \upsilon_{j_1,t,k}$. Since starting a renegotiation is costless, j_2 rejects the $k-th$ session agreement and demands a raise and, indeed, the firm accepts. From the firm's perspective, this informational spill-over constitutes a strategic advantage. Suppose that a worker demands a wage $(w_{j,t,k}, \upsilon_{j,t,k})$ such that $w_{j,t,k} \cdot \upsilon_{j,t,k} > l_p(\Lambda_{t,k})$, where $l_p(\Lambda_{t,k})$ is the lower bound on the support of the workers' belief $\Lambda_{t,k}$ about the firm's productivity $p_{i,t}$. If the firm accepts the demand $(w_{j,t,k}, \upsilon_{j,t,k})$, it reveals to the rest of the workforce that $p_{i,t} \geq w_{j,t,k} \cdot \upsilon_{j,t,k}$ and the informational externality induces a chain of costly renegotiations. On the other hand, the cost of rejecting the wage demand $(w_{j,t,k}, \upsilon_{j,t,k})$ is to delay the conclusion of the agreement with just one worker and to reduce the productivity of one match. Similarly, if the firm accepts a wage demand $(w_{j,t,k}, \upsilon_{j,t,k})$ such that $w_{j,t,k} > l_b(\Lambda_{t,k})$, where $l_b(\Lambda_{t,k})$ is the lower bound of the workers' beliefs about the long-term productivity b_i of the firm, it signals that $b_i \geq w_{j,t,k}$. The information constraints the future wage policy of the firm and induces a costly long-term distorsion. Since the firm trades with a conitnuum of workers, a no-screening strategy is a sequentially rational choice for a worker. Therefore, the equilibrium wage outcome coincides with the outcome of a full-information bargaining game between the worker and a firm with productivity $(b, \rho) = \left(l_b(\Lambda_{t,1}), (l_b(\Lambda_{t,1}))^{-1} l_p(\Lambda_{t,1})\right)$. This argument is formalized in Proposition [1].

While workers do not learn about their employer at the bargaining stage, the initial wage offer $m_{i,j,t}$ is informative about $\left(b_i, \rho_{i,t}\right)$. In fact, at the signaling stage there is a non-monotonic relationship between the wage paid by the firm and the fraction of contacted workers that elect firm i as their employer. Moreover, high productivity firms have a stronger preference for the size of the workforce than low productivity firms. Because different types have different preferences, there are equilibria of the cheap-talk game where the initial wage offer is informative and affects the lower bounds of the support of workers' beliefs and, in turn, the final wage outcome of the bargaining game. While the companion paper (Menzio, 2004) studies the case of endogenous noise, in this paper I introduce a small measure ζ of "noisy traders," whose signaling strategy is taken as given. Given this technical assumption, the firm can fine-tune the amount of information to be disclosed and can precisely control the relevant statistics of the workers' posterior beliefs. Without loss of generality, the optimal signaling strategy can be restricted to generic announcements, i.e. $m_{i,j,t} = m_{i,t}$, if workers learn about each other's wage offers. Moreover, since the wage history of the firm is observable by newly hired workers, the signaling strategy can be restricted to $m^1_{i,t} \geq m^1_{i,t-1}$, where the first dimension of the message is a lower bound on the permanent component of the firm's productivity.

2.3 Empirical Evidence

The *Non-Discrimination Rule* is the implication of the contractual and informational environment that is most critical to the development of the argument behind the theory of wage rigidity advanced in Section [3]. But is there any empirical evidence of intra-firm pay compression? Using data from the BLS Industry Wage Survey, Groshen (1991) shows that occupation and establishment identity alone can explain over 90 percent of wage variation among blue collar workers. The wage variation attributable to worker-specific wage differentials within an establishment-occupation cell lies between 3 and 7 percent of the total in those sectors where explicit incentive contracts are relatively uncommon (i.e. plastic products, industrial chemicals, steel as opposed to textiles). In a survey study, Medoff and Abraham (1980) find evidence suggesting that between-job earning differences are more important than within-job differences. Baker, Gibbs and Holmstrom (1994) analyze the personnel records for all management employees of a medium-sized U.S. firm in a service industry over the years 1969-1988. They find strong evidence for the importance of job levels as determinants of wages and a strong positive correlation between changes in the entering cohort and incumbent salary.

The view that pay is attached to a job rather than tailored to the identity of a specific worker is common in the personnel economics literature. Wage setting, as described by Doeringer and Piore (1971), is a rather bureaucratic process where a job is evaluated and "priced" to reflect its relative value in the overall production process. The relative position between the firm's pay structure and the market wage distribution is a strategic choice that trades off the gains from a lower turnover rate and the wage-bill costs. While the inefficiencies of a pay-per-job system are acknowledged, they are preferred to the haggling costs triggered by internal pay dispersion. This view is a recurrent theme in Bewley's survey (1999) who refers to it as the "morale problem" arising from internal inequities. Managers describe explicitly their experience with pay differentials:

"There were questions about why is he making more than me. Arguments over small amounts of money took up too much time", (Bewley, p. 81)

"There was envy, suspicion, and a feeling of being devalued. "My work is as good as hers. I am better in some ways. Why am I being paid less?". I spent four hours talking with her about this." (Bewley, p. 80)

"In setting the pay of new hires, we try to fit people with their peers-people who have similar years of experience. We don't pay them more than comparable people at the same location. To pay them less would be a problem too. New hires learn quickly what other people are paid. [...] Jealousy is a real management problem." (Bewley, p. 134)

3 Wage Rigidity

Given the reduced-form model of wage determination, presented in Section [2] and formally derived in Appendix [A], it is possible to formalize the players' problem and to define an equilibrium. When the equilibrium firm-wage policy is acyclical, the analysis of the model is sufficiently simple to be carried out in closed-form. In this scenario, the characterization of the workers' problem returns the optimal employment strategy (Lemma [3]) and a discount factor that applies to the transitory component of a wage offer. The characterization of the firm's problem contains the main result: the firm-wage policy function does not respond to shocks whose expected duration is small relative to the length of the employment relationship (Proposition [3]). On the other hand, the firm-wage is a non-degenerate function of the permanent component of the wage. The general equilibrium properties of the model are derived in the final subsection. In Proposition [4] derives the equilibrium wage distribution and shows that, under a technical condition, the distribution increases (in the sense of first order stochastic dominance) with the efficiency of the search process. Proposition [5] derives three sets of sufficient conditions under which the wage offer distribution is acyclical.

3.1 Rigid-Wages Sequential Equilibrium

The objective of the paper is to identify a set of sufficient conditions under which the optimal firm-wage policy is independent from the realization of transitory shocks to productivity. Denoting with $(w^*, v^*) : X \mapsto \mathbb{R}^2$ the wage policy function, the objective that we desire to meet is

$$(w^*(e, x, \rho; b), v^*(e, x, \rho; b)) = (w(b), 1) \tag{13}$$

for every (e, x, ρ) in the ergodic set $X^e(b)$, and for all $b \in [\underline{b}, \overline{b}]$. When [13] holds, the aggregate wage offer distribution $F(w)$ is independent of z. In turn, this implies that the worker's employment strategy is stationary and so are the wage distribution $G(w)$ and the unemployment rate u.

3.1.1 Worker's Problem

A worker contacts a firm with probability λ independently of her employment status. Under the assumption of random search, the identity i of the contacted firm is a realization from a uniformly distributed random variable $\tilde{\imath} \sim U(0,1)$. Given the conjecture in [13], the wage posted by the firm is a vector $(w_i, 1)$, where the persistent component of the firm has a cumulative distribution $F(w)$, defined by

$$F(w) \equiv \Pr\left[i : w(b_i) \leq w\right] \tag{14}$$

The support of the wage offer distribution $F(w)$ lies within the interval $\left[\underline{b}, \overline{b}\right]$, because of the *Consistency* constraint [9a] on the firm-wage policy. If the worker accepts the offer of firm i, a match is created and the relationship ends either due to exogenous displacement or when the worker meets a more desirable employer. In the first case the worker moves into unemployment, in the latter case the worker directly moves from one production location to another.

Denote with U the worker's expected value when unemployed and with $V(w)$ the value of holding a job that pays the wage $(w, 1)$. The functions $\{U, V(.)\}$ can be defined recursively as

$$U = w_0 + \delta \left\{ \lambda \left(\int_{\underline{b}}^{\overline{b}} \max\{U, V(w)\} dF(w) \right) + (1-\lambda) U \right\} \tag{15}$$

$$V(w) = \\ w + \delta \left\{ \lambda \left(\int_{\underline{b}}^{\overline{b}} \max\{U, V(w), V(\tilde{w})\} dF(\tilde{w}) \right) + \sigma U + (1-\sigma-\lambda) \max\{U, V(w)\} \right\} \tag{16}$$

The appearance of both functional equations is standard, but their correct interpretation has to be related to the signaling game underlying the reduced-form model. In fact, equation [16] assumes that the worker expects a firm that currently pays $(w_{i,t}, 1)$ to offer the same wage in the future. The expectation is correct along the equilibrium path, because of the conjecture [13]. The expectation is also correct after an off-equilibrium move by the firm:

given the worker's off-equilibrium conjectures, the optimal signaling strategy for any firm type in the support of workers' beliefs is given by $(w_{i,t+j}, \upsilon_{i,t+j}) = (w_{i,t}, 1)$, for $j = 1, 2, \ldots.$.

Next, we have to formalize the expected value of moving to a firm i that offers the off-equilibrium wage $(w_{i,t}, \upsilon_{i,t})$, such that $\upsilon_{i,t} > 1$. At date t, the worker expects a payoff $w_{i,t} \cdot \upsilon_{i,t}$. In the continuation game, the worker expects the firm to play an equilibrium strategy, conditional on the increased firm-size and updated worker's belief. Given Assumption [2] in the appendix and the conjecture on the optimal firm-wage policy [13], the firm is expected to offer $(w_{i,t+j}, \upsilon_{i,t+j}) = (w_{i,t}, 1)$, for $j = 1, 2, \ldots$. Therefore, the expected value of holding a job with a firm that currently offers (w, υ) is

$$V^{+}(w, \upsilon) = \\ w\upsilon + \delta \left\{ \lambda \left(\int_{\underline{b}}^{\overline{b}} \max\{U, V(w), V(\tilde{w})\} dF(\tilde{w}) \right) + \sigma U + (1 - \sigma - \lambda) \max\{U, V(w)\} \right\} \quad (17)$$

Given the standard form of the Bellman equations [15]-[17], it is possible to invoke a result in Mortesen and Pissarides (1999) in order to conclude that U and $V(w)$ are unique. Therefore, it is simple to verify that the function V is strictly increasing in w and $V(w_2) - V(w_1) \geq w_2 - w_1$, for $w_2 \geq w_1$. The function V^{+} is strictly incraesin in both arguments and maps a vector $(b, \rho) \in [\underline{b}, \overline{b}] \times [1, \overline{\rho}]$ into the co-domain $\left[V^{+}(\underline{b}, 1), V^{+}(\overline{b}, \overline{\rho})\right]$. From the previous observations, it follows that for every $(w, \upsilon) \in [\underline{b}, \overline{b}] \times [1, \overline{\rho}]$ there exist a unique number $A \geq 0$, such that $V^{+}(w, \upsilon) = V(w + A)$. Therefore, the equation

$$V^{+}(w, \upsilon) = V(w + A(w, \upsilon)) \quad (18)$$

implicitely defines an *Annuity Function* $A : [\underline{b}, \overline{b}] \times [1, \overline{\rho}] \to \mathbb{R}_{+}$, which returns the baseline salary equivalent of a temporary bonus υ.

On the one hand, the *Annuity Function* allows for a simple characterization of the worker's optimal choice between employers as a comparison between one-dimensional wage offers. On the other hand, the assumption that search is equally efficient off and on the job implies that a worker leaves unemployment for work if and only if the wage offer is larger than the unemployment benefit. If two employment alternatives happen to deliver the same expected value to the worker, I assume that the worker choses a job over unemployment and her old employer over a new employer.[6] These arguments lead to the following lemma.

[6] The reader can verify that none of the results depends on this particular tie-breaking rule.

Lemma 3 *(Worker's Employment Strategy)* Given the tie-breaking rule, the optimal employment strategy of a worker is specified under (i) and (ii) below.

(i) If at the beginning of date t, worker j is unemployed, she accepts an offer $(w_{i,t}, \upsilon_{i,t})$ iff $w_{i,t} + A(w_{i,t}, \upsilon_{i,t}) \geq w_0$;

(ii) If at the beginning of date t, worker j is employed at firm i_1 for a wage $(w_{i_1,t}, \upsilon_{i_1,t})$, she accepts an offer $(w_{i_2,t}, \upsilon_{i_2,t})$ iff $w_{i_2,t} + A(w_{i_2,t}, \upsilon_{i_2,t}) > w_{i_1,t} + A(w_{i_1,t}, \upsilon_{i_1,t})$.

Proof. *(i) The monotonicity of the value function $V(w)$ implies that a reservation wage R exists, with the property that $V(w) > U \iff w > R$ and $V(w) < U \iff w < R$. By solving the equation $V(R) = U$ with respect to R, we find that the unique solution is $R = w_0$. Part (i) follows from the definition of $A(w, \upsilon)$. (ii) A worker moves from i_1 to i_2 only if $V^+(w_{i_2,t}, \upsilon_{i_2,t}) > V^+(w_{i_1,t}, \upsilon_{i_1,t})$. From [18] follows that $V^+(w_{i,t}, \upsilon_{i,t}) = V(w_{i,t} + A(w_{i,t}, \upsilon_{i,t}))$ for $i = i_1, i_2$. Therefore, part (ii) follows from the strict monotonicity of $V(.)$.* ■

In light of Lemma [3], the value functions $\{U, V(.)\}$ can be re-expressed as

$$U = w_0 + \delta \left\{ \lambda \int_{w_0}^{\bar{b}} V(w) dF(w) + (1 - \lambda) U \right\} \tag{19}$$

$$V(w) = w + \delta \left\{ \lambda \int_{w}^{\bar{b}} V(\tilde{w}) dF(\tilde{w}) + \sigma U + (1 - \sigma - \lambda \overline{F}(w)) V(w) \right\} \tag{20}$$

where $\overline{F}(.) = 1 - F(.)$.

3.1.2 Firm's Problem

At any date t, a firm with permanent productivity b_i is described by a triple $\left(e_{i,t}, x_{i,t}, \rho_{i,t}\right) \in X$. The first dimension in X denotes the measure e of employees at the onset of the period, the second refers to the baseline salary x that the firm has committed to in the past and the third is the current realization of the transitory productivity shock. The state-space X of the firm is a compact and convex set

$$X = [0, \lambda/\sigma] \times \left[\underline{b}, \overline{b}\right] \times [1, \overline{\rho}] \tag{21}$$

where λ/σ is the largest sustainable firm-size.

Given its position in X, the firm maximizes the expected value of the discounted sum of net profits [12]. At date t, the firm's choice variable is a combination of a baseline salary w_t and a transitory bonus υ_t, which are paid to every employee independently of their specific history (*Non-Discrimination Constraint*). The wage (w_t, υ_t) is also subject to the *Consistency Constraint* [9a], which is derived from the bargaining game underlying the reduced-form model. Given the chosen wage (w_t, υ_t), the quantity of labor is constrained by a labor supply schedule, which is obtained from the worker's employment strategy described in Lemma [3]. More specifically, the labor input that the firm commands is

$$n_t = e_t \left(1 - \sigma - \lambda \overline{F}(w_t + A(w_t, \upsilon_t))\right) + \lambda \left[u + (1-u) G^- \left(w_t + A(w_t, \upsilon_t)\right)\right] \tag{LS}$$

if $w_t + A(w_t, \upsilon_t) \geq R$ and $n_t = 0$ otherwise. The first term on the right hand side of [LS] is the product of e_t and the survival probability of an employment relationship $(1 - \sigma - \lambda \overline{F}(w_t + A(w_t, \upsilon_t)))$. The second term in [LS] is given by the measure λ of workers contacted at date t times the probability that the offer is accepted. In turn, this is the sum of the unemployment rate u and the measure of workers that are employed for a wage strictly below $w + A(w, \upsilon)$. Therefore $G^-(w)$ denotes the left limit of the wage distribution, i.e.

$$G^-(w) = \lim_{\varepsilon \to 0^-} G(w + \varepsilon)$$

that the firm's offer (w, υ) is accepted. Given (w_t, υ_t), the position of the firm at the beginning of date $t+1$ is given by

$$\left(e_{t+1}, x_{t+1}, \rho_{t+1}\right) = (n_t, w_t, \tilde{\rho}) \tag{22}$$

where $\tilde{\rho} \sim \Phi\left(\rho | z_{t+1}\right)$.

The maximum of the firm's problem is denoted by a function $\Pi : X \mapsto \mathbb{R}_+$, whose recursive representation is given by the following functional equation

$$\Pi\left(e, x, \rho; b\right) = \max_{(n, w, \upsilon)} n \max\left\{b\rho - w\upsilon, 0\right\} + \delta \sum_{z' \in Z} \Pr\left(z'\right) \left(\int_{\rho'} \Pi(n, w, \rho'; b) d\Phi\left(\rho' | z'\right)\right), \text{ s.t.} \tag{FP}$$

$$(w, \upsilon) \in D(x) = \{(\hat{w}, \hat{\upsilon}) \in [\underline{b}, \overline{b}] \times [1, \underline{\rho}] : \hat{w} \geq x\}$$

$$n \leq \begin{cases} e\left(1 - \sigma - \lambda \overline{F}(w + A(w, \upsilon))\right) + \lambda\left[u + (1-u) G^-\left(w + A(w, \upsilon)\right)\right] \\ \text{if } w + A(w, \upsilon) \geq R \\ \\ 0 \text{ otherwise} \end{cases} \tag{FC}$$

The functional equation [FP] describes a problem of optimal pricing for a firm that has some monopsony power in the labor market. Assuming that F and G^- are continous functions, the firm faces an elastic labor supply schedule. But unlike a standard model of monopolistic competition, the problem in [FP] is dynamic because the source of firm's market power lies in the presence of search frictions. Since it takes time for workers to find a new employer, any change in the wage affects current employment and shifts the future labor supply schedule. In fact, the position of [LS] depends on the state variable e_t, which is the result of the past wage policy of the firm.[7] A second consequence of search frictions is that the supply of labor depends not only on the current wage $w \cdot \upsilon$, but also on the wage that the firm is expected to offer in the future. Therefore, the composition of the gross wage into the persistent and transitory parts matters, as is apparent from [LS].

Before turning to the characterization of the firm's problem, it is useful to rule out equilibria featuring non-continuous wage offer distributions. The gist of the argument, which is directly borrowed from Burdett and Mortensen (1998), is to show that a profitable deviation exists for any firm that posts a mass-point wage $(\hat{w}, 1)$. In fact, if the wage offer distribution $F(w)$ has a mass point at $\hat{w}$, then the wage distribution is discontinuous at $\hat{w}$, and $G^-(w) < G^-(w + \epsilon)$ for any $\epsilon > 0$. Therefore, if the firm offers a wage $(\hat{w}, \upsilon)$, where $\upsilon > 1$, the hiring inflow increases dicretely. The argument is completed and formalized in Lemma [4].

Lemma 4 *(Continuous Distributions)* Consider an equilibrium that satisfies the wage rigidity condition [13] and $w(b) \leq b$. Then the wage offer distribution $F(w)$ has no mass points.

Proof. *By contradiction. First, notice that the value of a firm who offers $(w_1, 1)$, where $w_1 < w_0$, is zero. Therefore, without loss of generality we can disregard equilibria where the lower bound on the support of $F(w)$ is below w_0.*

Next, suppose that there is an equilibrium whose wage offer distribution $F(w)$ has a mass point at $w_1 \in [w_0, \bar{b}]$. Denote the measure of firms posting $(w_1, 1)$ by

$$F(w_1) - F^-(w_1) = \Pr(i : w(b_i) = w_1) = k \tag{24}$$

[7] Robert Hall pointed out that formally the firm's problem [FP] is very similar to the one studied by Phelps and Winter (1970). Unlike in my model, they do not derive the labor supply schedule and its dynamics from the optimizing behavior of workers. For this reason, they do not discuss the role of workers' expectations for the optimal firm's strategy.

In the putative equilibrium, the measure of workers attracted by firms paying w or less is $\lambda u \cdot F(w)$. On the other hand, the outflow of workers from firms paying w or less is $(1-u)G(w)\left(\sigma+\lambda\overline{F}(w)\right)$. By equating inflows and outflows, we obtain the expression

$$G(w)=\frac{\lambda uF(w)}{(1-u)\left(\sigma+\lambda\overline{F}(w)\right)} \tag{25}$$

Inspecting [25], we conclude that the wage distribution has a mass point at $w=w_1$

$$G(w_1)-G^{-}(w_1)=$$

$$\frac{\lambda uF(w_1)}{(1-u)\left(\sigma+\lambda\overline{F}(w_1)\right)}-\frac{\lambda uF^{-}(w_1)}{(1-u)\left(\sigma+\lambda\overline{F}^{-}(w_1)\right)}>0$$

$G(w_1)-G^{-}(w_1)>0$ *if and only if* $F(w_1)-F^{-}(w_1)$.

Next, notice that there exists a firm i that offers $(w_{i,t},\upsilon_{i,t})=(w_1,1)$ and has productivity $(b_i,\rho_{i,t})$, such that $b_i\rho_{i,t}>w_1$. Consider the deviation from the putative equilibrium where firm i offers $(\hat{w}_{i,t},\hat{\upsilon}_{i,t})=(w_1,\epsilon)$, for some $\epsilon\in[1,\overline{\rho}]$. The wage offer $(\hat{w}_{i,t},\hat{\upsilon}_{i,t})$ is feasible and the firm-size is

$$n(\epsilon)=e\left(1-\sigma-\lambda\overline{F}(w_1+A(w_1,\epsilon))\right)+\lambda\left[u+(1-u)G^{-}\left(w_1+A(w_1,\epsilon)\right)\right] \tag{26}$$

The function $n(\epsilon)$ is discontinuous: $\lim_{\epsilon\to1^+}n(\epsilon)>n(1)$. Therefore, the firm's payoff is

$$\tilde{\Pi}(\epsilon)\geq n(\epsilon)\max\{b\rho-w_1\epsilon,0\}+\delta\sum_{z'\in Z}\Pr(z')\left(\int_{\rho'}\Pi(n_{i,t+1},w_1,\rho';b)d\Phi(\rho'|z')\right) \tag{27}$$

and $\lim_{\epsilon\to1^+}\tilde{\Pi}(\epsilon)>\tilde{\Pi}(1)$. ■

We are now in the position to prove the existence and the monotonicity of the value function $\Pi(.)$.

Lemma 5 (*Characterization of Firm's Value Function*) Suppose that $F(w)$ and $G(w)$ are continuous distributions, then

(i) the solution to the functional equation [FP] exists and is unique;

(ii) the value function $\Pi(e,x,\rho;b)$ is continuous, weakly increasing in e and ρ and weakly decreasing in x.

Proof. *(i) Let* $\overline{D} : X \mapsto [0, \lambda/\sigma] \times \left[\underline{b}, \overline{b}\right] \times [1, \overline{\rho}]$ *denote the correspondance from the state* (e, x, ρ) *to the set of feasible* (n, w, υ)*, implicitely defined by the constraints in [FP].Define the operator* T *on the space of bounded continuous functions* $B(X)$ *by*

$$(Tf)(e, x, \rho) = \max_{(n,w,\upsilon)\in\overline{D}(e,x,\rho)} n \max\{b\rho - w\upsilon, 0\} + \delta \sum_{z'\in Z} \Pr(z') \left(\int_{\rho'} f(n, w, \rho'; b) d\Phi(\rho'|z')\right) \quad (28)$$

The function $f(.)$ *is bounded by assumption, therefore the integral in the second line of [28] is well defined. Moreover, if* $f(.)$ *is increasing in* n *and decreasing in* w*, so is the expression in the second line of [28].*

The objective function of the maximization problem is a continuous function. The correspondance $\overline{D}$ *is compact-valued and continuous. Therefore, the Theoreom of the Maximum is applicable and we can conclude that* Tf *is a bounded continuous function. The operator* $T : B(X) \mapsto B(X)$ *satisfies the monotonicity and discounting properties defined in the Blackwell's sufficient conditions for a contraction (see Stokey and Lucas (1983), Theorem 3.3) and* T *has a unique fixed point* Π.

(ii) The objective function of the maximization problem is weakly increasing in ρ *and* ρ *does not affect the feasible set* $\overline{D}$*. Therefore* $\Pi(.)$ *is weakly increasing in* ρ*. The objective function is non-descreasing in the initial firm-size* e *and* $\overline{D}(e_1, x, \rho) \subseteq \overline{D}(e_2, x, \rho)$*, for* $e_2 \geq e_1$*. Therefore* $\Pi(.)$ *is weakly increasing in* e*. Finally, notice that the objective function is independent of* x*, but the feasible set is monotonically decreasing, i.e.* $\overline{D}(e, x_2, \rho) \subset \overline{D}(e, x_1, \rho)$*, for* $x_2 > x_1$*. Therefore* $\Pi(.)$ *is weakly decreasing in* x*.* ■

3.1.3 Definition of an Equilibrium

We are now in the position to formally define a rigid-wages equilibrium for the model economy.

Definition 2 A stationary *Rigid-Wages Sequential Equilibrium* is: (i) a wage function $w(b)$, such that $(w_{i,t}, \upsilon_{i,t}) = (w(b_i), 1)$ and $x_{i,t} = w(b_i)$ for $t = 0, 1, 2, ...$;(ii) a firm-size function $n(w)$, such that $e_{i,t} = n(w(b_i))$ for $t = 0, 1, 2, ...$; (iii) a continuous wage offer distribution $F(w)$, such that $F_t(w) = F(w)$ for $t = 0, 1, 2, ...$; (iv) a continuous wage distribution $G(w)$, such that $G_t(w) = G(w)$ for $t = 0, 1, 2, ...$; (v) an unemployment rate u, such that $u_t = u$ for $t = 0, 1, 2, ...$ These elements have to satisfy the following conditions:

(a) *Optimality of Wages.* Given $\{F(w), G(w), u\}$, the vector $(w(b), 1)$ is a solution to the firm's optimization problem

$$(w(b), 1) = (w^*(n(w(b_i)), w(b_i), \rho; b), \upsilon^*(n(w(b_i)), w(b_i), \rho; b)) \tag{29}$$

for any $\rho \in [1, \overline{\rho}]$ and every $b \in [\underline{b}, \overline{b}]$.

(b) *Consistency of Firm's Dynamics.* Given $\{F(w), G(w), u\}$, the firm-size $n(w)$ solves equation [LS]. It follows that

$$n(w) = \frac{\lambda [u + (1-u)G(w)]}{\sigma + \lambda \overline{F}(w)} \tag{30}$$

(c) *Consistency of Aggregate Dynamics.* Given the firm-wage policy $w(b)$ and the distribution $\Gamma(b)$, the wage offer distribution $F(w)$ is given by

$$F(w) = \int_{b: w(b) \leq w} d\Gamma(b) \tag{31}$$

Given $\{F(w), u\}$, the law of motion for the wage distribution

$$(1-u) G_{t+1}(w) = (1-u) G_t(w) \left(1 - \sigma - \lambda \overline{F}(w)\right) + \lambda u F(w) \tag{32}$$

holds for $G_{t+1}(w) = G_t(w) = G(w)$. It follows that

$$G(w) = \frac{\lambda u F(w)}{(1-u)\left(\sigma + \lambda \overline{F}(w)\right)} \tag{33}$$

The law of motion for the rate of unemployment

$$u_{t+1} = u_t + (1-u_t)\sigma - \lambda u_t \tag{34}$$

holds for $u_{t+1} = u_t = u$. It follows that

$$u = \frac{\sigma}{\lambda + \sigma} \tag{35}$$

(d) *Consistency of Beliefs.* The belief updating rule conjectured in Appendix [A] is consistent with Bayes' Law:

$$w(b) \leq b, \text{ for all } b \in [\underline{b}, \overline{b}] \tag{36}$$

Condition (d) in Definition [2] requires the baseline salary $w(b_i)$ offered by firm i being smaller or equal than the persistent component productivity b_i. This condition is necessary to vindicate the belief updating rule conjectured in the technical appendix. But if $w(b_i) \leq b_i$, then a firm i such that $b_i < w_0$ fails to attract worker and makes null profits. Therefore, if $b_i < w_0$ either condition (d) or condition (a) is violated. In order to sideline this problem, I adopt the following assumption.

Assumption 3 *(Efficient Employment)* The lower bound on $\Gamma(b)$ is $\underline{b} = w_0$.

3.2 The Optimal Firm-Wage Policy

In this subsection, I guess and verify the form of the firm's value function $\Pi(.)$ over a subset $X^*(b)$ of X, where

$$X^*(b) = [n(w(b)), \lambda/\sigma] \times [w(b), \overline{b}] \times [1, \overline{\rho}] \tag{37}$$

The subset $X^*(b)$ is invariant with respect to the firm-wage. More specifically, for every triple (e_t, x_t, ρ_t) in $X^*(b)$, the point $\left(e_{t+1}, x_{t+1}, \rho_{t+1}\right)$ lies in $X^*(b)$ with probability one

$$\Pr\left(\left(e_{t+1}, x_{t+1}, \rho_{t+1}\right) \in X^*\left(b\right) \mid \left(e_t, x_t, \rho_t\right), \left(w_t, \upsilon_t\right)\right) = 1$$

for all $(w_t, \upsilon_t) \in D(x_t)$. Also notice that the set $X^*\left(b\right)$ contains the segment $\left(n\left(w\left(b\right)\right), w\left(b\right), \rho\right)$, which is the ergodic set of a firm b in the stationary *RWSE*. Therefore, it is safe to restrict attention to $X^*(b)$ for the purpose of verifying the existence of conditions (a)-(d) in Definition [2].

The conjectured value of $\Pi : X^*(b) \mapsto \mathbb{R}_+$ is computed as the expected value of a firm that follows the wage policy

$$\begin{aligned} w^*(e, x, \rho; b) &= x, && (38a) \\ \upsilon^*(e, x, \rho; b) &= 1 && (38b) \end{aligned}$$

for all $(e, x, \rho) \in X^*(b)$. Denote the hiring inflow $h(.)$ and the separation rate $\sigma(.)$ for a firm that offers the wage (w, υ) by

$$\begin{aligned} h(w, \upsilon) &= \lambda\left[u + (1-u)G(w + A(w, \upsilon))\right] && (39a) \\ \sigma(w, \upsilon) &= \sigma + \lambda\overline{F}(w + A(w, \upsilon)) && (39b) \end{aligned}$$

and denote with $\hat{\rho}$ the unconditional mean of the transitory shock ρ

$$\hat{\rho} = \sum_{z \in Z} \Pr\left(z\right) \rho d\Phi\left(\rho | z\right) \tag{40}$$

Given these definitions and [38a] [38b], the conjectured value function $\Pi : X^*(b) \mapsto \mathbb{R}_+$ is

$$\begin{aligned} &\Pi(e, x, \rho; b) = \\ &\left(e\left(1 - \sigma(x,1)\right) + h(x,1)\right)\left(\max\{(b\rho - x), 0\} + \delta\frac{\left(1-\sigma(x,1)\right)E_\rho\left[\max\{(b\rho - x), 0\}\right]}{1 - \delta\left(1 - \sigma(x,1)\right)}\right) + \\ &\qquad \frac{\delta h(x,1)}{1-\delta}\left(\frac{E_\rho\left[\max\{(b\rho - x), 0\}\right]}{1 - \delta\left(1 - \sigma(x,1)\right)}\right) \end{aligned}$$

for all $b \in [\underline{b}, \overline{b}]$.

In order to verify the conjecture, I check that the solution of the firm's objective function

$$Obj(e, x, \rho, w, \upsilon; b) = \max_{(w,\upsilon)\in D(x)} n \max\{b\rho - w\upsilon, 0\} + \delta \sum_{z'\in Z} \Pr(z') \left(\int_{\rho'} \Pi(n, w, \rho'; b) d\Phi(\rho'|z') \right), \text{ s.t.} \tag{41}$$

$$n = e\left(1 - \sigma - \lambda\overline{F}(w + A(w, \upsilon))\right) + \lambda\left[u + (1 - u)G(w + A(w, \upsilon))\right]$$

is the wage $(w^*, \upsilon^*) = (x, 1)$. optimization problem of the firm given that the continuation value is Π. This analysis leads to identify a set of sufficient conditions under which [38a]-[38b] are optimal and, therefore, the firm-wage policy is acyclical. Assuming that the firm's problem is pseudo-concave,[8] the conjecture is satisfied if the following conditions hold

$$\frac{\partial Obj(e, x, \rho, w, \upsilon; b)}{\partial w}|_{\{w=x,\upsilon=1\}} \leq 0 \tag{42a}$$

$$\frac{\partial Obj(e, x, \rho, w, \upsilon; b)}{\partial \upsilon}|_{\{w=x,\upsilon=1\}} \leq 0 \tag{42b}$$

for every $(e, x, \rho) \in X^*(b)$.

3.2.1 The First Order Approach

The inspection of [41] reveals that the two wage instruments w_t and υ_t are not perfect substitutes. In fact, while both w_t and υ_t affect the current cost of labor, they enter asymmetrically in the labor supply schedule and in the continuation game. Formally, the transitory component of the wage is discounte by the annuity function A in [LS] and the baseline wage constraints the future firm-wage policy since $x_{t+1} = w_t$. Economically, by increasing the baseline salary, the firm reveals information about the permanent productivity component b_i and affects the worker's beliefs over the long-run. In turn, the belief update lead to a persistent change in the worker's bargaining strategy and in the wage outcome. Since worker correctly anticipate the long-term effects of the baseline salary, an increase in w_t affects the value of the job more than an equal increase in υ_t. The derivation of $\partial Obj/\partial w$ and $\partial Obj/\partial \upsilon$ allows to precisely identify the static and dynamic differences between w_t and υ_t.

[8] A function $f(x)$ in $X \subset \mathbb{R}^n$ is pseudo-concave if

$$\nabla f(x)(y - x) \leq 0 \Longrightarrow f(y) \leq f(x)$$

The Effect of a Change in the Persistent Component of the Wage The derivative of the objective function with respect to the baseline salary w can be divided into the sum of three effects. As the firm increases the persistent component of the wage, the probability of hiring a contacted worker increases both on impact and in the future (*hiring effect*), employees accept outside offers less frequently (*separation effect*) and the wage-bill raises (*wage effect*).

Formally, the hiring effect HE_1 is defined as

$$\begin{aligned}&HE_1(e,x,\rho,w,\upsilon;b)=\\&\quad h_1(w,1)\left(\max\{b\rho-w,0\}+\frac{\delta\,(1-\sigma(w,1))}{1-\delta\,(1-\sigma(w,1))}E_\rho\left[\max\{b\rho-w,0\}\right]\right)+\\&\quad\quad\frac{\delta}{1-\delta}h_1(w,1)\frac{E_\rho\left[\max\{b\rho-w,0\}\right]}{1-\delta\,(1-\sigma(w,1))}\end{aligned}$$

The first line on the right hand side represents the product of the marginal increase in hiring and the discounted value of a worker currently employed. The second line is the discounted sum of the future gains from hiring.

The separation effect SE_1 is defined as

$$\begin{aligned}&SE_1(e,x,\rho,w,\upsilon;b)=\\&\quad-\sigma_1(w,1)e\left(\max\{b\rho-w,0\}+\frac{\delta\,(1-\sigma(w,1))}{1-\delta\,(1-\sigma(w,1))}E_\rho\left[\max\{b\rho-w,0\}\right]\right)\\&\quad-\sigma_1(w,1)\left(\frac{\delta}{1-\delta}h(w,1)+e\,(1-\sigma(w,1))+h(w,1)\right)\left(\frac{\delta E_\rho\left[\max\{b\rho-w,0\}\right]}{(1-\delta\,(1-\sigma(w,1)))^2}\right)\end{aligned}$$

The first line is the product of the increase in the current employment level attributable to a reduced separation rate and the expected value of a worker. The second line measures the gains from the increase in the length of the employment relationships.

Finally, the wage effect WE_1 is given by the discounted sum of current and future wage increases, i.e.

$$\begin{aligned}&WE_1(e,x,\rho,w,\upsilon;b)=\\&\quad\frac{e\,(1-\sigma(w,1))+h(w,1)}{1-\delta\,(1-\sigma(w,1))}\mathbf{1}\left[b\rho\geq w\right]+\frac{\delta}{1-\delta}\frac{h(w,1)}{1-\delta\,(1-\sigma(w,1))}\Pr\left[b\;\tilde{\rho}\geq w\right]\end{aligned}$$

Summing up, we have that

$$\frac{\partial Ob(e,x,\rho,w,\upsilon;b)}{\partial w}=HE_1(.)+SE_1(.)-WE_1(.)\tag{43}$$

The Effect of a Change in the Transitory Component of the Wage The derivative of the objective function with respect to the transitory component of the wage can be broken down in a similar way, identifying hiring, separation and wage-bill effects. But the effects of a transitory wage raise are temporally limited and, consequently, workers do not value υ as much as w. From the point of view of the firm, the workers' discount factor on υ determines the hiring and separation effects of a transitory wage raise. Formally, the partial derivatives of the separation rate are respectively

$$\begin{aligned}\sigma_1(w,\upsilon) &= -\lambda F'(w+A(w,\upsilon))\,(1+A_1(w,\upsilon)) \\ \sigma_2(w,\upsilon) &= -\lambda F'(w+A(w,\upsilon))A_2(w,\upsilon)\end{aligned}$$

and their relationship can be expressed as

$$\frac{\sigma_2(w,\upsilon)}{\sigma_1(w,\upsilon)} = \frac{A_2(w,\upsilon)}{1+A_1(w,\upsilon)} \tag{45}$$

where the ratio $A_2(w,\upsilon)\,(1+A_1(w,\upsilon))^{-1}$ is the workers' discount factor. A similar relationship holds between $h_1(.)$ and $h_2(.)$:

$$\frac{h_2(w,\upsilon)}{h_1(w,\upsilon)} = \frac{A_2(w,\upsilon)}{1+A_1(w,\upsilon)} \tag{46}$$

The workers' discount factor is readily derived from [18]. By totally differentiating this identity, we obtain

$$V_2^+(w,\upsilon) = V'(w+A(w,\upsilon))A_2(w,\upsilon) \tag{47a}$$

$$V_1^+(w,\upsilon) = V'(w+A(w,\upsilon))\,(1+A_1(w,\upsilon)) \tag{47b}$$

From [16] and [17] follows that

$$V'(w) = \frac{1}{1-\delta(1-\sigma(w,1))} \tag{48a}$$

$$V_1^+(w,\upsilon) = \upsilon+\delta(1-\sigma(w,1))V'(w) \tag{48b}$$

$$V_2^+(w,\upsilon) = w \tag{48c}$$

Substituting [48] into [47], we conclude that

$$\frac{A_2(w,\upsilon)}{1+A_1(w,\upsilon)} = w\frac{1-\delta(1-\sigma(w,1))}{\upsilon(1-\delta(1-\sigma(w,1)))+\delta(1-\sigma(w,1))} \tag{49}$$

Around $\upsilon = 1$, the discount factor evaluates at

$$\frac{1}{w}\frac{A_2(w,1)}{1+A_1(w,1)} = 1-\delta(1-\sigma(w,1)) \in (1-\delta,1) \tag{50}$$

As the intuition would suggest, the transitory wage component is discounted more heavily the longer is the expected duration of the job $1/\sigma(w,1)$.

Substituting out for [45] and [46], we can express the hiring, separation and wage-bill effects of a transitory wage raise as

$$HE_2(e,x,\rho,w,\upsilon;b) = w\ h_1(w,1)\left(\max\{b\rho-w,0\}+\frac{\delta\,(1-\sigma(w,1))}{1-\delta\,(1-\sigma(w,1))}E_\rho\left[\max\{b\rho-w,0\}\right]\right)\left[\frac{1}{w}\frac{A_2(w,1)}{1+A_1(w,1)}\right] \tag{51}$$

$$SE_2(e,x,\rho,w,\upsilon;b) = -w\ \sigma_1(w,1)e\left(\max\{b\rho-w,0\}+\frac{\delta\,(1-\sigma(w,1))}{1-\delta\,(1-\sigma(w,1))}E_\rho\left[\max\{b\rho-w,0\}\right]\right)\left[\frac{1}{w}\frac{A_2(w,1)}{1+A_1(w,1)}\right] \tag{52}$$

$$WE_2(e,x,\rho,w,\upsilon;b) = \frac{(e\,(1-\sigma(w,1))+h(w,1))}{1-\delta\,(1-\sigma(w,1))}w\ \mathbf{1}\left[b\rho\geq w\upsilon\right] \tag{53}$$

and

$$\frac{\partial Ob(e,x,\rho,w,\upsilon;b)}{\partial\upsilon} = HE_2(.)+SE_2(.)-WE_2(.) \tag{54}$$

By comparing [54] with [43] the mechanism behind the theory of high-frequency wage rigidity can already be identified. While the hiring and separation effects of a transitory wage raise are discounted by the annuity factor, the wage-bill effect is not. Therefore, [54] is a one-period version of [43] only on the cost side. On the side of benefits, [54] is less then proportional to [43]. This intuition is fully developed in the next pages.

3.2.2 High-Frequency Wage Rigidity

A direct inspection of [54] and [43] reveals that both expressions are non-decreasing in ρ. Therefore, it is sufficient to verify [42a] and [42b] for every $(e,x,\overline{\rho})\in X^*(b)$. Secondly, if the cross-derivative of the objective function between wage (w,υ) and firm-size e is negative,[9]

[9] These two conditions on the cross-derivatives are verified for $\sigma+\lambda\leq 1/2$ and the derivation of this general equilibrium result is included in the proof of Proposition [5] in Appendix [B]. In the remaining of the section, I will assume that this condition is satisfied.

it is sufficient to verify that [42a] and [42b] for every $(n(w(b)), x, \overline{\rho}) \in X^*(b)$. Given the assumption on pseudo-concavity, then conditions [42a] and [42b] are satisfied whenever

$$\frac{\partial Obj(n(w(b)), w(b), \overline{\rho}, w, \upsilon; b)}{\partial w}|_{\{w=w(b), \upsilon=1\}} \leq 0 \tag{55a}$$

$$\frac{\partial Obj(n(w(b)), w(b), \overline{\rho}, w, \upsilon; b)}{\partial \upsilon}|_{\{w=w(b), \upsilon=1\}} \leq 0 \tag{55b}$$

The wage rigidity condition [55b] can be conveniently re-expressed using the inflow-outflow identity $n(w)\sigma(w,1) = h(w,1)$ for a firm that offers a constant wage $(w,1)$. The derivative of the objective function with respect to υ is non-positive in a neighborhood of $(n, w, \rho) = (n(w(b)), w(b), \rho)$ if and only if

$$n \geq \left[\lambda(1-u)G'(w) + \lambda n F'(w)\right] \left[(1 - \delta (1 - \sigma(w,1)))\right] \left[\frac{(b\hat{\rho} - w)}{1 - \delta (1 - \sigma(w,1))} + b(\rho - \hat{\rho})\right] \tag{WRC}$$

The left hand side of inequality [WRC] represents the marginal cost of increasing the short-term component of the wage. Because of the non-discriminatory principle, the cost is proportional to the measure of firm's employees rather than to the inflow of workers. The right hand side of [WRC] is the marginal benefit and can be described as the product of three effects. The first effect, represented by the first term of the product, is the increase of the firm's size induced by a marginal increase in the permanent component of the wage w. The second term is a discount factor [50] applied to the temporary component of the wage. Finally, the third term is the expected value of a worker. Overall, the marginal benefit depends positively on the measure of workers contacted in a period of time λ, on the density of the wage offer and earning distribution functions at w, on the realization of the transitory shock to productivity ρ and on the length of the transitory shock relative to the employment duration.

As the title of this paper suggests doing, the fundamental economic insight in [WRC] is gained by studying its dependence from the expected duration Δ of a transitory shock. Formally this exercise is carried over by multiplying the players' payoffs by Δ and expressing the discount, contact and separation rates as exponential arrival rates

$$\delta(\Delta) = e^{-\delta\Delta} \tag{56a}$$

$$\sigma(\Delta) = 1 - e^{-\sigma\Delta} \tag{56b}$$

$$\lambda(\Delta) = 1 - e^{-\lambda\Delta} \tag{56c}$$

Using the approximation $1 - e^{-x\Delta} \simeq x\Delta$, the wage rigidity condition becomes

$$n\Delta \geq \kappa_1 \Delta^2 \left[\lambda(1-u)G'(w) + \lambda n F'(w)\right] \left[\delta + \sigma + \lambda \overline{F}(w)\right] + \kappa_2 \Delta^3 \tag{57}$$

where κ_1 and κ_2 are positive constants. Since the marginal cost is linear in Δ and the marginal benefit is a function of higher powers of Δ, the inequality [57] holds when the duration of a transitory shock is sufficiently small. Moreover, if the approximation $1 - e^{-x\Delta} \simeq x\Delta$ is accurate, there is a critical cutoff Δ^* such that [57] holds if and only if $\Delta < \Delta^*$. When the firm announces a transitory wage raise in response to a productivity shock, the increase in the workers' evaluation of the job depends on how long they expect the higher wage offer to shock to persist. Therefore the discount factor on a transitory wage raise is proportional to the expected duration of the underlying shock. Moreover, the wage raise only affects the employment decision of workers contacted during the period when the raise is offered. Therefore, the marginal benefit is quadratic in Δ. On the other hand, the marginal cost is given by the product of firm's size and the expected duration of the shock. If transitory shocks to productivity are short-lived, the firm announces the lowest realization independently of the true realization of ρ and the second dimension of the firm's signal is uninformative, does not affect the beliefs of the worker and, in turn, leaves the final wage outcome unchanged.

To understand the critical elements of the high-frequency wage rigidity result, we can refer again to the expression in [57]. Without the *Non-Discrimination Constraint*, the left hand side of [57] would be proportional to Δ^2 as the product between the inflow of workers and the length of the shock Δ. *Search frictions* and *equilibrium wage dispersion* guarantee that the right hand side of [57] is finite and proportional to Δ^2. Given these three features of the environment, there is a negative relationship between the expected duration of a wage raise and the derivative of the firm's objective function. In turn, the relationship between the duration of a wage change and a productivity shock follows from rational expectations. For Δ sufficiently small, the inequality [57] holds for every $\rho \in [1, \overline{\rho}]$ and the signaling strategy of the firm is independent of the realization ρ. As the firm pretends that the worst realization of the transitory shock has occurred, the wage outcome is driven to the minimum wage that employees are willing to accept given their beliefs about the long-term productivity component b of the firm.

A critical element to the result, one that does not appear explicitly in [57], is the firm's lack of commitment. In solving for the optimal state-contingent wage policy, the firm would internalize that part of the return on a transitory wage raise that is obtained *before* the raise itself is paid. In fact, a credible state-contingent wage raise improves the firm's hiring rate even before it materializes. It is easy to show that the first-best firm-wage policy depends non-trivially on the observable state of the economy and that it is *time inconsistent*. In the contractless environment studied by this paper, the firm has an *imperfect substitute for commitment* in the mechanism of information disclosure. When it reveals information about the idiosyncratic realization of an observable aggregate shock, the firm manages to influence the workers' beliefs, their bargaining strategy and the wage outcome until the state of the economy changes. Similarly, by revealing information about the acyclical component of productivity, the firm can commit to a part of the wage that is independent of transitory shocks. On the contrary, the firm cannot credibly disclose information about the idiosyncratic realization of a future aggregate shock and therefore cannot precommit to a state-dependent wage policy.

The condition [55a] on the derivative of the objective function with respect with the long-term component of the wage reads

$$\frac{n}{1-\delta} \geq \frac{[\lambda(1-u)G'(w) + \lambda n F'(w)]}{1-\delta}\left[\frac{(b\hat{\rho}-w)}{1-\delta\left(1-\sigma(w,1)\right)} + (1-\delta)b\left(\overline{\rho}-\hat{\rho}\right)\right] \quad (58)$$

Temporarily let's assume that the baseline salary w is such that condition [58] is satisfied for $n = n(w)$. If [58] holds, then a sufficient condition for wage rigidity is obtained by substituting the second term of [58] into the left hand side of [WRC]. The revised wage rigidity condition is

$$\left[\frac{b\hat{\rho}-w}{1-\delta\left(1-\sigma(w,1)\right)} + b(\overline{\rho}-\hat{\rho})(1-\delta)\right] \geq \left[\frac{b\hat{\rho}-w}{1-\delta\left(1-\sigma(w,1)\right)} + b(\overline{\rho}-\hat{\rho})\right] \cdot \left[1-\delta\left(1-\sigma(w,1)\right)\right] \quad \text{(WRC2)}$$

According to [WRC2], wage rigidity obtains when the derivative of the firm's objective function with respect to the long-term component of the wage is (weakly) greater than the derivative with respect to υ (for every realization of $\tilde{\rho}$). The relative advantage of increasing υ over w is flexibility, because υ does not constraint the wage policy in the continuation game. The relative advantage of w over υ is static efficiency, because an increase in w affects the expectations of contacted workers and leads to a larger increase in size. The cost of constraining the future wage-policy depends on how volatile the firm's productivity is. The efficiency gains from adjusting the baseline salary are larger the longer is the expected duration of the employment relationship with respect to a transitory shock. Proposition [3] summarizes the analysis.

Proposition 3 *(High-Frequency Wage Rigidity)* Let $w \in [w_0, b]$ be a wage that satisfies [58] given $\{F(w), G(w), n(w)\}$. If one condition among (a)-(b) is satisfied, then the optimal wage policy is

$$(w^*(n(w), w, \rho; b), \upsilon^*(n(w), w, \rho; b)) = (w, 1)$$

for every $\rho \in [1, \overline{\rho}]$.

(a) *Long Employment Relations*: the separation rate $\sigma(w, 1)$ is smaller or equal than σ^*, where $\sigma^* \in (0, 1)$ is implicitly defined by the only positive root of

$$(b\hat{\rho} - w)(1 - \sigma^*) - b(\overline{\rho} - \hat{\rho})\sigma^*(1 - \delta(1 - \sigma^*)) = 0$$

(b) *Low Volatility*: the largest realization of the productivity shock $\overline{\rho}$ is smaller or equal than $\overline{\rho}^*$, where $\overline{\rho}^* \in (\hat{\rho}, +\infty)$ is defined by

$$\overline{\rho}^* = \hat{\rho} + \frac{1 - \sigma(w, 1)}{\sigma(w, 1)} \frac{1}{b} \frac{b\hat{\rho} - w}{1 - \delta(1 - \sigma(w, 1))}$$

(c) *High-Frequency Shocks:* the duration Δ of a transitory shock is smaller or equal than a cutoff $\Delta^* > 0$.

Proof. *The first two conditions are derived from rearranging [WRC2]. The latter condition is derived by substituting [56a]-[56c] into [WRC2] and checking that.for* $\Delta \to 0$ *[WRC2] holds.* ■

Empirically, the notion of *high-frequency wage rigidity* has found some support. In a recent analysis of an Italian matched employer-employee dataset, Guiso Pistaferri and Schiavardi (2003) are able to identify transitory and persistent shocks to value added at the firm level. While wage changes appear to be orthogonal to short-term shocks (MA(2)), they are correlated with unit-root shocks. Even though the authors stress the wage-insurance explanation, their finding fits the prediction of my theory as well.

3.3 General Equilibrium: Wage Dispersion and Acyclical Wages

In this final subsection, I show that the optimal firm-wage policy is a non-degenerate function of the persistent component of productivity b and derive the equilibrium wage distribution (Proposition [4]). Given the closed-form solution for $w(b)$ and $F(w)$, Proposition [5] contains a general equilibrium version of Proposition [3].

Before turning to the general equilibrium analysis, we have to address a technical issue. In the Signal Posting model of wage determination, the current choice of the long-term wage w constrains the feasible firm-wage choices in the future. The history dependence of firm-wages creates a multiplicity of equilibria, indexed by the (binding) constraint x. Nevertheless, some of these equilibria are not convincing because they assume that the firm has committed to a wage that it would have never been optimal to offer. Assumption [4a] formalizes this refinement and Proposition [4] shows that the surviving set of RWSE is a singleton.

Assumption 4 (*Equilibrium Refinement: No Over-Commitment*) For every $b \in \left[\underline{b}, \overline{b}\right]$, there is a state (e^*, x^*, ρ^*) in the ergodic set

$$X^e(b) = \{(e, x, \rho) \in X_1(b) : e = n(w(b)), x = w(b)\} \tag{59}$$

where the constraint on the wage w is not binding, i.e.

$$\frac{\partial Obj(e^*, x^*, \rho^*, w, \upsilon; b)}{\partial w}|_{\{w=w(b),\upsilon=1\}} \geq 0 \tag{ER}$$

3.3.1 Wage Dispersion

The derivative of the objective function with respect to w is non positive, because of the first order condition [55a], and is not always strictly negative, because of the equilibrium refinement in Assumption [4]. By combining the two conditions, we can conclude that [55a] holds with equality at $(n(w), w, \overline{\rho})$ and that

$$n = \left[\lambda(1-u)G'(w) + \lambda n(w)F'(w)\right]\left[\frac{(b\hat{\rho} - w)}{1 - \delta\left(1 - \sigma(w, 1)\right)} + (1-\delta)b\left(\overline{\rho} - \hat{\rho}\right)\right]$$

Substitute $G(w)$, $n(w)$, u, and $G'(.)$ from [33] [30] [35] and the previous expression becomes

$$2\lambda F'(w)\left(\frac{b\hat{\rho} - w}{1 - \delta\left(1 - \sigma - \lambda\overline{F}(w)\right)} + b(\overline{\rho} - \hat{\rho})(1-\delta)\right) - 1 = 0 \tag{60}$$

The left hand side of [60] in strictly increasing in b. Therefore if w_1 solves [60] for $b = b_1$, then the derivative of the objective function is strictly positive at $w = w_1$ for any $b_2 > b_1$. If the firm's problem is quasi-concave, the equilibrium wage function $w(b)$ is strictly increasing. The measure of firms paying a wage smaller or equal to w is

$$F(w(b)) = \Gamma(b) \tag{61}$$

The total differentiation of [61] returns a relationship between the wage offer density and the ratio between the density function of productivity and the local derivative of the wage function

$$F'(w)w'(b) = \Gamma'(b) = \gamma(b) \tag{62}$$

After substituting [61] and [62] into [60], we obtain an ordinary differential equation for the wage function

$$w'(b) = 2\lambda\gamma(b)\left(\frac{b\hat{\rho} - w(b)}{1 - \delta\left(1 - \sigma - \lambda\overline{\Gamma}(b)\right)} + b(\overline{\rho} - \hat{\rho})(1 - \delta)\right). \tag{63}$$

The initial condition for [63] is $w(\underline{b}) = w_0$, because $w(\underline{b}) > w_0$ would violate Assumption [4] and $w(\underline{b}) < w_0$ would violate individual rationality.

The explicit solution of the wage function $w(b)$ is

$$w(b) = b\hat{\rho} + (w_0 - \underline{b}\hat{\rho})\left(\frac{1 - \delta\left(1 - \sigma - \lambda\overline{\Gamma}(b)\right)}{1 - \delta\left(1 - \sigma\right)}\right)^{\frac{2}{\delta}} - \hat{\rho}\int_{w_0}^{b}\left(\frac{1 - \delta\left(1 - \sigma - \lambda\overline{\Gamma}(b)\right)}{1 - \delta\left(1 - \sigma - \lambda\overline{\Gamma}(x)\right)}\right)^{\frac{2}{\delta}} dx +$$
$$2\lambda(\overline{\rho} - \hat{\rho})(1 - \delta)\int_{w_0}^{b}\gamma(x)x\left(\frac{1 - \delta\left(1 - \sigma - \lambda\overline{\Gamma}(b)\right)}{1 - \delta\left(1 - \sigma - \lambda\overline{\Gamma}(x)\right)}\right)^{\frac{2}{\delta}} dx \tag{64}$$

As in Burdett-Mortensen, the wage function is non-degenerate if and only if there is a strictly positive probability that a worker has two wage offers at the same time (i.e. if and only if $\lambda > 0$).[10] More generally, the efficiency λ of the search process determines the competitiveness of the labor market by increasing the average acceptance wage of a contacted worker and pushes the wage distribution to the right (in the sense of first-order stochastic dominance).

Proposition 4 *(Wage Dispersion and Profits)* Under the equilibrium refinement in Assumption [4], in any RWSE the firm-wage policy is

$$w^*(n(w(b)), w(b), \rho; b) = w(b)$$

(i) The wage function $w(b)$ is strictly increasing in b, if and only if $\lambda > 0$;

(ii) The wage offer distribution $F(w)$ is non-degenerate, if and only if $\lambda > 0$;

(iii) There is a $\delta^* \in (0, 1)$ such that, for every $\delta \geq \delta^*$ and every $b > \underline{b}$, the wage $w(b)$ is strictly increasing in λ.

[10] Indeed, for $\delta \to 1$ and $\overline{\rho} = \hat{\rho} = 1$, the wage function $w(b)$ coincides with the solution of the original Burdett-Mortensen wage posting game.

Proof. *Parts (i),(ii) are an immediate implication of [64]. The derivative of* $w(b)$ *with respect to* λ *is the sum of three strictly positive and one negative term associated with the last term of [64]. The sign of the derivative is unambiguously positive whenever* $\delta \geq 1 - \hat{\rho}\left(2\hat{b}(\overline{\rho} - \hat{\rho})\right)^{-1}$, *where* $\hat{b}$ *is the mean of* b. ■

Proposition [4] shows that the firm-wage policy $(w^*(.), v^*(.))$ is a non-trivial function of the long-term component b of labor productivity. This result is a direct implication of high-frequency wage rigidity, if we interpret the productivity component b as a persistent shock to productivity. According to [57], if the expected duration of a shock is above a critical cutoff, the optimal wage policy is interior and, therefore, depends (positively) on the realization of the shock.

3.3.2 Acyclical Wages

The mechanism behind high-frequency wage rigidity is that, if the expected duration of the underlying productivity shock is short, the firm's optimal strategy is to announce that the lowest realization has occurred independently of the true realization of ρ. Consistently, workers consider the announcement uninformative and the resulting wage is not a function of ρ. When the transitory productivity shock is the idiosyncratic response to an economy-wide shock, workers learn about the firm's productivity based on the public signal z even when the firm's announcements are uninformative. But unlike information that is voluntarily disclosed by the firm, public information does not necessarily affect the wage outcome of the bargaining game and, in the environment described in Section [2], it does not. The literature on asymmetric information bargaining (see for instance Fudenberg, Levine and Tirole (1983) and Grossman-Perry (1986b)) shows that the equilibrium terms-of-trade converge to the outcome of a full information bargaining game between the the lowest type of firm and the worker, as the delay between successive offers shrinks. Even if the average of productivity is procyclical, this information does not translate into procyclical wages.

Letting $F(w)$ and $w(b)$.be determined by [61] and [64], Proposition [5] derives three sets of sufficient conditions under which there exists a Rigid Wages Sequential Equilibrium. The three conditions are more restrictive versions of conditions (a)-(c) in Proposition [3], where the extra qualifications are introduced to guarantee that the wage function satisfies the sequential equilibrium restriction $w(b) \leq b$.

Proposition 5 *(Existence of a Rigid-Wages Sequential Equilibrium)* Under the equilibrium refinement in Assumption [4], there exists a RWSE if one of the conditions (a)-(c) is satisfied.

(a) *High-Frequency Business Cycle.* The duration Δ of an aggregate shock belongs to the non-empty interval $(0, \Delta^*(\lambda)]$ and the job-finding rate is $\lambda \in (0, \lambda^*]$.

(b) *Long Employment Relations.* For any rate of unemployment $u^* \in (0, 1)$, the separation rate σ belongs to the non-empty interval $(0, \sigma^*]$.

(c) *Small Shocks.* The upper bound on the productivity shock $\overline{\rho}$ belongs to the non-empty interval $(\hat{\rho}, \overline{\rho}^*(\lambda)]$ and the job-finding rate is $\lambda \in (0, \lambda^*]$.

Proof. *In Appendix [B].* ■

When the conditions in Propositions [4] and [5] are simultaneously satisfied, the model predicts wage dispersion and acyclical wages. Both results are general equilibrium implications of the same high-frequency wage rigidity property of the firm-wage policy, when aggregate shocks are not too persistent with respect to employment relationships and firms are heterogeneous in the long-term component of productivity. Not only a single theory about wage determination captures two important stylized facts about the labor market[11], it also shows how one supports the other. In fact, the high-frequency wage rigidity argument, formalized in [57], requires the distribution of workers' acceptance wages being dispersed and independent of Δ. Indeed, the allocative effect of a wage raise associated with a business cycle shock falls as the frequency of the cycle grows only if the dispersion in the value of job offers remains constant.

Both Proposition [4] and [5] are relevant to the main macroeconomic application of the theory: the increase in the volatility of the return from filling a vacancy. While a model featuring wage rigidity constitutes a qualitative improvement with respect to a procyclical wages benchmark, quantitatively its amplification power depends on the level of wages. The closer the acyclical wage is to the productivity of labor, the larger is the impact of a productivity shock on the value of filling a vacancy for a firm. Proposition [5] shows that the wage level depends on the efficiency of the job finding rate and Proposition [4] shows that the cyclical behavior of wages is controlled by the persistence of aggregate shocks. In principle the model is compatible with wages that are high and acyclical.

[11] There is a wealth of empirical evidence that similar workers are paid different wages and a survey of recent findings is provided in the introductory chapter of Mortensen (2003). Even less controversial is the view that wages are nearly acyclical (among many others see Kydland (1995) or Shimer (2005)).

4 Endogenous Vacancies: A Quantitative Analysis

The previous section has shown that the optimal firm-wage policy is independent of the realization of short-term shocks to productivity, and the equilibrium wage distribution depends positively on the degree of competitiveness of the labor marker as measured by the job-finding rate. Since those results are obtained under the assumption of exogenous search intensity, they are not immediately relevant to address the issue of unemployment fluctuations.

This section generalizes the model economy of Section [3] by endogenizing the search intensity of the firm. Following Mortensen (2000), I assume that the firm chooses the measure of vacancies to advertise and faces an increasing and convex cost function. In equilibrium, the job-finding rate λ_t depends on the aggregate level of vacancy. Even though this generalization is conceptually obvious and does not affect the qualitative results derived in Section [3], it dramatically complicates the analytic derivation of the closed-form solution of the model because the wage distribution $G_t(w)$ enters as a state variable of the economy. The model can still be solved computationally by approximating the wage distribution with a finite number of statistics.

After the steady-state of the economy is solved for, the model is linearized and calibrated to the U.S. data. The summary statistics of the model are then compared to their empirical counterparts. Three main findings obtain from this quantitative exercise. First, the calibrated parameters of the model satisfy the wage rigidity condition and the simulated semi-elasiticty of wages to unemployment is consistent with the nearly acyclical behavior of wages observed in the data. Secondly, wage rigidity increases the volatility in the rate of return from filling a vacancy, which in turn induces wider fluctuations in the job-finding rate. The simulated semi-elasticity of average labor productivity to unemployment is close to its empirical counterpart. Finally, the model induces a strong negative correlation between unemployment and vacancies (-0.95 at one quarter lag) that is consistent with the empirical Beveridge curve.

4.1 The Reduced-Form Model

To obtain interesting business cycle predictions out of the assess theory of high-frequency wage rigidity, the model in Section [3] has to be generalized to allow for endogenous search intensity and for positively correlated aggregate shock. Since both modifications are conceptually straightforward, I only discuss the main differences they introduce in the reduced-form model.

Correlated Aggregate Shocks. In order to calibrate the model to quarterly data, it is obviously necessary to introduce some persistence in the process of aggregate shocks. Specifically, I consider the two-state Markov switching process represented by the following transition matrix

z_t/z_{t+1}	z_1	z_2
z_1	π	$1-\pi$
z_2	$1-\pi$	π

(65)

In words, the aggregate shock takes on two values $\{z_1, z_2\}$ and switches from one state to the other with probability $1-\pi$.

In turn, the aggregate state z_t affects the probability distribution $\Phi(\rho|z_t)$ of firm specific shocks

$$\rho_{i,t} = \begin{cases} \rho_{i,t-1} & \text{if } z_t = z_{t-1} \\ \tilde{\rho} \sim \Phi(\rho|z_t) & \text{if } z_t \neq z_{t-1} \end{cases} \tag{66}$$

To simplify the analysis is useful to approximate firm heterogeneity by the permanent productivity type b only. The approximation gets arbitrarily accurate as we let $\Phi(\rho|z)$ converge to a degenerate distribution with a mass point at the mean $\hat{\rho}(z)$. This is the limit case discussed in this section.

Endogenous Search Intensity. In order to verify the intuition that wage rigidity amplifies productivity shocks, it is necessary to allow for endogenous search intensity on the firm side. Following Mortensen (2000), I posit a vacancy cost function $c(.) : \mathbb{R}_+ \mapsto \mathbb{R}_+$ that satisfies the standard convexity and interiority conditions. Specifically, the function is such that $c(0) = 0$, $c'(0) = 0$, $c'(v) > 0$ for all $v \in \mathbb{R}_{++}$, and $c''(v) > 0$ for all $v \in \mathbb{R}_+$.

The firm-specific vacancies v_i are aggregate by integration

$$v = \int_0^1 v_i di \tag{67}$$

The resulting economy-wide vacancy v is the input of a meeting function $\lambda(.) : \mathbb{R}_+ \mapsto \mathbb{R}_+$ that returns the measure of encounters occurring in one period. Formally, I let $\lambda(v)$ be

$$\lambda(v) = \min\{Av^\alpha, 1\} \tag{68}$$

The parameter $A > 0$ controls the efficiency of the search process and $\alpha \in [0, 1]$ measures the negative externality (congestion effect) caused by vacancies on each other. The meeting function is bounded above by the measure of workers searching for jobs (which coincides with the entire population and is normalized to one), to guarantee that every worker receives at most one job offer in every period. Besides being the measure of contacts created in a period of time, $\lambda(v)$ is also the probability that a worker receives a job-offer because of the random-search assumption.

Aggregate Dynamics. Unlike in Section [3], the state of the economy ω is non-degenerate. The sufficient statistics of the economy at date t are the unemployment level u_t, the allocation of workers across firms $G_t(w)$ and the realization of the productivity shock z_t:

$$\omega = (u, G(w), z) \tag{69}$$

In order to derive the aggregate laws of motion for [69], it is useful to denoting with $w(b)$ and $v(b;\omega)$ the wage and vacancy policy functions of a firm b in state ω. The unemployment dynamics are given by

$$u'(\omega) = u + (1-u)\sigma - \lambda\left(v\left(\omega\right)\right)u \tag{70}$$

where $v(\omega)$ denotes total vacancies in state ω. The law of motion for the wage distribution is given by the following expression

$$\left(1-u'(\omega)\right)G'(w;\omega) = (1-u)G(w)(1-\sigma(w,1;\omega)) + h(w_0,1;\omega)\int_{\underline{b}}^{b^{-1}(w)} v\left(b;\omega\right)d\Gamma(b) \tag{71}$$

where $\sigma(w;\omega)$ is the separation rate at a firm that pays the wage $(w,1)$ and $h(w_0;\omega)$ is the hiring rate (per-vacancy) at a firm that attracts unemployed workers only. More specifically, the hiring and separation functions are respectively

$$\begin{aligned} h(w,v;\omega) &= \frac{\lambda\left(v(\omega)\right)}{v(\omega)}\left(u+(1-u)G(w+A\left(w,v;\omega\right))\right) \\ \sigma(w,v;\omega) &= \frac{\lambda\left(v(\omega)\right)}{v(\omega)}\int_{b^{-1}(w+A(w,v;\omega))}^{\bar{b}} v\left(b;\omega\right)d\Gamma(b) \end{aligned}$$

The Problem of the Worker. The functional equations describing the value of the employment status of a worker are self-explanatory in light of the analysis in Section [3]:

$$U(\omega) = w_0 + \delta E_{\omega'}\left[\lambda\left(\omega'\right)\int_{w_0}^{\bar{b}} V(\tilde{w};\omega')dF(\tilde{w}) + \left(1-\lambda\left(\omega'\right)\right)U(\omega)\right]$$

$$V\left(w;\omega\right) =$$
$$w + \delta E_{\omega'}\left[\sigma U(\omega') + \lambda\left(\omega'\right)\int_{w}^{\bar{b}} V(\tilde{w};\omega')dF(\tilde{w}) + \left(1-\sigma-\lambda\left(\omega'\right)\bar{F}(w)\right)V\left(w;\omega'\right)\right]$$

Finally, the value of an off-equilibrium job offer (w, υ) is denoted with $V^+(w, \upsilon; \omega)$ and

$$V^+(w, \upsilon; \omega) =$$

$$w\upsilon + \delta\pi E_{\omega'}\left[\begin{array}{l} \sigma U(\omega') + \lambda(\omega')\int_{w+A(w,\upsilon;\omega')}^{\bar{b}} V(\tilde{w};\omega')dF(\tilde{w}) + \\ \left(1-\sigma-\lambda(\omega')\bar{F}(w+A(w,\upsilon;\omega'))\right)V^+(w,\upsilon;\omega')\,|z'=z \end{array}\right] +$$

$$\delta(1-\pi)E_{\omega'}\left[\begin{array}{l} \sigma U(\omega') + \lambda(\omega')\int_{w}^{\bar{b}} V(\tilde{w};\omega')dF(\tilde{w}) + \\ \left(1-\sigma-\lambda(\omega')\bar{F}(w)\right)V(w;\omega')\,|z'\neq z \end{array}\right]$$

The second term on the right-hand-side of the expression above is the continuation value when the aggregate state z' keeps its previous value z and the third term is the continuation value for $z' \neq z$. In the first case the firm is expected to offer (w, υ), while it reverts to $(w, 1)$ in the second case. As in Section [3], the annuity value of the deviation is implicitly given by

$$V(w + A(w, \upsilon; \omega); \omega) = V^+(w, \upsilon; \omega)$$

The Problem of the Firm. A simple generalization of the arguments developed in the Appendix leads to the following constraint on the firm-wage policy

$$D(x_1, x_2) = \{(w, \upsilon) \in [\underline{b}, \bar{b}] \times [1, \overline{\rho}] : (w, \upsilon) \geq (x_1, x_2)\} \tag{72}$$

With respect to the constraint in the i.i.d. model, both components of the wage are downwardly rigid. Intuitively, when the firm increases the transitory component of the wage by disclosing information about the value of ρ, it affects the bargaining strategy of the worker for as long as the underlying aggregate shock z remains unchanged.

The recursive formulation of the firm's optimization problem is

$$\begin{aligned} \Pi(e, x_1, x_2, \rho; \omega, b) &= \max_{\{v,w,\upsilon\}} -c(v) + n\max\{\rho b - w\upsilon, 0\} + \delta E\left[\Pi(e', x_1', x_2', \rho'; \omega', b)\right] \quad (73) \\ (w, \upsilon) &\in D(x_1, x_2) \\ x_1' &= w \\ x_2' &= \upsilon \text{ if } z' = z,\ x_2' = \upsilon \text{ if } z' \neq z \\ e' &= n = e(1 - \sigma(w, \upsilon; \omega)) + h(w, \upsilon; \omega)v \end{aligned}$$

where $\sigma(w, \upsilon; \omega)$ and $h(w, \upsilon; \omega)$ are the hiring and separation rates for a wage offer (w, υ).

Solution Method. The first step involves resolving the non-stochastic steady-state of the model, where the aggregate shock is a constant $\hat{z} = (z_1 + z_2)/2$ and the transitory shock to productivity is $\hat{\rho} = (\hat{\rho}(z_1) + \hat{\rho}(z_2))/2$. The solution of the stationary model follows closely the procedure described in Section [3] with the exception of the wage selection rule. Under Assumption [4], the wage is pinned down by verifying that the constraint x is just binding over the the ergodic set of the firm state-space. In a non-stationary environment, this procedure is computationally burdensome. Therefore, I replace Assumption [4] with a refinement based on an entry/exit tremble of the model. The wage is determined by the optimal choice of an unconstrained firm that enters the market in steady-state.

Next, the model is linearized about the non-stochastic steady-state and the resulting linear dynamical system is resolved by identifying the unique non-explosive path. This exercise is not standard because the wage distribution $G(w)$ is a state variable of the model. Therefore, I discretize $[\underline{b}, \overline{b}]$ into K intervals with cutoffs $\{b_k\}_{k=1}^{K+1}$, that satisfy the restriction $b_1 = \underline{b}$ and $b_{K+1} = \overline{b}$. A wage distribution $G(w)$ is approximated by the parametric form

$$\hat{G}\left(w; \{G_k\}_{k=1}^{K+1}\right) = \sum_{k=1}^{K} \mathbf{1}(w, b_k, b_{k+1}) \left[G_k + (G_{k+1} - G_k) \frac{w - w(b_k)}{w(b_{k+1}) - w(b_k)}\right] \tag{74}$$

where $\mathbf{1}(w, b_k, b_{k+1})$ is an indicator function that takes the value 1 iff $w \in (b_k, b_{k+1}]$. Using the linear approximation [74] and noticing that $G_1 = 0$ and $G_{K+1} = 1$, the dimensionality of the state of the economy can be reduced to a subset of $\mathbb{R}^{K+1}$. The laws of motion for $\{G_{t,k}\}_{k=2}^{K}$ are derived by applying [74] to [71] for $\{w(b_k)\}_{k=2}^{K}$. Similarly, the vacancy policy $v(b; \omega)$ can be approximated by

$$\hat{v} = \sum_{k=1}^{K} \mathbf{1}(w, b_k, b_{k+1}) \left[v_k + (v_{k+1} - v_k) \frac{b - b_k}{b_{k+1} - b_k}\right] \tag{75}$$

where $v_k = v\left(b_k; u, \hat{G}\left(w; \{G_k\}_{k=1}^{K+1}\right), z\right)$.

In the approximated model, there are $K+1$ state variables and $K+1$ control variables given by the vacancy policy of firms $\{b_k\}_{k=1}^{K+1}$. The firm's search intensity is determined by the first order condition of [73] with respect to v:

$$c'(v) = h(w, 1; \omega) \left\{b\rho - w + \delta E_{\omega', \rho'} \left[\Pi_1(e', x_1', x_2', \rho'; \omega', b)\right]\right\} \tag{76}$$

The expression above leads to an intertemporal Euler Equation. By the envelope theorem, the derivative of the value function with respect to a worker is

$$\Pi_1(e, x_1, x_2, \rho; \omega, b) = (1 - \sigma(w, 1; \omega)) \left\{b\rho - w + \delta E_{\omega', \rho'} \left[\Pi_1(e', x_1', x_2', \rho'; \omega', b)\right]\right\} \tag{77}$$

Substituting [77] into [76], we obtain the expression

$$\Pi_1(e, x_1, x_2, \rho; \omega, b) = c'(v) \frac{(1 - \sigma(w, 1; \omega))}{h(w, 1; \omega)}$$

and the intertemporal Euler Equation

$$c'(v(b;\omega)) = h(w,1;\omega)\left\{b\rho - w + \delta E_{\omega',\rho'}\left[c'(v(b;\omega'))\frac{(1-\sigma(w,1;\omega'))}{h(w,1;\omega')}\right]\right\} \tag{EE}$$

The linearization of [EE] for $\{b_k\}_{k=1}^{K+1}$ returns $K+1$ equations that, along with the laws of motion for the $K+1$ state variables, constitute a dynamical system $x_{t+1} = Bx_t$. The $K+1$ control variables in x_t are uniquely pinned down by the transversality conditions associated to the $K+1$ unstable roots of the system. In the final step of the analysis, the wage rigidity condition is numerically verified for every firm type b.

4.2 A Quantitative Exercise

The calibration of the dynamic model of on-the-job search discussed above requires identifying more parameters than a standard matching-and-bargaining model.[12] On the other hand, allowing for job-to-job transitions and firm-wage heterogeneity induces a richer set of implications.

4.2.1 Parametrization of the Model

In Table [2], the parameters of the model are listed together with the selected value and a brief motivation. In this subsection, the interested reader can find a more detailed description of the evidence and logic behind the choices that have been made.

Preferences. As discussed in Section [2], the assumption of risk-neutral agents is justified by a perfect credit market and the discount rate δ on future payoffs is identified by the inverse of the interest rate. The quarterly discount factor is set to $\delta = 0.9847$ in order to match an annual interest rate of 0.06.

When a worker is unemployed, she receives a benefit and can consume home-produced goods. The value of this alternative to the labor market is denoted by w_0 and is normalized to 1.

Search Frictions. In the standard matching-and-bargaining model, only unemployed workers search for jobs. Given the measure of unemployed and total vacancies, the number of jobs created in a period is given by a matching function. A large number of empirical studies has estimated the matching function in different countries and with different identifying

[12] The matching-and-bargaining model (dubbed DMP after Diamond, Mortensen and Pissarides) is presented in Appendix [C].

assumptions. On the contrary, there is very little empirical work that studies the properties of contact or matching functions between workers (employed and not) and firms[13].

In order to select the parameters in [68], we can appeal to a theoretical argument. As noticed in Petrongolo and Pissarides (2001), an estimate of the elasticity of matches (or meetings) to vacancies that uses only data about the flows from unemployment is biased downward if the real meeting function is constant returns to scale. Therefore, Hall's (2003) point estimate of 0.765 can be used as a lower bound on the coefficient α in [68]. The argument can be strengthened in the specific case of this paper because [68] returns contacts rather than new matches. In a contact model, vacancies create a congestion effect by increasing the endogenous separation rate of other matches and reducing the acceptance probability of a job offer. On the other hand, a matching function is a reduced-form that summarizes the positive and negative externalities of the search process. For this reason, setting $\alpha = 1$ in [68] seems the most theoretically consistent approach.

The parameter A, that controls the efficiency of information transmission in the labor market, is set to 4.3 in order to match a steady-state unemployment rate of 6.5 (the long-term average for the U.S. reported in Merz 1995). In order to parametrize the exogenous displacement rate σ, I match σ to the average (and remarkably acyclical) layoff rate reported in Hall (2005) from JOLTS data.

Vacancy Costs and Productivity. Another important difference between the model studied in this paper and the matching-and-bargaining benchmark regards the formalization of vacancy costs. In the context of this paper, a linear cost function induces an equilibrium where only the most productive firm posts vacancies and there is ex-post full information. To allow for substantial heterogeneity, it is useful to let the vacancy cost function be strictly concave. More specifically, I consider the case of a function $c(v)$ with first derivative

$$c'(v) = \left[\left(\frac{1}{1-v} \right)^{\eta} - 1 \right]$$

defined on the domain $[0, 1]$. The general equilibrium implications of the model appear to be extremely robust to the specific value of η, the parameter that controls the convexity of $c(v)$. In the baseline simulation I tentatively set $\eta = 0.5$.

Mortensen (2003) reports the firm-size density observed in the Danish dataset IDA. It is apparent that in the Danish labor market the vast majority of firms are small: 62 percent have four or fewer employees and only 3 percent have more than 50 workers. Given

[13] A contact or meeting function returns the number of meetings between workers and firms created in a period of time. On the other hand, a matching function returns a number of jobs.

the prediction of this model on the relationship between productivity and size, a Pareto distributions $\Gamma(b;\psi) = 1 - b^{-\psi}$ should not give a bad approximation of the productivity distribution. The parameter ψ is chosen in order to capture the extent of the critical feature of the empirical wage distribution: dispersion. Among others, Bowlus and Neumann (2004) report that the empirical wage distribution for white male high-school graduates ten years after exiting school is significantly dispersed. The third and first quartiles, w_{75} and w_{25}, satisfy approximately the relationship $\left(F(w_{75}) - F(w_{05})\right)\left(F(w_{25}) - F(w_{05})\right)^{-1} \simeq 2$ (see Bowlus and Neumann, Figure 1). By setting $\psi = 10$, the model matches this empirical stylized fact[14]. Finally, following Hall (2003), I introduce a constant (per-worker) intermediate good cost in order to match an average labor share of 0.55.

Aggregate Shocks. The volatility and persistence of the aggregate shocks are calibrated to match the standard deviation and the quarterly autocorrelation of output, respectively. Specifically, the ratio $\hat{\rho}(z_2)/\hat{\rho}(z_1)$ is such that the simulated percentage standard deviation of the detrended logarithm of aggregate output matches its detrended and HP-filtered empirical counterpart. This statistic is 1.87 based on Citibase data over the period 1954:I-1988:II (see Merz (1995) or Kydland (1995)). The persistence π of the aggregate shock is calibrated so that the model's predicted autocorrelation of aggregate output is 0.875, the value reported by Merz for the 1954:I-1988:II period.

4.2.2 Simulation Results

The Steady-State The main finding of the steady-state analysis is that profits-per-worker are small (relative to output) for the vast majority of firms. Theoretically, as discussed in BM98 and in Section [3], on-the-job search induces an element of competition between firms and drives the wage distribution up (in the sense of first order stochastic dominance). Quantitatively, the efficiency of the search process is indirectly identified by the exogenous separation rate σ and the steady-state level of unemployment. In the baseline simulation, I have set $\sigma = 0.04$ per quarter based on the JOLTS data on layoffs reported in Hall (2004). Shimer's analysis of the employment to unemployment transition in the CPS dataset is very close to Hall's estimate. Graphically, Figure [3] reports the wage function $w(b)$ which has the familiar concave shape. The lower curve represents the CDF of firm types b weighted for search intensity. The wage offer distribution, derived from the

[14] The selected statistic is the ratio of the third quartile net of the fifth percetile and the first quartile net of the fifth percentile. The reader might be surprised that I have chosen this statistic over the third to first quartile ratio. Since in the model the reservation wage w_0 is normalized to one, the correction is necessary.

vacancy-weighted productivity distribution and the wage function, is depicted in Figure [4].

The job-to-job transition probability depends on the current employment status of a worker. The simulated average quitting probability is 11.2 percent per quarter, which is not too far from the 9 percent probability of employment-to-employment transitions found by Nagypál (2004) in the CPS after correcting for short-term transitions from employment to out of the labor force back into employment. In the model, the quitting probability is decreasing with the worker's position in the wage ladder and ranges from 58 percent for a worker employed at the reservation wage to less than one percent for employees above the 9th decile of the wage offer distribution.

Layoffs and quits transforms the wage offer distribution into the earned wage distribution, as illustrated in Figure [4]. The simulated wage distribution first order stochastically dominates the offer distribution, which is a correct empirical prediction. Quantitatively, the ratio of the median of the wage distribution over the median of the offer distribution is about 30 percent. The equilibrium wage distribution matches the interquartile ratio observed in the data, even though the underlying productivity distribution is very concentrated. While the vast majority of firms are small and offer a low wage, the few high-wage firms account for a disproportionately large fraction of employment since they search more intensely and effectively attract workers from low-wage employers.

Given the steady-state wage distribution $G(w)$, it is possible to compute the return on posting a vacancy and decompose it into three determinants: the acceptance rate of a posted vacancy, the profit rate on each employee and the expected duration of the employment relationship. Figure [5] plots the return on a posted vacancy as a function of the firm type b. The function is strictly monotonic and approximately convex: the return-on-vacancy for a firm at the 90th percentile of the wage distribution is six times the return at the 10th percentile. Since more productive firms offer higher wages, the acceptance rate per contact is increasing and the separation rate decreasing in b. The profit rate is 1.5 percent for low productivity firms and grows (non-monotonically) up to 5 percent at the top percent of the distribution.

Dynamics The model economy is approximated by an 11-point interpolation of the wage distribution and then linearized about the non-stochastic steady-state. Then, the cyclical properties of the dynamical system are analyzed by performing two numerical exercises. First, the system is simulated for 10,000 periods and sample statistics are derived and compared to their empirical counterparts. Secondly, the steady-state is hit with a one percent productivity shock and the resulting response functions are derived to gauge the amplification and propagation properties of the model. The results of these two exercises are reported in Figures [6]-[14] and in Tables [3] and [4].

Acyclical Wages. As discussed at length in Section [3], wage rigidity obtains when aggregate shocks are sufficiently small and short-lived. In the calibrated version of the model, these two features of the driving force are pinned down by the empirical autocorrelation and volatility of output. In order to match the quarterly autocorrelation of GNP in the U.S. data, the aggregate shock z must have a 0.75 autocorrelation. In order to match the standard deviation of GNP, the standard deviation of the logarithm of the productivity innovation $\log(z_{t+1} - z_t)$ has to be 0.0046. Given this process for aggregate shocks, the wage rigidity condition is satisfied for every productivity type $b \in \left[\underline{b}, \overline{b}\right]$.

Even though the wage policy of each firm is acyclical, the aggregate wage is somewhat responsive to productivity shocks because of composition effects, as illustrated by the impulse response function in Figure [6]. On impact, the wage responds to a positive productivity shock by falling by 0.2 percent and later it slightly overshoots the steady-state level. At the beginning of the expansion, there are larger-than-normal inflows of unemployed workers that tend to shift the wage distribution down. At the same time, the increase in high-paying jobs induces faster-than-normal job-to-job transitions. The net effect is negative in the first year after the shock and positive afterwards.

In the model, the simulated semi-elasticity of the wage to the unemployment rate (formally, the coefficient on a regression of the log deviation of the wage from its trend on the absolute deviation of the unemployment rate from its trend) is 0.13. The average wage increases by 0.13 percent in response to one percentage point increase in the unemployment rate. When I compute the coefficient on the first difference regression, the estimated semi-elasticity is 0.0001 and not significantly different from zero. Empirically, Shimer (2003) studies the semi-elasticity of wages to unemployment for various definitions of labor compensation: -0.53 for the Average Hourly Earnings (from the CES), -0.13 for the Average Hourly Compensation (NIPA), 0.10 for the Employment Cost Index (BLS). Indeed, the model's predictions about the wage are consistent with the data and are a significant improvement over the matching-and-bargaining benchmark, which predicts a semi-elasticity of -4.89.

Procyclical Vacancies. The empirical negative relationship between vacancies and unemployment is a well-established fact known as the "Beveridge Curve." Using the Conference Board help-wanted advertising index as a proxy for vacancies, the quarterly contemporaneous correlation between the two variables is -0.95. In the calibrated model, the correlation between vacancies and the unemployment rate lagged by one quarter is -0.94. The contemporaneous correlation is only -0.61, a phenomenon that is attributable to the discreteness of time in the simulation. In the DMP model, the contemporaneous correlation is small (-0.11) and the one-lag correlation is -0.57. In Merz' matching-and-bargaining model with physical capital[15], the contemporaneous correlation is small -0.15 and, in the dynamics, the negative relation between vacancies and lagged unemployment is missing (see Table [3]).

As recently stressed by Hall (2003), recruiting effort is remarkably volatile and it is not uncommon for advertising to fall by 50 percent from peak to trough. The ratio of the standard deviation of vacancies to GDP is 7.31, which is well approximated by the prediction of my model 7.83. Consistently, Figure [7] shows that aggregate vacancies increase by 20 percent in response to a one percent increase in productivity[16]. The response in recruiting effort to productivity shocks varies markedly with the position of a firm in the productivity ladder. The profit-rate at a low-productivity firm is the smallest, and the productivity shock has the largest impact on the return-on-vacancy. Moreover, a low-productivity firm searches at the lowest intensity and therefore movements of the return are amplified the most into vacancy movements. On impact, the number of vacancies posted by the least productive firm increases by 38 percent. This impulse response function is represented by the V00 curve in Figure [7]. The behavior of a high-productivity firm is indeed quite different: the number of vacancies posted by a firm in the top percentile of the productivity distribution increases by a mere 0.5 percent.

Sullying Recessions and Propagation. Nagypál (2004) finds that expansions are times when the job-to-job transition rate is above average. Given the assumption that vacancies do not create (direct) congestion effects, the model is consistent with the empirical procyclical behavior of job-to-job transition rates. Moreover, since workers move from low to high paying jobs and the wage function $w(b)$ is increasing in productivity, the increase in job-to-job transitions is associated with a positive composition effect. This phenomenon had been previously identified by Barlevy (2002) and dubbed “sullying effect” of a recession. The

[15] Merz' model is an anomalous example of search model because it introduces capital in the environment. In the Appendix, she shows that the social planner problem has an equivalent representation in terms of a decentralized economy where wages are set as the outcome of a bargaining game between the marginal worker and the firm. This is not the most obvious extension of the Nash Bargaining Solution when firms deal with multiple workers as clearly shown by Stole and Zwiebel (1996 a,b). Nonetheless, the wealth of simulation data in Merz makes it a quite complete point of comparison.

[16] In the DMP mode, the vacancy rate grows by 6 percent in respose to a 1 percent increase in productivity. In Merz' model, the vacancy respose is4.5 percent.

beginning of an expansion is also characterized by a negative composition effect, because the relative response of low and high productivity firms induces a fall in the wage offer distribution at a time when the inflow from the unemployment pool is higher than normal. This phenomenon is the well-known "cleansing effect" of a recession. The dynamics of the composition effect are reported in Figure [9]. The three curves represent the impulse response function in the percentage of workers employed at a wage lower than the cutoffs w_{10}, w_{50} and w_{90}, where $w_{10} < w_{50} < w_{90}$ are the first, fifth and ninth deciles of the steady-state wage distribution. Two quarters after the shock, the percentage of workers employed at a wage below w_{10} is 0.105 and the percentage of workers employed for w_{90} or less is 0.901. The wage distribution is lower than normal, in the sense of first order stochastic dominance. After six quarters, the percentage of workers employed at w_{10} or less is below steady-state as the sullying effect takes over the initial cleansing effect. After twelve quarters, the wage distribution first order dominates the steady-state distribution.

The slow-moving dynamics of the composition effect account for the internal propagation of the model. Setting an autocorrelation of productivity shocks at 0.75 per quarter, the resulting autocorrelation of output is 0.875. This result contrasts favorably with the finding Merz' matching-and-bargaining model, where the persistence of the driving force is 0.95 and the autocorrelation of output is only 0.781, or with the DMP model where the output autocorrelation is 0.74. The Signal Posting model displays more propagation than the frictionless model with indivisible labor. In the calibration presented by King and Rebelo (1999), the persistence of GNP is matched by setting the autocorrelation of the driving force at 0.9892.

Unemployment Fluctuations and Amplification. As pointed out by Shimer (2005) and Hall (2005), wage rigidity increases the volatility of unemployment by increasing the elasticity of the return-on-vacancy to productivity shocks. Quantitatively, the amplification power of rigid wages depends on the steady-state profit rate and on the congestion effects induced by an increase in total vacancies. As discussed earlier, the search process is efficient and the profit rates are very small for all firms. The congestion effects created by the 20 percent increase in vacancies in reponse to a 1 percent increase in productivity are represented in Figure [10] and [11]. In Figure [10], the curves represent the impulse response functions (as absolute deviations from the steady-state) in the separation rate at three firms that are, respectively, at the second fifth and eigth decile of the steady-state wage distribution. As expected the lower is the position of a firm in the wage ladder, the stronger is the increase in separation rates caused by an expansion. Similarly, Figure [11] reports the effect of the boom on the acceptance rate faced by these three different firms.

Notwithstanding the congestion effects, the amplification induced by wage rigidity is quite powerful. Figure [6] shows that the unemployment rate falls by 0.8 percentage points in response to a 1 percent increase in productivity and, as a result, output grows by . The estimated semi-elasticity of average labor productivity with respect to unemployment is -0.82, a value closer to Shimer's estimate of -0.34 than the prediction of the standard Nash Bargaining model, which belongs to the range $[-5, -2.5]$ for all reasonable parametrizations. Another way to evaluate the amplification mechanism of the model is to compare simulation and estimates of the relative standard deviation of unemployment and labor productivity with respect to the standard deviation of output. In the baseline calibration, the relative volatility of unemployment is 6.45 and the relative volatility of average labor productivity is 0.64, which are both very close to their empirical counterparts 6.11 and 0.68 (see Table [4]). On the other hand, the DMP model predicts a much smaller volatility of unemployment (2.56) and a higher volatility of labor productivity (0.89).[17]

5 Conclusions

In this paper, I have advanced a novel theory of wage determination in frictional labor markets. Technically the wage is determined as the stable outcome of an intra-firm bargaining game under asymmetric information, but the model has a reduce-form where the firm chooses a wage subject to a non-discrimination and consistency constraints. The characterization of the optimal wage policy leads to the fundamental result of the paper: high-frequency wage rigidity. In a model where workers search off and on the job, the allocative power of a transitory wage raise falls with the expected duration of the underlying productivity shock. On the other hand, the non-discrimination principle keeps the marginal cost of a wage increase strictly positive and proportional to the size of the firm's workforce. The optimal firm-wage policy is independent of the realization of productivity shocks whose persistence falls below a critical threshold. The common wage is the minimum that workers are willing to accept, given the information about the acyclical component of productivity that the firm has disclosed in the past.

[17] For the sake of completeness, Merz's model predicts 4.63 and 0.74 for the relative volatility of unemployment and labor productivity.

References

[1] Abowd, J., F. Kramarz, D. Margolis, 1999. *High Wage Workers and High Wage Firms*. Econometrica, 67.

[2] Acemoglu, D., 1995. *Asymmetric Information Bargaining and Unemployment Fluctuations*. International Economic Review, 36

[3] Baker, G., M. Gibbs, B. Holmstrom, 1994. *The Wage Policy of a Firm*. Quarterly Journal of Economics, 109 (4).

[4] Barlevy, G., 2002. *The Sullying Effect of Recessions.* Review of Economic Studies, 69 (1).

[5] Bewley, T., 1999. *Why Wages Don't Fall During a Recession*. Harvard University Press, Cambridge, Massachusetts.

[6] Binmore, K., A. Rubinstein, A. Wolinsky, 1986. *The Nash Bargaining Solution in Economic Modeling*, Rand Journal of Economics, 17.

[7] Bowlus, A., G. Neumann, 2004. *The Job Ladder*. Mimeo, University of Western Ontario.

[8] Burdett, K., K. Judd, 1983. *Equilibrium Price Dispersion*. Econometrica, 51 (4).

[9] Burdett, K., D. Mortensen, 1998. *Wage Differentials, Employer Size, and Unemployment*. International Economic Review, 39.

[10] Christensen, B., R. Lentz, D. Mortensen, G. Neumann, A. Werwatz, 2001. *On the Job Search and Wage Distribution.* Northwestern University, Working Paper.

[11] Cole, H., R. Rogerson, 1999. *Can the Mortensen-Pissarides Matching Model Match the Business-Cycle Facts?* International Economic Review, 40.

[12] Crawford, V., J. Sobel, 1982. *Strategic Information Transmission.* Econometrica, 50 (6).

[13] Diamond, P., 1971. *A Model of Price Adjustment*. Journal of Economic Theory, 3.

[14] Doeringer, P., M. Piore, 1971. *Internal Labor Markets and Manpower Analysis.* Heath Lexington Books.

[15] FUDENBERG, D., D. LEVINE, J. TIROLE, 1983. *Infinite-horizon models of bargaining with one-sided incomplete information*, in "Bargaining with Incomplete Information", ed. A. Roth, Cambridge University Press.

[16] FUDENBERG, D., J. TIROLE, 1991. *Game Theory*. MIT Press.

[17] GROSHEN, E., 1991. *Sources of Intra-Industry Wage Dispersion: How Much Do Employers Matter?* Quarterly Journal of Economics, 106 (3).

[18] GROSSMAN, S., M. PERRY, 1986a. *Perfect Sequential Equilibrium*. Journal of Economic Theory, 39.

[19] GROSSMAN, S., M. PERRY, 1986b. *Sequential Bargaining under Asymmetric Information*. Journal of Economic Theory, 39.

[20] GUISO, L., L. PISTAFERRI, F. SCHIAVARDI, 2003. *Insurance Within the Firm*. CEPR Discussion Paper 2793.

[21] HALL, R., 2003. *Wage Determination and Employment Fluctuations*. Mimeo, Stanford University.

[22] HALL, R., 2004. *The Labor Market is the Key to Understanding the Business Cycle*. Mimeo, Stanford University.

[23] HALL, R., 2005. *Employment Fluctuations with Equilibrium Sticky Wages*. American Economic Review, forthcoming.

[24] KENNAN, J., 2004. *Private Information, Wage Bargaining and Employment Fluctuations*. Mimeo, University of Winsconsin-Madison.

[25] KING, R., S. REBELO, 1999. *Resuscitating Real Business Cycle*. Handbook of Macroeconomics.

[26] KYDLAND, F., 1995. *Business Cycles and Aggregate Labor Fluctuations*. In T. Cooley, ed., Frontiers of Business Cycle Research. Princeton University Press.

[27] KYLE, A., 1985. *Continuous Auction and Insider Trading*. Econometrica, 53 (6).

[28] MEDOFF, J., K. ABRAHAM, 1980. *Experience, Performance, and Earnings*. Quarterly Journal of Economics, 95.

[29] MENZIO, G., 2003. *Signal Posting*. Mimeo, Northwestern University.

[30] Merz, M., 1995. *Search in the labor market and real business cycle*. Journal of Monetary Economics, 36.

[31] Mortensen, D., 2000. *Equilibrium Unemployment with Wage Posting: Burdett-Mortensen Meet Pissarides*. In H. Bunzel, B.J. Christensen, P. Jensen, N.M. Kiefer, and D. Mortensen, eds., Panel Data and Structural Labour Market Model. Elsevier, Amsterdam.

[32] Mortensen, D., 2003. *Wage Dispersion. Why Are Similar Workers Paid Differently?*. MIT Press.

[33] Mortensen, D., C. Pissarides, 1999. *New Developments in Models of Search in the Labor Market*. In O. Ashenfelter, D. Card. eds., Hnadbook of Labor Economics, vol. 3B, Elsevier, Amsterdam.

[34] Moscarini, G., 2004. *Job-to-Job Quits and Corporate Culture*. Mimeo, Yale University.

[35] Muthoo, A., 1999. *Bargaining Theory with Applications*. Cambridge University Press.

[36] Nagypal, E., 2004. *Worker Reallocation over the Business Cycle: The Importance of Job-to-Job Transitions*. Mimeo, Northwestern University.

[37] Petrongolo, B., C. Pissarides, 2001. *Looking into the Black Box: A Survey of the Matching Function*, Journal of Economic Literature, 39.

[38] Phelps, E., S. Winter, 1970. *Optimal Price Policy under Atomistic Competition*. In E. Phelps, ed., Microeconomic Foundations of Employment and Inflation Theory, Norton & Company, NY.

[39] Rubinstein, A., 1982. *Perfect Equilibrium in a Bargaining Model*. Econometrica, 50.

[40] Shimer, R., 2005. *The Cyclical Behavior of Unemployment and Vacancies*. American Economic Review, forthcoming.

[41] Stevens, M., 2004. *Wage-Tenure Contracts in Frictional Labour Market: Firms' Strategies for Recruitment and Retention*. Review of Economic Studies, 71.

[42] Stokey, N., R. Lucas Jr., 1989. *Recursive Methods in Economics Dynamics*. Harvard University Press.

[43] Stole, L., J. Zwiebel, 1996a. *Intra-firm Bargaining under Non-binding Contracts*. Review of Economic Studies, 63.

[44] Stole, L., J. Zwiebel, 1996b. *Organizational Design and Technology Choice under Intrafirm Bargaining*. American Economic Review.

A The Derivation of the Reduced-Form Model

A.1 The Bargaining Game

In what follows, I characterize a Sequential Equilibrium of the bargaining game between firm i and its employees $j = 1, 2, ...n_{\Delta}$. The first step of the analysis is to conjecture an equilibrium for the subgame starting with the $k-th$ bargaining session, under the assumption that every worker has identical prior beliefs about the productivity (b, ρ) of the firm. Next, I verify that the conjectured strategies constitute a Sequential Equilibrium of the subgame, together with the specified belief updating rule. In conclusion, I discuss the properties of the stable outcome profile and its uniqueness.

A.1.1 Preliminaries

Before turning to the characterization of the bargaining outcome, it is necessary to introduce some notation to describe the workers' beliefs about the productivity of the firm (b, ρ).[18] Denote with $\Lambda(b, \rho)$ the beliefs of the worker about the firm's productivity components (b, ρ). Let Λ_b be the marginal distribution of b associated with $\Lambda(b, \rho)$ and Λ_p the marginal distribution of $p = b \cdot \rho$. The lower bound on the support of the marginal distribution of b is defined as

$$l_b(\Lambda) = \inf\{b \in \left[\underline{b}, \overline{b}\right] : \Lambda_b(b) > 0\} \tag{78}$$

Similarly, the lower bound on the support of the marginal distribution of p is defined as

$$l_p(\Lambda) = \inf\{p \in [\underline{b}, \overline{b}\overline{\rho}] : \Lambda_p(\rho) > 0\} \tag{79}$$

Finally, it is useful to denote the potential support of the belief Λ as a set $S(\Lambda)$ as

$$S(\Lambda) = \left\{(b, \rho) \in \left[\underline{b}, \overline{b}\right] \times [1, \overline{\rho}] : b \geq l_b(\Lambda) \wedge b\rho \geq l_p(\Lambda)\right\} \tag{80}$$

The analysis of the bargaining game is greatly simplified when the worker's belief Λ has full support, as defined below.

Definition 1 *(Full Support)* We say that the probability distribution Λ has full support if, for any $(b, \rho) \in S(\Lambda)$ and any $\epsilon > 0$, there is a vector (b', ρ') such that:

$$\left\|(b', \rho') - (b, \rho)\right\| \leq \epsilon \text{ and } \Lambda(b', \rho') > 0$$

[18] Throughout the Appendix, the object of worker's beliefs is exclusively the productivity of the firm. On and off the equilibrium path, the worker takes the behavior of her co-workers as given and dictated by [82] (as specified in the main text). Formally, worker j expects that $(w_{-j}, v_{-j}) = \left(l_b(\Lambda_j), (l_b(\Lambda_j))^{-1} l_p(\Lambda_j)\right)$.

When a distribution Λ has full support, the lower bounds on Λ_b and Λ_p are compatible. More specifically, there is a postive density in a neighborhood of the point $\left(l_b(\Lambda), (l_b(\Lambda))^{-1} l_p(\Lambda)\right)$. Secondly, if a worker revises her prior beliefs to rule out the types $b < b^*$ and $p < p^*$, the posterior belief Λ' has full support.

A.1.2 Equilibrium Strategies and Updating Rule

The subgame of interest is the one that begins with the $k-th$ bargaining session between the firm i and its workers $j = 1, 2, ...n_\Delta$. The state variables of the subgame are the following: (i) the worker's belief $\{\Lambda_{j,k}\}_{j=1}^{n_\Delta}$, (ii) the distribution of wage offers previously accepted by the firm $\{\Omega_{j,k-1}\}_{j=1}^{n_\Delta}$, (iii) the distribution of initial wage offers from the firm $\{m_{j,t}\}_{j=1}^{n_\Delta}$, (iv) the most recent public signal $I_{t,k-1}$.

It is useful to restate the details of the game. First, each worker j and the firm i independently decides whether to begin or not a renegotiation (R) of a previous agreement $\Omega_{j,k-1}$ or nor (NR). If every party accepts his past agreement $\Omega_{j,k-1}$, then the game ends immediately and the wages defined in $\Omega_{j,k-1}$ are paid. If some workers or the firm demand a renegotiation, then a bargaining session is initiated. Every worker simultaneously and independently demands a wage $(w_{j,k,0}, \upsilon_{j,k,0})$. The firm observes the profile of wage demands $\{(w_{j,k,0}, \upsilon_{j,k,0})\}_{j=1}^{n_\Delta}$ and responds by accepting Y or rejecting N each individual demand. If the demand of worker j is accepted, the bargaining round between i and j comes to an end. If the demand is rejected, the worker gets to advance another demand $(w_{j,k,1}, \upsilon_{j,k,1})$ and the productivity of the match is discounted at the rate β.

In this section, we restrict attention to the case where the following conditions are met: (a) every worker has identical beliefs[19] $\Lambda_{j,k} = \Lambda_{=j,k} = \Lambda_k$, and Λ_k has full support; (b) $l_b(\Lambda_k) \leq I_{t,k}^1$ and $l_p(\Lambda_k) \leq I_{t,k}^2$, where $\left(I_{t,k}^1, I_{t,k}^2\right)$ are defined as

$$\begin{aligned} I_{t,k}^1 &= \sup_{\theta \leq t, \kappa \leq k} \max\left\{h_{\theta,\kappa}^1, (\overline{\rho})^{-1} h_{\theta,\kappa}^2\right\} \\ I_{t,k}^2 &= \sup_{\kappa \leq k} \max\left\{h_{t,\kappa}^2, I_{t,k}^1\right\} \end{aligned}$$

In words, $I_{t,k}^1$ is the highest baseline salary that the firm has ever paid and $I_{t,k}^2$ is the highest wage that the firm has accepted (or offered) during the negotiations at date t.

Let $s_w^* : \Lambda \times \mathbb{R}^2 \mapsto \{R, NR\} \times \mathbb{R}^2$ denote the conjectured equilibrium strategy of the worker. The arguments in s_w^* are: (i) the belief $\Lambda_{k,\tau}$ held by worker at the $\tau - th$ round

[19] In what follows, I say that the probability distributions Λ_1 and Λ_2 are equal iff: (i) Λ_1 and Λ_2 have full-support, (ii) $l_b(\Lambda_1) = l_b(\Lambda_2)$, (iii) $l_p(\Lambda_1) = l_p(\Lambda_2)$. It is obvious from the forthcoming analysis that the only payoff-relevant statistics of Λ are the two lower bounds.

of the $k-th$ bargaining session and (ii) the agreement $\left(\Omega^1_{t,k-1},\Omega^2_{t,k-1}\right)$ previously reached with the firm. The function s^*_w returns: (a) the renegotiation choice $\{R,S\}$ at the beginning of the game and (b) the wage demand $(w_{k,\tau},\upsilon_{k,\tau})$ at the $\tau-th$ round. The strategy of the firm is a vector of functions $\{s^*_{f,j}\}_{j=1}^{n_\Delta}$. Each function is a map

$$s^*_{f,j}:\Lambda\times\left(\mathbb{R}^2\right)^{n_\Delta}\times\left(\mathbb{R}^3\right)^{n_\Delta}\longmapsto\{R,NR\}\times\{Y,N\}$$

The arguements of each function are: (i) the common prior belief of the workers Λ_k, (ii) the distribution of agreement $\left\{\left(\Omega^1_{t,k-1},\Omega^2_{t,k-1}\right)\right\}_{\tilde{j}=1}^{n_\Delta}$ previously reached with the workers, (iii) the distribution of wage demands $\left\{\left(\hat{w}_{\tilde{j},k,\tau},\hat{\upsilon}_{\tilde{j},k,\tau}\right)\right\}_{\tilde{j}=1}^{n_\Delta}$ at the $\tau-th$ bargaining round,[20] (iv) the firm type (b,ρ). The function $s^*_{f,j}$ tells whether the agreement $\Omega_{t,j,k-1}$ is renegotiated by the firm and whether the wage demand $(\hat{w}_{j,k,\tau},\hat{\upsilon}_{j,k,\tau})$ is accepted or not (given that the bargaining table is open). The functional form for s^*_w and $s^*_{f,j}$ is

$$s^*_w\left(\Lambda_{k,\tau},\Omega^1_{t,k-1},\Omega^2_{t,k-1}\right)=$$

$$\begin{cases} R & \text{iff } \Omega^1_{t,k-1}\cdot\Omega^2_{t,k-1}<l_p(\Lambda_{k,\tau}) \\ (w_{k,\tau},\upsilon_{k,\tau})= & \left(l_b(\Lambda_{k,\tau}),\frac{l_\rho(\Lambda_{k,\tau})}{l_b(\Lambda_{k,\tau})}\right)\end{cases}$$

$$s^*_{f,j}\left(\Lambda_k,\left\{\left(w_{\tilde{j},k,\tau},\upsilon_{\tilde{j},k,\tau}\right)\right\}_{\tilde{j}=1}^{n_\Delta},\left\{\Omega_{\tilde{j},k-1}\right\}_{\tilde{j}=1}^{n_\Delta};b,\rho\right)= \tag{82}$$

$$\begin{cases} R & \text{iff } \Omega^1_{t,j,k-1}\cdot\Omega^2_{t,j,k-1}>l_p(\Lambda_k) \\ \\ Y & \text{if } \{w_{j,k,\tau}\le l_b(\Lambda_k)\wedge w_{j,k,\tau}\upsilon_{j,k,\tau}\le l_p(\Lambda_k)\wedge b\rho\ge w_{j,k,\tau}\upsilon_{j,k,\tau}\} \\ \\ N & \text{if } \{w_{j,k,\tau}>l_b(\Lambda_k)\vee w_{j,k,\tau}\upsilon_{j,k,\tau}>l_p(\Lambda_k)\vee b\rho>w_{j,k,\tau}\upsilon_{j,k,\tau}\} \\ & \text{and } (w_{-j,k,\tau},\upsilon_{-j,k,\tau})=\left(l_b(\Lambda_k),\frac{l_\rho(\Lambda_k)}{l_b(\Lambda_k)}\right) \\ \\ f_j(.) & \text{otherwise}\end{cases}$$

According to [82], the worker's strategy is to ask for a renegotiation if and only if she is

[20] If the $k-th$ bargaining round between i and j_2 has concluded with the agreement $(\hat{w}_{j_2},\hat{\upsilon}_{j_2})$, we let $(w_{j_2,k,\tau},\upsilon_{j_2,k,\tau})=(\hat{w}_{j_2},\hat{\upsilon}_{j_2})$. Similalrly, if the the deal $\Omega_{j_2,k-1}$ is not renegotiated, we let $(w_{j_2,k,\tau},\upsilon_{j_2,k,\tau})=(\Omega^1_{j_2,k-1},\Omega^2_{j_2,k-1})$.

certain that the firm can afford a wage raise. The wage demanded is the lower bound on firm's productivity. The firm's strategy is to accept any wage demand $(w_{j,k,\tau}, \upsilon_{j,k,\tau})$ that does not exceed the productivity of the lowest type compatible with Λ (i.e. $w_{j,k,\tau} \leq l_b(\Lambda)$ and $w_{j,k,\tau} \cdot \upsilon_{j,k,\tau} \leq l_p(\Lambda)$) and that induces non-negative profits (i.e. $b\rho \geq w_{j,k,\tau} \cdot \upsilon_{j,k,\tau}$). If a single worker j deviates from the equilibrium and demands a wage $(w_{j,k,\tau}, \upsilon_{j,k,\tau})$ such that either $w_{j,k,\tau} > l_b(\Lambda)$ or $w_{j,k,\tau} \cdot \upsilon_{j,k,\tau} > l_p(\Lambda)$, the firm rejects the offer. The function $f_j(.) \mapsto \{Y, N\}$ describes the firm's strategy after histories where more than one worker demands a "high" off-equilibrium wage. Identifying the exact form of $f_j(.)$ is unimportant.

A wage demand (w_j, υ_j), such that $\{w_j \leq l_b(\Lambda) \wedge w_j \cdot \upsilon_j \leq l_p(\Lambda)\}$, is accepted by every type (b, ρ) on the support of worker's belief Λ. Similalrly, a demand $\{w_j > l_b(\Lambda) \vee w_j \cdot \upsilon_j > l_p(\Lambda)\}$ is rejected by every firm $(b, \rho) \in S(\Lambda)$. After either one of these two actions is observed, the worker's posterior belief Λ' is obtained by applying Bayes' Law. It is immediate to verify that $\Lambda' = \Lambda$. At the end of the $k - th$ bargaining session, the worker observes the public signal $\left(I^1_{t,k+1}, I^2_{t,k+1}\right)$ about the wage of her employees. By assumption, every worker has identical beliefs about the firm's productivity and the only realization of the signal along the equilibrium path is $\left(I^1_{t,k+1}, I^2_{t,k+1}\right) = (l_b(\Lambda), l_p(\Lambda))$. The associated updating rule is $\Lambda' = \Lambda$. On the other hand, an inspection of [82] shows that some firm's actions are never observed in equilibrium. Therefore, in order to complete the characterization of a Sequential Equilibrium, it is necessary to specify the belief updating rule after any of these off-equilibrium moves. The only restriction imposed by the notion of SE[21] on the selection of off-equilibrium conjectures is that the support of the posterior has to be contained within the support of the prior, i.e. $S(\Lambda') \subset S(\Lambda)$. The indeterminacy of the off-equilibrium conjectures leads, in general, to a multiplicity of equilibria. Rather than advancing a refinement, I specify a particular conjecture and argue that it is a sensible choice in the context of this game.[22] Assumption [1] contains the updating rule of beliefs after on and off equilibrium actions are observed.

Assumption 1 (*Updating Rules I: Bargaining*) Let Λ describe the full-support beliefs of the worker prior to observing an off-equilibrium move.

[21] For a formal definition of Sequential Equilibrium, the reader can refer to the textbook treatment in Fudenberg and Tirole (1991).

[22] The multiplicity of equilibria in bargaining games of asymmetric information is a well-recognized problem. Gorssman and Perry (1986b) argue that the "Intuitive Criterion" is a refinement with no bite in this context and advance a more stringent "Perfect Sequential Equilibrium." In the companion paper (Menzio 2004), I adopt the PSE concept to characterize the bargaining game. In this paper, the presence of noisy traders makes this approach unmanageable.

(a) If a wage demand (w, υ) is accepted by the firm and either $w > l_b(\Lambda)$ or $w \cdot \upsilon > l_p(\Lambda)$, then the full-support posterior belief is a probability distribution[23] Λ' such that

$$\left(l_b(\Lambda'), l_p(\Lambda')\right) = \left(\max\left\{l_b(\Lambda), w, (\overline{\rho})^{-1} w \cdot \upsilon\right\}, \max\left\{l_p(\Lambda), w \cdot \upsilon, l_b(\Lambda')\right\}\right) \quad (83)$$

(b) If a demand (w, υ) is rejected by the firm and both $w \le l_b(\Lambda)$ and $w \cdot \upsilon \le l_p(\Lambda)$, then the full-support posterior belief Λ' coincides with the prior Λ.

(c) If the worker's posterior belief at the end of the $k-th$ negotiation session is Λ_k and the public signal is a vector $\left(I^1_{t,k}, I^2_{t,k}\right)$ and $I^1_{t,k} > l_b(\Lambda)$ or $I^2_{t,k} > l_p(\Lambda)$, then the updated belief is a probability distribution Λ'_k such that

$$\left(l_b(\Lambda'_k), l_p(\Lambda'_k)\right) = \left(\max\left\{l_b(\Lambda), I^1_{t,k}\right\}, \max\left\{l_p(\Lambda), I^2_{t,k}\right\}\right)$$

(d) If the signal $\left(I^1_{t,k}, I^2_{t,k}\right)$ is such that both $I^1_{t,k} \le l_b(\Lambda)$ and $I^2_{t,k} \le l_p(\Lambda)$, then there is no belief update.

Consider a firm that is demanded the wage (w_j, υ_j), such that $w_j \upsilon_j > l_p(\Lambda)$. The firm can either accept and trade immediately, or reject and let the productivity of the match decay. If the bargaining game was played by the two parties alone, a firm with productivity $p < w_j \upsilon_j$ would never accept the demand. Therefore, it seems natural to assume that the updated belief puts probability one on the event $p \ge w_j \upsilon_j$. Secondly, I assume that the composition of the wage offer between the short and long term components is "taken seriously" by the workers. Therefore, if the firm accepts a demand for a baseline salary $w_j > l_b(\Lambda)$, I assume that the worker revises her beliefs so that $l_b(\Lambda') = w_j$. On the other hand, if the firm rejects the equilibrium wage demand, the worker should revise his beliefs downwards. Given the SE restriction that $S(\Lambda') \subset S(\Lambda)$, a passive belief revision $\Lambda' = \Lambda$ seems a sensible choice. Finally, Assumption [1](c)-(d) imply that every worker interprets the off-equilibrium actions in the same way.

A.1.3 Vindicating the Conjectured Equilibrium

Along, the (conjectured) equilibrium path, either all workers demand a renegotiation, the firm demands one with every employee or the outcome profile $\{\Omega_{j,k-1}\}_{j=1}^{n_\Delta}$ is stable. If a renegotiation is started, worker j demands $\left(l_b(\Lambda_k), (l_b(\Lambda_k))^{-1} l_\rho(\Lambda_k)\right)$ and an agreement is immediately reached. The public signal revealed at the end of the $k-th$ bargaining

[23] The exact specification of the posterior Λ' is not a factor in the analysis. The reader can safely intrepret the posterior as the appropriate truncation of the prior.

session is $I_{t,k+1}$, such that $\left(I^1_{t,k+1}, I^2_{t,k+1}\right) = \left(l_b\left(\Lambda_k\right), l_p\left(\Lambda_k\right)\right)$. The workers' posterior belief is $\Lambda_{j,k+1} = \Lambda_k$ for $j = 1, 2, ...n_\Delta$. The outcome profile $\{\Omega_{j,k-1}\}_{j=1}^{n_\Delta}$ is stable and $I_{t+1,0} = I_{t,k+1}$. The complete payoffs for the two trading parties are

$$\begin{aligned} & \beta^{\tau_{j,k-1}} l_p\left(\Lambda_k\right) + \delta\tilde{V}\left(I^1_{t+1,0}\right) && \text{for } j = 1, 2, ..n_\Delta \\ & \sum_{j=1}^{n_\Delta} \frac{n}{n_\Delta} \beta^{\tau_{j,k-1}} \max\left\{\left(p - l_p\left(\Lambda_k\right)\right), 0\right\} + \delta\tilde{\Pi}\left(n, I^1_{t+1,0}; b\right) \end{aligned} \tag{84}$$

where $\tilde{V}$ and $\tilde{\Pi}$ denote the expected discounted payoff for the worker and the firm, respectively. More specifically, $\tilde{V} : \left[\underline{b}, \overline{b}\right] \mapsto \mathbb{R}$ is a strictly increasing function of $I^1_{t+1,0}$ and $\tilde{\Pi} : \mathbb{R}_+ \times \left[\underline{b}, \overline{b}\right]^2 \mapsto \mathbb{R}$ is linearly increasing in the firm-size n and strictly decreasing in $I^1_{t+1,0}$.[24]

Proposition 1 *(No Screening)* If $n_\Delta \to \infty$, the strategies [82] constitute a Sequential Equilibrium of the bargaining game, together with the belief updating rule specified in Assumption [1].

Proof.

Claim (1): Worker j demands a renegotiation if and only if $l_p(\Lambda_k) > \Omega^1_{j,t,k-1} \cdot \Omega^2_{j,t,k-1}$.

Proof. Suppose that at the end of the $(k-1)$-th bargaining session, worker j has agreed to a wage $\left(\Omega^1_{j,t,k-1}, \Omega^2_{j,t,k-1}\right) = (\hat{w}, \hat{\upsilon})$ such that $\hat{w} \cdot \hat{\upsilon} < l_p(\Lambda_k)$. Because $\Pr\left[p \geq l_p(\Lambda_k) | \Lambda_k\right] = 1$*, the worker expects the firm to accept a demand*

$$(w, \upsilon) = \left(l_b\left(\Lambda_k\right), \left(l_b\left(\Lambda_k\right)\right)^{-1} l_p(\Lambda_k)\right)$$

This implies that starting a renegotiation is optimal if $l_p(\Lambda_k) > \Omega^1_{j,t,k-1} \cdot \Omega^2_{j,t,k-1}$. Proving the converse is immediate.

Claim (2): The firm's strategy in [82] is sequentially rational.

Proof. First consider the situation where $(w_{-j}, \upsilon_{-j}) = \left(l_b\left(\Lambda_k\right), \left(l_b\left(\Lambda_k\right)\right)^{-1} l_p(\Lambda_k)\right)$. It is useful to split the analysis in two cases depending on the ordering of $l_p\left(\Lambda_k\right)$ and p.

Case I: $p \geq l_p\left(\Lambda_k\right)$.

[24] The conjecture for $\tilde{V}$ and $\tilde{\Pi}$ is verified in Section [3].

Case Ia: The wage demand is such that $w_{j,k,\tau} \leq l_b(\Lambda_k)$ *and* $w_{j,k,\tau}\upsilon_{j,k,\tau} \leq l_\rho(\Lambda_k)$. *If the firm accepts the demand of workers* j *and* $-j$*, the common posterior belief is* $\Lambda_{k+1} = \Lambda_k$. *During the* $(k+1)-th$ *bargaining session, worker* j *demands a renegotiation if* $w_{j,k,\tau}\upsilon_{j,k,\tau} < l_\rho(\Lambda_k)$ *and the final payoff for the firm is*

$$\beta^{\tau+\tau_{j,k-1}} \frac{n}{n_\Delta} \left(p_t - l_p\left(\Lambda_k\right)\right) + \sum_{-j} \beta^{\tau_{-j,k-1}} \frac{n}{n_\Delta} \left(b\rho_t - l_p(\Lambda_k)\right) + \delta\tilde{\Pi}(n, l_b\left(\Lambda_k\right))$$

If the firm rejects the wage demand of j*, her posterior belief is* $\Lambda'_j = \Lambda_k$. *At round* $\tau+1$*, worker* j *demands* $(w_{j,k,\tau+1}, \upsilon_{j,k,\tau+1}) = \left(l_b\left(\Lambda_k\right), \left(l_b\left(\Lambda_k\right)\right)^{-1} l_p(\Lambda_k)\right)$ *and the firm accepts. The wage outcome is stable and the payoff to the firm is*

$$\beta^{\tau+1+\tau_{j,k-1}} \frac{n}{n_\Delta} \left(p_t - l_p\left(\Lambda_k\right)\right) + \sum_{-j} \beta^{\tau_{-j,k-1}} \frac{n}{n_\Delta} \left(b\rho_t - l_p(\Lambda_k)\right) + \delta\tilde{\Pi}(n, l_b\left(\Lambda_k\right))$$

It follows that the best response of the firm is to accept any demand such that $w_j \leq l_b(\Lambda_k)$ *and* $w_j\upsilon_j \leq l_\rho(\Lambda_k)$.

Case Ib: The wage demand is such that $w_{j,k,\tau}\upsilon_{j,k,\tau} > l_\rho(\Lambda_k)$. *By accepting* $(w_{j,k,\tau}, \upsilon_{j,k,\tau})$*, the firm signals that* $p \geq w_{j,k,\tau}\upsilon_{j,k,\tau}$. *More specifically, the posterior belief for worker* j *is* $\Lambda'_j = \Lambda'_{-j} = \Lambda_{k+1}$ *such that*

$$l_p\left(\Lambda_{k+1}\right) = w_{j,k,\tau}\upsilon_{j,k,\tau}$$

and

$$l_b\left(\Lambda_{k+1}\right) = \max\{l_b\left(\Lambda_{k+1}\right), w_{j,k,\tau}, \left(\overline{\rho}\right)^{-1} w_{j,k,\tau}\upsilon_{j,k,\tau}\}$$

In the $(k+1)-th$ *bargaining session, worker* $-j$*.demands for a renegotiation. Overall, the firm's payoff if the demand* $(w_{j,k,\tau}, \upsilon_{j,k,\tau})$ *is accepted is*

$$\beta^{\tau+\tau_{j,k-1}} \frac{n}{n_\Delta} \left(p - w_{j,k,\tau}\upsilon_{j,k,\tau}\right) + \\ \beta^{\tau_{-j,k-1}} \left(n - \frac{n}{n_\Delta}\right) \max\left\{\left(p - w_{j,k,\tau}\upsilon_{j,k,\tau}\right), 0\right\} + \delta\tilde{\Pi}(n, \hat{I}^1_{t+1,0})$$

where $\beta^{\tau_{-j,k-1}} = \sum_{j_2 \neq j} \beta^{\tau_{j_2,k-1}}$ *is the average delay accumulated by the remaining work-force and* $\hat{I}^1_{t+1,0} = l_b\left(\Lambda_{k+1}\right)$. *If the firm rejects the demand* $(w_{j,k,\tau}, \upsilon_{j,k,\tau})$*, worker* j *demands* $(w_{j,k,\tau+1}, \upsilon_{j,k,\tau+1}) = \left(l_b\left(\Lambda_k\right), \left(l_b\left(\Lambda_k\right)\right)^{-1} l_p(\Lambda_k)\right)$. *At the end of the* $k-th$ *bargaining session, a stable agreement is reached and the firm's payoff is*

$$\beta^{1+\tau+\tau_{j,k-1}} \frac{n}{n_\Delta} \left(p - l_p(\Lambda_k)\right) + \beta^{\tau_{-j,k-1}} \left(n - \frac{n}{n_\Delta}\right) \left(p - l_p(\Lambda_k)\right) + \delta\tilde{\Pi}(n, l_b\left(\Lambda_k\right)) \quad (85)$$

Rejecting $(w_{j,k,\tau}, \upsilon_{j,k,\tau})$ *is sequentially rational whenever [**??**] is smaller than [85]. This condition is satisfied when*

$$\beta^{\tau_{j,k-1}} \frac{n}{n_\Delta} (1-\beta)(p - w_{j,k,\tau}\upsilon_{j,k,\tau}) \leq$$
$$\beta^{\tau_{-j,k-1}} \left(n - \frac{n}{n_\Delta}\right) \left[(p - l_p(\Lambda_k)) - \max\{(p - w_{j,k,\tau}\upsilon_{j,k,\tau}), 0\}\right] +$$
$$\delta \left[\tilde{\Pi}(n, l_b(\Lambda_k)) - \tilde{\Pi}(n, \hat{I}^1_{t+1,0})\right]$$

Taking the limit for $n_\Delta \to \infty$*, the expression above becomes*

$$\beta^{\tau_{-j,k-1}} n \max\{(p - w_{j,k,\tau}\upsilon_{j,k,\tau}), 0\} \leq$$
$$\beta^{\tau_{-j,k-1}} n (p - l_p(\Lambda_k)) + \delta \left[\tilde{\Pi}(n, l_b(\Lambda_k)) - \tilde{\Pi}(n, \hat{I}^1_{t+1,0})\right]$$

which is always satisfied because

$$p - l_p(\Lambda_k) > \max\{(p - w_{j,k,\tau}\upsilon_{j,k,\tau}), 0\}$$

and

$$\tilde{\Pi}(n, l_b(\Lambda_k)) \geq \tilde{\Pi}(n, \hat{I}^1_{t+1,0})$$

Case Ic: The wage demand $(w_{j,k,\tau}, \upsilon_{j,k,\tau})$ *is such that* $w_{j,k,\tau}\upsilon_{j,k,\tau} \leq l_\rho(\Lambda_k)$ *but* $w_{j,k,\tau} > l_b(\Lambda_k)$*. If the firm accepts the offers* $(w_{j,k,\tau}, \upsilon_{j,k,\tau})$ *and* (w_{-j}, υ_{-j})*, it obtains a payoff*

$$\beta^{\tau+\tau_{j,k-1}} \frac{n}{n_\Delta} (p - l_p(\Lambda)) + \beta^{\tau_{-j,k-1}} \left(n - \frac{n}{n_\Delta}\right) \max\{(p - l_p(\Lambda_k)), 0\} + \delta\tilde{\Pi}(n, \hat{I}^1_{t+1,0})$$

On the other hand, if the firm rejects $(w_{j,k,\tau}, \upsilon_{j,k,\tau})$*, the payoff is*

$$\beta^{\tau+1+\tau_{j,k-1}} \frac{n}{n_\Delta} (p - l_p(\Lambda)) + \beta^{\tau_{-j,k-1}} \left(n - \frac{n}{n_\Delta}\right) \max\{(p - l_p(\Lambda)), 0\} + \delta\tilde{\Pi}(n, l_b(\Lambda))$$

where $l_b(\Lambda) < \hat{I}^1_{t+1,0}$*. Rejecting* $(w_{j,k,\tau}, \upsilon_{j,k,\tau})$ *is sequentially rational.*

It remains to verify that the firm optimally accepts a wage demand

$$(w_{-j}, \upsilon_{-j}) = .\left(l_b(\Lambda_k), (l_b(\Lambda_k))^{-1} l_p(\Lambda_k)\right)$$

independently of the demand of worker j*. By assumption [1] and because of the restriction (b) on the initial state of the subgame, the firm reveals no information by accepting the wage* (w_{-j}, υ_{-j})*. Moreover, the posterior beleifs of worker* $-j$ *are non-decreasing and the firm will never receive a wage demand below* $\left(l_b(\Lambda_k), (l_b(\Lambda_k))^{-1} l_p(\Lambda_k)\right)$*.*

Both statically and dynamically, accepting $\left(l_b(\Lambda_k), (l_b(\Lambda_k))^{-1} l_p(\Lambda_k)\right)$ *is at least as profitable as rejecting.*

Case II: $p < l_p(\Lambda_k)$*. The conjectured equilibrium payoff of the firm is*

$$0 + \delta\tilde{\Pi}(n, l_b(\Lambda_k))$$

The equilibrium payoff is attained by rejecting every wage demand. Because of Assumption [1] and tthe restriction (b) on the initial state of the subgame, after every history of play $I^1_{t+1,0} \geq l_b(\Lambda_k)$*. Therefore, for a deviation from the putative equilibrium to be profitable, there has to be some worker* j *who trades at* $(w_{j,k'}, \upsilon_{j,k'})$ *and* $w_j \upsilon_j < p < l_p(\Lambda_k)$*. Since* $\Lambda_{j,k'} = \Lambda_{-j,k'}$ *after every history of play, the subgame at* k' *is described by the strategies in [82]. Therefore,* $(w_{j,k'}, \upsilon_{j,k'})$ *cannot be part of a stable agreement.*

When $(w_{-j}, \upsilon_{-j}) \neq . \left(l_b(\Lambda_k), (l_b(\Lambda_k))^{-1} l_p(\Lambda_k)\right)$*, the optimal strategy of the firm is unimportant and it is represented by some function* $f(.)$*..*

Claim (3): Firm i *demands a renegotiation with worker* j *if and only if* $l_p(\Lambda_k) > \Omega^1_{j,t,k-1} \cdot \Omega^2_{j,t,k-1}$*.*

Proof. Trivial.

Claim (4): The worker's bargaining strategy in [82] is sequentially rational.

Proof. The worker takes the behavior of $-j$ *as given and expects that* $(w_{-j}, \upsilon_{-j}) = \left(l_b(\Lambda_k), (l_b(\Lambda_k))^{-1} l_p(\Lambda_k)\right)$*. Therefore, when a worker* j *demands* $(w_{j,k,\tau}, \upsilon_{j,k,\tau})$ *such that either* $w_{j,k,\tau} > l_b(\Lambda_k)$ *or* $w_{j,k,\tau}\upsilon_{j,k,\tau} > l_p(\Lambda_k)$*, she expects a rejection with probability one. Her payoff is*

$$\beta^{1+\tau+\tau_{j,k}} l_p(\Lambda_k) + \delta\tilde{V}(l_b(\Lambda_k))$$

By demanding $(w_{j,k,\tau}, \upsilon_{j,k,\tau}) = \left(l_b(\Lambda_k), (l_b(\Lambda_k))^{-1} l_p(\Lambda_k)\right)$*, the worker obtains*

$$\beta^{\tau+\tau_{j,k}} l_p(\Lambda_k) + \delta\tilde{V}(l_b(\Lambda_k))$$

Finally, if worker j *demands* $(w_{j,k,\tau}, \upsilon_{j,k,\tau})$ *such that* $w_{j,k,\tau} \leq l_b(\Lambda_k)$ *or* $w_{j,k,\tau}\upsilon_{j,k,\tau} \leq l_p(\Lambda_k)$ *and one of the inequalities is strict, he obtains the payoff*

$$\beta^{\tau+\tau_{j,k}} l_p(\Lambda_k) + \delta\tilde{V}(l_b(\Lambda_k))$$

by initiating a renegotiation during the $(k+1)-th$ *bargaining session..* ■

According to Proposition [1], the strategies in [82] are a Sequential Equilibrium of the bargaining session, together with the specified updating rule for beliefs. In this equilibrium, the wage outcome coincides with the solution of the full-information bargaining game between worker j and a firm of productivity $l_p(\Lambda_k)$. Moreover, a stable agreement is reached without delay. The firm can successfully mimic the lowest productivity type because of the mechanism of informational spillovers created by the public signal $I_{t,k+1}$. When the firm accepts a wage demand (w_j, v_j), such that $w_j \cdot v_j > l_p(\Lambda_k)$, it signals to the entire workforce that its productivity p is higher than $l_p(\Lambda_k)$ and this leads to a sequence of renegotiations. The cost of accepting the demand (w_j, v_j) is proportional to the size of the firm n, while the cost of rejecting the demand is to reduce the productivity of a single worker. For $n_\Delta \to \infty$, the second effect is infinitesimal with respect to the first.[25]

The No-Screening result in Proposition [1] is reminiscent of the finding in Schmidt (1995). In his model, a worker and a firm play a "split-the-cake" game repeatedly over time, and the firm succesfully imitates the low type. The spillover of information across time palys the same role in Schimdt's paper that the inter-personal spillover has in mine. On the other hand, the mechanism behind Proposition [1] is different from the "Coase Conjecture," even though the result is somewhat similar. According to the conjecture, as the delay between sucessive vanishes (as $\beta \to 1$), every firm type trades for a wage that is arbitrarly close to $l_p(\Lambda_k)$ and the equilibrium number of delays τ converges to zero. Intuitively, the worker cannot credibly threaten the firm with long delays if she is allowed to advance a new offer soon after the previous one has been rejected. The "Coase Conjecture," which has been verified by Fudenberg, Levine and Tirole (1983), is based on the lack of commitment of the worker rather than on the presence of external effects.

A.1.4 Uniqueness

If firm i and workers $j = 1, 2, ...n_\Delta$ play according to [82], the stable outcome of the bargaining game is

$$\left\{\left(w^*_{j,1}, v^*_{j,1}\right)\right\}_{j=1}^{n_\Delta} = \left\{\left(l_b(\Lambda), (l_b(\Lambda))^{-1} l_p(\Lambda)\right)\right\}_{j=1}^{n_\Delta}$$

[25] If the firm bargains with a finite number of workers, a No-Screening result would still hold for n sufficiently large. In fact, as n increases, the maximum wage demand that the firm is willing to accept to avoid a delay falls and converges to $l_p(\Lambda)$ for $n \to \infty$. When n is finite (but sufficiently large), the worker has no incentives to bear the risk of a delay in order to obtain a small wage raise.

In this subsection, I want to argue that there exists a set of restrictions on the family of Sequential Equilibria of the bargaining game such that the stable outcome $\left\{\left(w_{j,1}^{*}, \upsilon_{j,1}^{*}\right)\right\}_{j=1}^{n_{\Delta}}$ is unique. A complete analysis of the issue is beyond the scope of this paper, yet the *informal arguments* below might convince the reader that the equilibrium described in Proposition [1] is of particular interest.

Consider any Sequential Equilibrium of the bargaining game. Let $(\underline{w}, \underline{\upsilon})$ the lowest (acceptable) wage demand of a worker on and off the equilibrium path and suppose that $\underline{w} \cdot \underline{\upsilon} < l_p(\Lambda)$. Suppose that instead of demanding $(w_j, \upsilon_j) = (\underline{w}, \underline{\upsilon})$, worker j demands $(\hat{w}, \hat{\upsilon}) = (\underline{w}, \underline{\upsilon} + \epsilon)$, for some $\epsilon > 0$. More specifically, it is possible to pick an ϵ such that the following inequality holds

$$b\rho - \underline{w}\left(\underline{\upsilon} + \epsilon\right) > \beta\left(b\rho - \underline{w}\underline{\upsilon}\right)$$

for every $(b, \rho) \in S(\Lambda)$. It seems sensible to rule out equilibria where a worker updates her beliefs in response to an action that every firm would take, if the workers were not to revise their beliefs. If this refinement concept is adopted, then the only posterior belif after $(\hat{w}, \hat{\upsilon})$ is accepted is $\Lambda' = \Lambda$. Therefore, if worker j demands $(\hat{w}, \hat{\upsilon})$, the firm accepts with probability one. We have identified a violation of sequential rationality and we can conclude that $\underline{w} \cdot \underline{\upsilon} \geq l_p(\Lambda)$.

Secondly, suppose that there exists a Sequential Equilibrium where screening occurs along the equilibrium path. In such equilibrium, there exists a worker j who demands a wage (w_j, υ_j) that is rejected by a set of firms $S_a \neq \varnothing$ and accepted by others $S(\Lambda)/S_a \neq \varnothing$. Therefore, there is a positive probability that worker j's demand is rejected and the productivity of her match falls. If we restrict attention to equilibria where workers follow symmetric strategies, then worker j can improve her payoff by demanding $(\hat{w}, \hat{\upsilon})$, such that $\hat{w} \cdot \hat{\upsilon} \leq l_p(\Lambda)$. The demand is accepted with probability one and worker j learns about the firm from the public signal. If she can start a renegotiation costlessly, her optimal strategy is to wait. In equilibrium, workers do not learn about their employer during the bargaining game and the unique pooling wage is (w, υ), such that $w \cdot \upsilon \leq l_p(\Lambda)$.

By combining the two results, we can conclude that in any SE trade occurs only for $w \cdot \upsilon = l_p(\Lambda)$. If the division of the total wage into baseline salary and bonus is taken as informative, then $w = l_b(\Lambda)$.

A.2 Endogenous Constraints on the Signaling Strategy

In this subsection, the characterization of the bargaining game obtained in Proposition [1] is used to derive a set of constraints on the signaling strategy of the firm. Formally, I am interested in the game that starts with the decision node where the firm makes the wage offers $m_{i,j,t}$. The state of the game is given by: (i) the number n^c of workers with which the firm is in contact, (ii) the worker's belief distribution $\{\Lambda_{j,t,-1}\}_{j=1}^{n_\Delta^c}$, (iii) the information about the outside offers of the contacted workers that firm i might have, (iv) the signal $I_{t,0}$. The firm's signaling strategy is represented by a vector of functions $\{m_{f,j}\}_{j=1}^{n_\Delta^c}$, where each function $m_{f,j}$ maps the state of the game into a wage offer $m_{i,j,t} \in \left[\underline{b}, \overline{b}\right] \times [1, \overline{\rho}]$. The analysis is restricted to the case where the initial state satisfies the condition $l_b\left(\Lambda_{j,t,-1}\right) \leq I_{t,0}^1$ and $l_p\left(\Lambda_{j,t,-1}\right) \leq I_{t,0}^2$, for $j = 1, 2, ...n_\Delta^c$. Moreover, throughout this subsection, the interpretation of the firm's offers $m_{i,j,t}$ is taken as given. If a worker j, who has a prior (full-support) belief Λ_j, receives an offer $m_{i,j,t}$, then the posterior is a full-support probability distribution Λ_j' such that

$$\left(l_b(\Lambda_j'), l_p(\Lambda_j')\right) = \left(\max\left\{l_b(\Lambda_j), m_{i,j,t}^1\right\}, \max\left\{l_p\left(\Lambda_j\right), m_{i,j,t}^1 \cdot m_{i,j,t}^2\right\}\right) \tag{86}$$

Given these restrictions, Lemma [1b] shows that there is a Sequential Equilibrium of the signaling game where the firm does not advance worker-specific offers. Secondly, Lemma [2] shows that restricting the firm's strategy space to $m_{i,t} \geq m_{i,t-1}$ is without loss of generality. Finally, Proposition [2] summarizes the results and derives the reduced-form wage posting model.

A.2.1 Intratemporal Constraints

The first step in the analysis is to evaluate the payoff of the firm given an arbitrary profile of wage offers $\{m_{i,j,t}\}_{j=1}^{n_\Delta^c}$. After observing the offer $m_{i,j,t}$, worker j updates her belief $\Lambda_{j,t,-1}$ to $\Lambda_{j,t,0}$, according to [86]. Next, the worker chooses whether to move to the location of firm i or not. It is shown in Section [3] that if the worker accepts an offer from a firm Λ_1, then she certainly accepts the offer from a firm Λ_2 such that $l_b\left(\Lambda_2\right) \geq l_b\left(\Lambda_1\right)$ and $l_p\left(\Lambda_2\right) \geq l_p\left(\Lambda_1\right)$. Once the worker has reached the firm, she observes the signal $I_{i,t,1}$, before the beginning of the first bargaining session. The posterior belief is denoted as $\Lambda_{j,t,1}$ and is determined according to the updating rules specified in Assumption [1].

Lemma 1a *(Generic Wage Offers, part I)* If $h_{i,t,0} = H\left(\{m_{i,j,t}\}_{j=1}^{n_\Delta}\right)$ and $I_{i,t,1} = I_{i,t,0} \cup h_{i,t,0}$, then the restriction $m_{i,j,t} = m_{i,t}$ is without loss of generality.

Proof. *The arguement of the proof is to show that whatever payoff the firm can obtain by sending worker-specific messages $\{m_{i,j,t}\}_{j=1}^{n_\Delta^c}$ can be replicated (or improved) by using a strategy where $\hat{m}_{i,j,t} = \hat{m}_{i,t}$. The public signal $I_{t,1}$ is such that $I_{t,1}^1 =$*

$\max\left\{I^1_{t,0}, m^1_{j,t}\right\}_{j=1}^{n_\Delta}$ *and* $I^2_{t,1} = \max\left\{I^2_{t,0}, m^1_{j,t} m^2_{j,t}\right\}_{j=1}^{n_\Delta}$. *If* j *is an old employee, then her prior belief* $\Lambda_{j,-1}$ *is such that* $l_b(\Lambda_{j,-1}) \leq I^1_{t,0}$ *and* $l_p(\Lambda_{j,-1}) \leq I^2_{t,0}$ *(by assumption) and her posterior belief at the beginning of the first bargaining session is* $\Lambda_{j,1}$ *such that*

$$(l_b(\Lambda_{j,1}), l_p(\Lambda_{j,1})) = \left(I^1_{t,1}, I^2_{t,1}\right)$$

If j *is a newly hired worker, her prior belief* $\Lambda_{j,-1}$ *is derived from the (common knowledge) type distribution and* $l_b(\Lambda_{j,-1}) = \underline{b} \leq I^1_{t,0}$ *and* $l_p(\Lambda_{j,-1}) = \underline{b} \leq I^2_{t,0}$. *Her posterior belief is*

$$(l_b(\Lambda_{j,1}), l_p(\Lambda_{j,1})) = \left(I^1_{t,1}, I^2_{t,1}\right)$$

Therefore, at the beginning of the first bargaining session every worker has an identical belief $\Lambda_{t,1}$, *such that*

$$\begin{aligned} \max\left\{m^1_{j,t}\right\}_{j=1}^{n_\Delta} &\leq l_b(\Lambda_{t,1}) = I^1_{t,1} \\ \max\left\{m^1_{j,t} m^2_{j,t}\right\}_{j=1}^{n_\Delta} &\leq l_p(\Lambda_{t,1}) = I^2_{t,1} \end{aligned}$$

Since the initial conditions for the bargaining game characterized in Proposition [1] are met, the payoff of the firm is given by

$$n \max\left\{(p - l_p(\Lambda_{t,1})), 0\right\} + \delta\tilde{\Pi}\left(n, I^1_{t+1,0}; b\right)$$

Suppose that the firm follows the alternative strategy

$$\hat{m}_{i,j,t} = \hat{m}_{i,t} = \left(I^1_{t,1}, \left(I^1_{t,1}\right)^{-1} I^2_{t,1}\right)$$

Then the payoff to the firm is

$$\hat{n} \max\left\{(p - l_p(\Lambda_{t,1})), 0\right\} + \delta\tilde{\Pi}\left(\hat{n}, I^1_{t+1,0}; b\right)$$

The number of employees is $\hat{n}$. *Moreover,* $\hat{n} \geq n$. *because* $\hat{m}_{i,t}$ is such that $\hat{m}^1_{i,t} \geq m^1_{i,j,t}$ *and* $\hat{m}^1_{i,t}\hat{m}^2_{i,t} \geq m^1_{i,j,t} m^2_{i,j,t}$, *for* $j = 1, 2, \ldots n_\Delta$. *The deviation is profitable.* ■

A.2.2 Intertemporal Constraint

The next lemma identifies an intertemporal restriction on the wage policy of the firm: the persistent component of the wage is non-decreasing over time. The result is an immediate consequence of the information transmission mechanism which allows for newly hired workers to observe the wage paid in the past by the firm. While this assumption might seem strong, some version of Lemma [2] holds under weaker communication rules. For instance, even if the history of wages cannot be directly observed, the firm would still have to displace the entire cohort of old employees to break the downward wage rigidity constraint.

Lemma 2 *(Downward Wage Rigidity)* The restriction $m_{i,t}^1 \geq I_{i,t,0}^1$ on the signaling strategy of the firm at date t is without loss of generality.

Proof. *Suppose that firm i offers a wage $(m_{i,t})$ such that $m_{i,t}^1 < I_{t,0}^1$. The common posterior belief at the beginning of the first bargaining session is $\Lambda_{t,1}$ such that*

$$(l_b(\Lambda_{t,1}), l_p(\Lambda_{t,1})) = \left(I_{t,1}^1, I_{t,1}^2\right)$$

$I_{t,1}^1 = \max\left\{I_{t,0}^1, m_t^1\right\}$ *and* $I_{t,1}^2 = \max\left\{I_{t,0}^2, m_t^1 m_t^2\right\}$. *The result follows from the definition of $I_{t,1}^1$..* ■

A.2.3 The Reduced-Form Model

Consider a game where firm i posts a two-part wage $(w_{i,t}, \upsilon_{i,t})$ at every date t. The wage offered by the firm is subject to the following constraints

$$D(x_{i,t}) = \{(w, \upsilon) \in [\underline{b}, \overline{b}] \times [1, \underline{\rho}] : w \geq x_{i,t}\} \tag{87}$$

and $x_{i,t+1} = w_{i,t}$. The n_t^c workers that are in contact with the firm observe the posted wage and interpret it as a message $(w_{i,t}, \upsilon_{i,t}) = m_{i,t}$, then they decide whether to trade with firm i or not. The date-t payoff to the firm is

$$n_t \max\left\{0, b_i \rho_{i,t} - w_{i,t} \upsilon_{i,t}\right\}$$

where $n_t \leq n_t^c$ is the number of workers who have elected firm i as a trading partner. The date-t (expected) payoff to a worker is $w_{i,t} \upsilon_{i,t}$.

Proposition 2 (*Wage Posting Equivalence*) Suppose that at date $t = 0$, $l_b(\Lambda_{j,t,-1}) \leq I_{t,0}^1$ and $l_p(\Lambda_{j,t,-1}) \leq I_{t,0}^2$ for all $j = 1, 2, ...n_\Delta^c$. Then the solution of the wage posting problem for $x_{i,0} = I_{t,0}^1$ coincides with the equilibrium outcome of the signaling-and-bargaining game.

Proof. *First, I prove that the equilibrium outcome of the signaling game is a feasible strategy of the wage-posting game if $x_0 = I_{t,0}^1$. Consider the signaling-and-bargaining game. At date $t = 0$, the conditions for Lemmata [1] and [2] are met and the optimal signal is a generic $m_0 \in [\underline{b}, \overline{b}] \times [1, \underline{\rho}]$ such that $m_0^1 \geq I_{0,0}^1$. During the bargaining game, every worker demands the wage $\left(m_0^1, m_0^2\right)$. The demand is immediately accepted by the firm. The public signal is $I_{1,0}$ such that $I_{1,0}^1 = m_0^1$ and $I_{1,0}^2 = m_0^1 m_0^2$. The payoff to the firm is $n_0 max\{0, b\rho_0 - m_0^1 m_0^2\}$. At the beginning of date $t = 1$, the old employees have a common belief $\Lambda_{1,-1}$, such that $l_b(\Lambda_{1,-1}) = m_t^1$ and $l_p(\Lambda_{1,-1}) =$*

m_t^1, where the last equality is obtained via Bayes' Law (the transitory component of productivity ρ_1 is stochastically independent of ρ_0). Newly contacted workers have a belief Λ^n about the firm, which is derived from the (common knowledge) properties of the stochastic process. By assumption, $l_b(\Lambda^n) = l_p(\Lambda^n) = \underline{b}$. Therefore, the conditions for Lemmata [1] and [2] are satisfied at date $t = 1$ as well. The optimal signal of the firm is a generic $m_1 \in [\underline{b}, \overline{b}] \times [1, \underline{\rho}]$ such that $m_1^1 \geq I_{1,0}^1 = m_0^1$. The argument proceeds by induction. By labelling $(m_t^1, m_t^2) = (w_t, \upsilon_t)$ and $m_t^1 = x_t$, it follows that the equilibrium outcome of the signaling is a feasible strategy of the wage-posting game. Similalrly, I can prove that every feasible strategy of the wage-posting game is a feasible strategy of the the signaling game. Therefore, the solution of the wage-posting game coincides with the equilibrium outcome of the signaling game. ■

A.3 The Signaling Game: Noisy Traders

In this subsection, I vindicate the conjecture about the workers' interpretation of signals held in the derivation of Lemmata [1]-[2] and of Proposition [2]. The informative content of a message $m \in M$ depends on the types of firm that select the action m in equilibrium, conditional on the observable history of play. In any equilibria where signals are not perfectly revealing about the firm's type, there exists partial pooling. Coversely, if the signaling strategy is strictly monotonic in the firm's type, the signal is invertible and workers can perfectly identify a firm's type from its wage offer m. In a related paper (Menzio, 2004), I study the set of equilibria of the Signal Posting game and show that they feature partial pooling of firm types. More specifically, the type-space is partitioned into intervals and every firm in the interval posts the same signal. The maximum fineness of the partition depends on the incentives that firms have to attract workers.[26] Technically, the characterization of the optimal firm's strategy in the context of a "pooling" equilibrium is quite complicated. First, because the consequences of posting a message depend on the set of firms that follows that strategy in equilibrium, there are typically multiple equilibria of the signaling game.[27] Secondly, the payoff of the firm changes discontinuously with the signal posted and first order conditions cannot be used to identify the equilibrium strategy. In order to sideline these two problems, I introduce some exogenous noise in the model. None of the qualitative

[26] The main result in Menzio (2004) is that the classic Burdett-Judd condition for equilibrium wage dispersion does not apply. In fact, when the probability that a worker is in contact with two firms is above a certain threshold, every firm type has an incentive to "brag" about the quality of the job opening. In equilibrium, job advertisements are not informative and the wage distribution is degenerate.

[27] On this point the reader can convice himself by referring to Crawford and Sobel (1982).

results derived in Section [3] relies on this assumption, but the characterization of the equilibrium is greatly simplified.

More specifically, I assume that a measure $\xi \in (0,1)$ of the firms populating the economy are "noisy traders." The productivity of each of these firms is a vector $\left(b_i, \rho_{i,t}\right)$, where b_i is the persistent component of productivity and $\rho_{i,t}$ is a transitory shock. As for a "strategic firm," b_i is the realization from the CDF $\Gamma(b)$ and $\rho_{i,t}$ is distributed according to $\Phi(\rho|z_t)$. But the signaling behavior of the noisy traders is not the given by the solution of an optimization problem, rather is exogenous. In particular, after any on-equilibrium history of play, the behavioral firm i follows a stationary strategy $\left(m^1_{i,t}, m^2_{i,t}\right) = (\tilde{m}_i, 1)$, where $\tilde{m}_i$ is a uniformly distributed random variable with support $[\underline{b}, b_i]$. After an off-equilibrium signal $\left(m^1_{i,t}, m^2_{i,t}\right) \neq (\tilde{m}_i, 1)$ is observed, the firm's signaling strategy is $m^{off}_i = \left(\max\left\{m^1_{i,t}, \tilde{m}_i\right\}, 1\right)$. On the other hand, as discussed in Section [3], strategic firms post the stationary signal $(m^1_{i,t}, m^2_{i,t}) = (w(b), 1)$ where $w(b)$ is a strictly increasing function such that $w(b) \leq b$.

When a newly contacted worker receives a wage offer $\left(m^1_{i,j,t}, m^2_{i,j,t}\right) = \left(m^1, 1\right)$, the posterior belief is a (full-support) distribution $\Lambda_{i,j,t}$ such that $l_b(\Lambda_{i,j,t}) = l_p(\Lambda_{i,j,t}) = m^1$. When the worker reaches the firm, he observes a signal $I_{i,t,0}$ such that

$$(I^1_{i,t,0}, I^2_{i,t,0}) = \left(m^1, m^1\right)$$

At date $t+1$, the firm posts the wage $\left(m^1_{i,j,t+1}, m^2_{i,j,t+1}\right) = \left(m^1, 1\right)$. For as long as the worker remains employed with firm i, her beliefs remain unchanged.

Assumption [1] has described the off-equilibrium conjectures during the bargaining game. Here, I follow the same logic to identify the belief updating rule after an off-equilibrium signal is sent.

Assumption 2 (*Updating Rules II: Signaling*) Let Λ describe the full-support beliefs of the worker prior to observing an off-equilibrium signaling move.

(a) If a wage offer $\left(m^1_{i,j,t}, m^2_{i,j,t}\right)$ is advanced by the firm and either $m^1_{i,j,t} > l_b(\Lambda)$ or $m^1_{i,j,t} \cdot m^2_{i,j,t} > l_p(\Lambda)$, then the full-support posterior belief is a probability distribution Λ' such that

$$\left(l_b(\Lambda'), l_p(\Lambda')\right) = \left(\max\left\{l_b(\Lambda), m^1_{i,j,t}\right\}, \max\left\{l_p(\Lambda), m^1_{i,j,t} \cdot m^2_{i,j,t}\right\}\right) \tag{88}$$

(b) If a wage offer $\left(m^1_{i,j,t}, m^2_{i,j,t}\right)$ is advanced by the firm and both $m^1_{i,j,t} \leq l_b(\Lambda)$ and $m^1_{i,j,t} \cdot m^2_{i,j,t} \leq l_p(\Lambda)$, then the full-support posterior belief is a probability distribution $\Lambda' = \Lambda$.

Given the technical assumptions about the behavior of noisy traders and the way workers interpret off-equilibrium messages, the conjecture about the informativeness of signals held in Lemmata [1]-[2] and Proposition [2] is verified. Therefore, the signaling-and-bargaining problem has a simple reduced-form in which the firm is free to choose its wage policy, subject to a *Non-Discrimination* and *Consistency* constraints. The first restriction requires every worker to receive the same wage independently of her particular employment history (tenure, previous employer,...). The *Consistency* restriction requires the offered wage to be a possible realization of firm's productivity, conditional on the information previously revealed by the firm. Section [3] characterizes the limit economy as $\xi \to 0$.

A.4 A Micro-Foundation for the Burdett-Mortensen model of Wage Dispersion

The *Signal Posting* model of wage determination is technically an intra-firm bargaining model à la Stole and Zwiebel, but economically is a wage posting model à la Burdett and Mortensen. In BM98, a firm i is characterized by a constant level of labor productivity b_i and chooses its wage w_i. The firm-wage w_i is paid to every worker that trades with the firm, irrespective of her tenure, inital employment status or outside offers. Formally, the wage strategy of the firm is exogenously constrained to the real line. A recent paper by Stevens (2004) shows that the restriction on the strategy space of the firm is not without loss of generality. The full-commitment wage strategy of the firm is a two-part wage. Upon hiring, the worker makes a payment to the employer and, from that date on, the worker recieves the entire product of the match. The intial payment addresses the division of rents between the two trading partners and the second part of the contract guarantees allocative efficiency.[28] Moreover, the first part of the contract should be contingent on the current employment status of a contacted worker.[29] In the *Signal Posting* model, the wage outcome is not worker-specific, precisely as it is assumed in BM98. Moreover, for $\overline{\rho} = 1$, $\delta \to 1$ and $\overline{b} > \underline{b} = w_0$, both models have the identical equilibrium wage dispersion. In this sense, the theory of wage determination described in this paper is a micro-foundation for the Burdett-Mortensen model. This result is of some interest on its own.

[28] Burdett and Coles (2003) characterize the optimal tenure-dependent wage in an environment where workers are risk-averse and cannot access the capital market. They find that the equilibrium contract features a positive tenure effect.

[29] This point has been recently stressed by Moscarini (2004).

B Proofs for Section [3]

Proof of Proposition [5] *Claim (1). The condition $w(b) \leq b$, for every $b \in \left[\underline{b}, \overline{b}\right]$, is satisfied in a neighborhood of $\lambda = 0$.*

Proof. *From [63], we can derive an upper bound $\overline{w}'$ on the derivative of the wage function*

$$w'(b) \leq \overline{w}' = 2\lambda\overline{\gamma}\left(\frac{\overline{b}\hat{\rho} - w_0}{1-\delta} + (1-\delta)\,\overline{b}\,(\overline{\rho} - \hat{\rho})\right)$$

The first part of the claim is then verified because $\overline{w}' < 1$ for λ small and $w(\underline{b}) = \underline{b}$.

*Claim (2). The wage rigidity condition [**??**] is satisfied under (a)-(c).*

Proof. *Using the equation $F(w(b)) = \Gamma(b)$, the condition [**??**] can be rewritten as*

$$\frac{b\;\hat{\rho} - w(b)}{1 - \delta\left(1 - \sigma - \lambda\overline{\Gamma}(b)\right)}\left(1 - \sigma - \lambda\overline{\Gamma}(b)\right) \geq b\,(\overline{\rho} - \hat{\rho})\left(\sigma + \lambda\overline{\Gamma}(b)\right) \tag{89}$$

The expression [89] constitutes the starting point for the derivation of all three results (a)-(c).

(b) Consider a sequence of economies characterized by a constant unemployment rate u^*. Therefore the job-finding probability is proportional to the displacement rate: $\lambda = k\sigma$, where $k = u^{*-1}(1-u^*)$. The wage rigidity condition [89] reads

$$\frac{b\;\hat{\rho} - w(b)}{1 - \delta\left(1 - \sigma\left(1 + k\overline{\Gamma}(b)\right)\right)}\left(1 - \sigma\left(1 + k\overline{\Gamma}(b)\right)\right) \geq b\,(\overline{\rho} - \hat{\rho})\,\sigma\left(1 + k\overline{\Gamma}(b)\right) \tag{90}$$

For $\sigma = 0$ the expression above holds as a strict inequality for every b because $w(b;\sigma,u^*) = w_0 = \underline{b}$. Given the boundedness of the wage function, the condition [90] holds whenever

$$\frac{b\;\hat{\rho} - \bar{w}'(b - \underline{b})}{1 - \delta\,(1 - \sigma\,(1+k))}\,(1 - \sigma\,(1+k)) \geq b\,(\overline{\rho} - \hat{\rho})\,\sigma$$

Since $\bar{w}'$ is decreasing in σ, there is a σ^* such that for any $\sigma \leq \sigma^*$ the condition [90] holds for every b.

(c) For $\overline{\rho} = \hat{\rho}$, the condition [89] holds for every b as a strict inequality. The difference between the left and the right hand side is bounded below by

$$\frac{\min_b(b\;\hat{\rho} - \bar{w}'(b - \underline{b}))}{1 - \delta\,(1 - \sigma - \lambda)}\,(1 - \sigma - \lambda) > 0$$

By continuity [89] holds for any $\overline{\rho}$ in a neighborhood of $\hat{\rho}$.

(a) Consider a family of economies, indexed by the duration Δ of a period. Expressing δ, λ and σ as exponential functions of Δ, the condition [89] becomes

$$\frac{(b\,\hat{\rho} - w(b;\Delta))\,\Delta}{1 - e^{-\delta\Delta}\,\left(e^{-\sigma\Delta} - (1 - e^{-\lambda\Delta})\,\overline{\Gamma}(b)\right)}\left(e^{-\sigma\Delta} - \left(1 - e^{-\lambda\Delta}\right)\overline{\Gamma}(b)\right) \geq b\Delta\,(\overline{\rho} - \hat{\rho})\left(1 - e^{-\sigma\Delta} + \left(1 - e^{-\lambda\Delta}\right)\overline{\Gamma}(b)\right) \quad (91)$$

and the wage function $w(b;\Delta)$ is [64] under the appropriate substitutions. In the limit for $\Delta \to 0$ the right hand side of [91] converges to zero, while the left hand side is bounded below by

$$\frac{\min_b\,(b\hat{\rho} - \bar{w}'(b - \underline{b}))}{\delta + \sigma + \lambda} > 0$$

Claim (3). The cross derivative of the objective function with respect to firm size and wage components are non-positive for any (σ, λ) such that $\sigma + \lambda \leq 1/2$.

Proof. *First notice that*

$$\left(\frac{\partial^2 Obj(e,x,\rho,w,\upsilon;b)}{\partial w \partial n} - \frac{\partial^2 Obj(e,x,\rho,w,\upsilon;b)}{\partial \upsilon \partial n}\right)|_{\{w=x,\upsilon=1\}} \geq 0$$

Then some manipulations lead to

$$\frac{\partial^2 Ob(e,x,\rho,w,\upsilon;b)}{\partial w \partial n} \leq$$
$$\lambda F'(w)\left(\frac{b\hat{\rho} - w(b)}{1 - \delta\left(1 - \sigma - \lambda\overline{\Gamma}(b)\right)} + b(\overline{\rho} - \hat{\rho})(1 - \delta)\right) - (1 - \sigma - \lambda) = \sigma + \lambda - \frac{1}{2}$$

where the second line makes use of [60]

C The DMP Model

In this part of the Appendix, I formally introduce the matching-and-bargaining model, which has been used in Section [4] as the point of comparison in the quantitative assesment of the high-frequency wage rigidity theory. In the matching-and-bargaining model, workers search for employment opportunities only when unemployed. The job finding rate depends on the vacancy rate v_t, through a constant returns to scale matching function

$$m\,(u_t, v_t) = Av_t^{\alpha}u_t^{1-\alpha}$$

The productivity of a match between a worker and a firm is given by a time-dependent random variable p_t, that follows the two-states Markov process described in the matrix [65], i.e.

p_t/p_{t+1}	p_1	p_2
p_1	π	$1-\pi$
p_2	$1-\pi$	π

On the other side of the market, a firm can post as many vacancies as desired at a constant unit cost c. The probability of filling a vacancy is given by $m(u_t, v_t)/v_t$. Finally, the model is clsoed by assuming that the wage w_t is set according to the Axiomatic Nash Bargaining Solution, taking the outside options of the two parties as the disagreement point.

Following the standard notation, let $\theta_t \equiv v_t/u_t$ denote the degree of market tightness and conjecture that $\theta_t = \theta_j$ if $p_t = p_j$, for $j = 1, 2$. The value of unemployment when the market is in state j is given by

$$U_j = w_0 + \delta\left[\pi\left(A\theta_j^\alpha W_j + \left(1 - A\theta_j^\alpha\right)U_j\right) + (1-\pi)\left(A\theta_{-j}^\alpha W_{-j} + \left(1 - A\theta_{-j}^\alpha\right)U_{-j}\right)\right]$$

The value to a worker of holding a job is

$$W_j = w_j + \delta\left[\pi\left(\sigma U_j + (1-\sigma)W_j\right) + (1-\pi)\left(\sigma U_{-j} + (1-\sigma)W_{-j}\right)\right]$$

From the firm's perspective, the value of posting a vacancy is

$$V_j = -c + \delta\left[\pi\left(A\theta_j^{\alpha-1}\Pi_j + \left(1 - A\theta_j^{\alpha-1}\right)V_j\right) + (1-\pi)\left(A\theta_{-j}^{\alpha-1}\Pi_{-j} + \left(1 - A\theta_{-j}^{\alpha-1}\right)V_{-j}\right)\right]$$

and the value of a worker to the firm is

$$\Pi_j = (p_j - w_j) + \delta\left[\pi\left(\sigma V_j + (1-\sigma)\Pi_j\right) + (1-\pi)\left(\sigma V_{-j} + (1-\sigma)\Pi_{-j}\right)\right]$$

Because of the Axiomatic Nash Bargaining Solution, the wages w_1 and w_2 are such that the worker obtains half of the surplus from the match

$$W_j - U_j = \frac{1}{2}\left(W_j + \Pi_j - U_j\right)$$

Finally, the cost of posting a vacancy is constant and therefore, the free entry conditions $V_j = V_{-j} = 0$ hold.

Calibration In order to make the DMP model comparable to the HFWR model, I mantain the same stochastic process across the two simulations. The other parameters are presented in Table [5]. In a recent analysis of the JOLTS dataset, Hall (2003) estimates the elasticity of the matching function with respect to vacancies and finds that $\alpha = 0.77$. The efficiency parameter A of the matching function is set so that the steady-state unemployment rate matches the historical average. The destruction rate σ is set to match the layoff rate reported in Hall (2005). Finally, the cost of posting a vacancy is calibrated so that the steady-state vacancy rate is 2.8 percent.

Table 1: Cyclical Behavior of U.S. Labor Market Aggregates, 1954:I-1991:II

		Cross-Correlation of Real GNP with:										
Variable	*Volatility (%SD)*	*X(-5)*	*X(-4)*	*X(-3)*	*X(-2)*	*X(-1)*	*X*	*X(+1)*	*X(+2)*	*X(+3)*	*X(+4)*	*X(+5)*
Real Gross National Product	1.72	-.02	.16	.38	.63	.85		.85	.63	.38	.16	-.02
Hours (Household Survey)	1.49	-.10	.05	.25	.46	.70	.86	.85	.74	.58	.38	.17
Employment	1.09	-.17	-.03	.16	.38	.63	.83	.88	.80	.65	.46	.25
Hours per Worker	0.54	.07	.20	.36	.49	.64	.70	.58	.42	.28	.12	-.02
Hours (Establishment Survey)	1.66	-.23	-.07	.14	.39	.67	.88	.91	.80	.63	.42	.22
GNP/Hours (Household Survey)	0.87	.12	.23	.33	.47	.50	.51	.22	-.01	-.24	-.32	-.34
Average Hourly Real Compensation (Business Sector)	0.93	.35	.39	.41	.43	.41	.35	.25	.16	.05	-0.7	-.18
Real Employee Compensation (NIPA)/Hours (Household Survey)	0.65	-.11	-.11	-.13	.06	.02	.10	.13	.14	.10	.08	.04
Real Employee Compensation (NIPA)	1.54	-.14	.00	.18	.41	.67	.88	.88	.76	.59	.38	.18
Employee Compensation (NIPA)/GNP		-.21	-.32	-.44	-.53	-.51	-.46	-.13	.13	.32	.39	.38
		*Cross-Correlation of *:*										
Employment and Average Labor Productivity** (X)	1.09			.73	.68	.57	.35	.09	-.15	-.32		
Vacancies and Unemployment (X)	12.54			-.36	-.61	-.82	-.95	-.93	-.77	-.54		
GNP and Labor Share (X)	1.07			-.61	-.73	-.78	-.74	-.48	-.22	-.00		

Source: Finn E. Kydland (1995), (*) Source: M. Merz (1995) using CITIBASE data for the period 1959:I-1988:II, (**) Average Labor Productivity is defined as Real GNP over Employment

Table 2: Parameter Values

Symbol	*Description*	*Value*	*Source*
δ	Discount Rate	0.9847	Calibration: match 6 percent annual interest rate
w_0	Unemployment Benefit	1	Normalization
α	Elasticity of Contacts to Vacancies	1	Theoretical Assumption: Constant Returns to Scale Technology
A	Efficiency of Matching Function	4.3	Calibration: match 0.065 stationary unemployment rate
σ	Displacement Rate	0.04	Micro-Evidence: JOLTS
η	Convexity of the Cost Function	0.5	Free
ψ	Coefficient of the Pareto Distribution $\Gamma(b)$	10	Calibration: match ratio of third to first quartile of the wage distribution
$\rho(z_2)/\rho(z)$	Relative Average Productivity in Expansion	1.019	Calibration: match 1.87 std of GNP
π^*	Shock Autocorrelation	0.75	Calibration: match 0.875 quarterly autocorrelation of GNP
g	Intermediate Goods	1.34	Calibration: match 0.55 steady-state labor share

Table 3: Dynamic Correlations

	Correlation	*X(-3)*	*X(-2)*	*X(-1)*	*X*	*X(+1)*	*X(+2)*	*X(+3)*
HFWR(*)	GNP and GNP (X)	**.54**	**.69**	**.87**	**1**	**.87**	**.69**	**.54**
U.S. Data		*.46*	*.68*	*.87*	*1*	*.87*	*.68*	*.46*
DMP		.45	.57	.74	1	.74	.57	.45
Merz		.27	.50	.78	1	.78	.50	.27
HFWR	Unemployment and Unemployment (X)	**.47**	**.63**	**.83**	**1**	**.83**	**.63**	**.47**
U.S. Data		*.43*	*.69*	*.90*	*1*	*.90*	*.69*	*.43*
DMP		.55	.71	.87	1	.87	.71	.55
Merz		.22	.42	.68	1	.68	.42	.22
HFWR	Unemployment and Vacancies (X)	**-.63**	**-.81**	**-.94**	**-.61**	**-.45**	**-.35**	**-.27**
U.S. Data		*-.53*	*-.77*	*-.93*	*-.95*	*-.82*	*-.61*	*-.36*
DMP		-.52	-.61	-.57	-.11	+.03	+.07	+.06
Merz		-.09	-.03	-.04	-.15	-.82	-.59	-.40
HFWR	GNP and Labor Share (X)	**-.55**	**-.70**	**-.88**	**-.96**	**-.72**	**-.54**	**-.41**
U.S. Data		*-.61*	*-.73*	*-.78*	*-.74*	*-.48*	*-.22*	*-.00*
DMP		+.36	+.49	+.68	+.94	+.73	+.56	+.44
Merz		-.02	-.09	-.23	-.77	-.51	-.33	+.22
HFWR	Employment and Average Labor Productivity (X)	**.83**	**.97**	**.68**	**.52**	**.42**	**.33**	**.27**
U.S. Data		*.73*	*.69*	*.58*	*.35*	*.10*	*-.15*	*-.32*
DMP		.53	.64	.71	.53	.33	.21	.14
Merz		.30	.59	.96	.60	.43	.28	.15

(*) HFWR is the benchmark calibration of the model in Section [4]. The row "U.S. Data" reports the correlations from the CITIBASE data bank. The row "DMP" refers to a version of the canonical matching-and-bargaining model subject to the same shocks as in the HFWR calibration. The data under "Merz" refer to the simulation with fixed search intensity in Merz (1995). The results in "Merz" are not directly comparable with mine, because the underlying productivity shocks have different variance and persistence.

Table 4: Second Moments

Statistic	σ_Y	σ_u/σ_Y	σ_{ALP}/σ_Y	σ_V/σ_Y
HFWR(*)	**1.87**	**6.45**	**0.64**	**7.89**
U.S. Data	*1.87*	*6.11*	*0.68*	*7.32*
DMP	1.53	2.56	0.89	2.68
Merz	1.07	4.63	0.74	6.38
Statistic	σ_{1-u}/σ_Y	σ_W/σ_Y	σ_{LS}/σ_Y	
HFWR(*)	**0.44**	**0.09**	**0.70**	
U.S. Data	*0.54*	*0.37*	*0.53*	
DMP	0.17	2.68	0.45	
Merz	0.36	0.34	0.47	

HFWR is the benchmark calibration of the model in Section [4]. The row "U.S. Data" reports the correlations from the CITIBASE data bank. The row "DMP" refers to a version of the canonical matching-and-bargaining model subject to the same shocks as in the HFWR calibration. The data under "Merz" refer to the simulation with fixed search intensity in Merz (1995).

Table 5: Parameter Values for the DMP model			
Symbol	*Description*	*Value*	*Source*
δ	Discount Rate	0.9847	Calibration: match 6 percent annual interest rate
w_0	Unemployment Benefit	1	Normalization
α	Elasticity of Contacts to Vacancies	0.77	Micro-Estimate (source Hall, 2003)
A	Efficiency of Matching Function	1.12	Calibration: match 0.065 stationary unemployment rate
σ	Displacement Rate	0.04	Micro-Evidence (source Hall, 2003)
c	Vacancy Cost	1.15	Calibration: match 0.028 stationary vacancy rate
E(p)	Average Labor Productivity	1.61	Comparability: maintain an average wage of 1.55
p_2/ E(p)	Relative Average Productivity in Expansion	1.019	Comparability
π^*	Shock Autocorrelation	0.75	Comparability
g	Intermediate Goods	1.34	Comparability

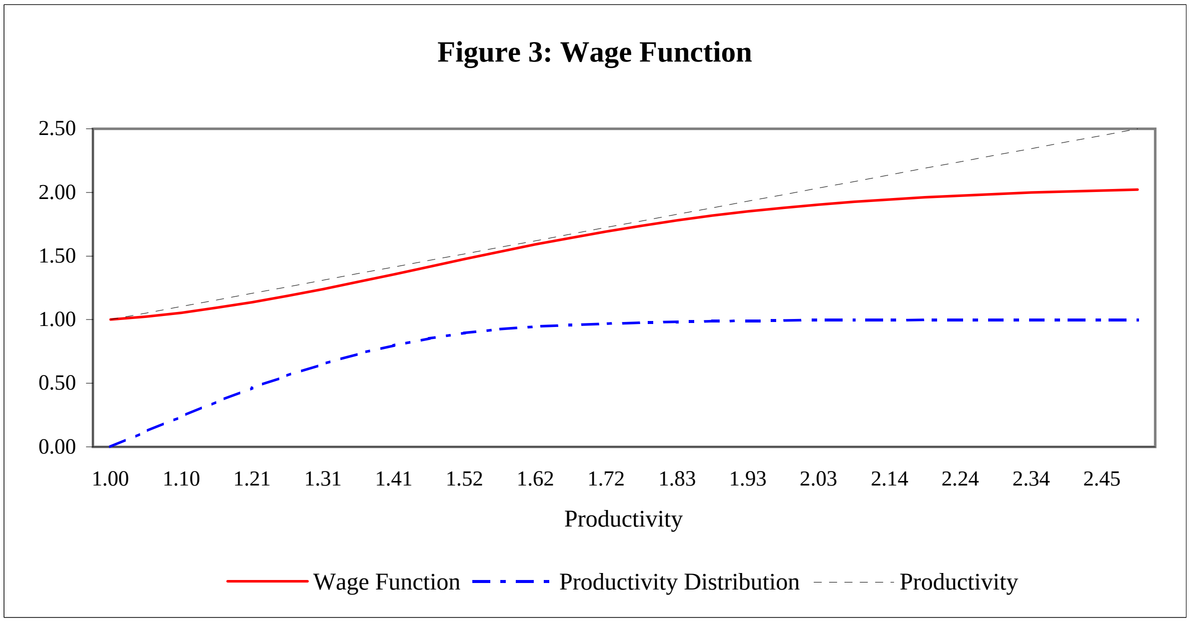


Figure 3: Wage Function
2.50
2.00
1.50
1.00
0.50
0.00
1.00
1.10
1.21
1.31
1.41
1.52
1.62
1.72
1.83
1.93
2.03
2.14
2.24
2.34
2.45
Productivity
Wage Function
Productivity Distribution
Productivity

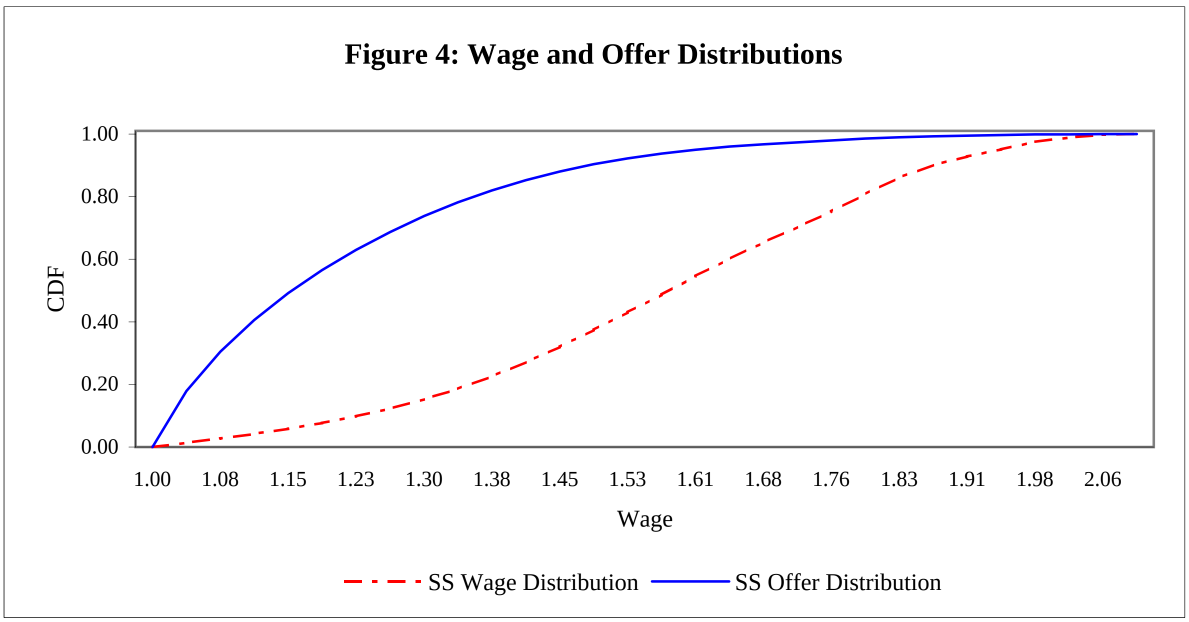


Figure 4: Wage and Offer Distributions
CDF
1.00
0.80
0.60
0.40
0.20
0.00
1.00
1.08
1.15
1.23
1.30
1.38
1.45
1.53
1.61
1.68
1.76
1.83
1.91
1.98
2.06
Wage
SS Wage Distribution
SS Offer Distribution

Figure 5: Return on Vacancy

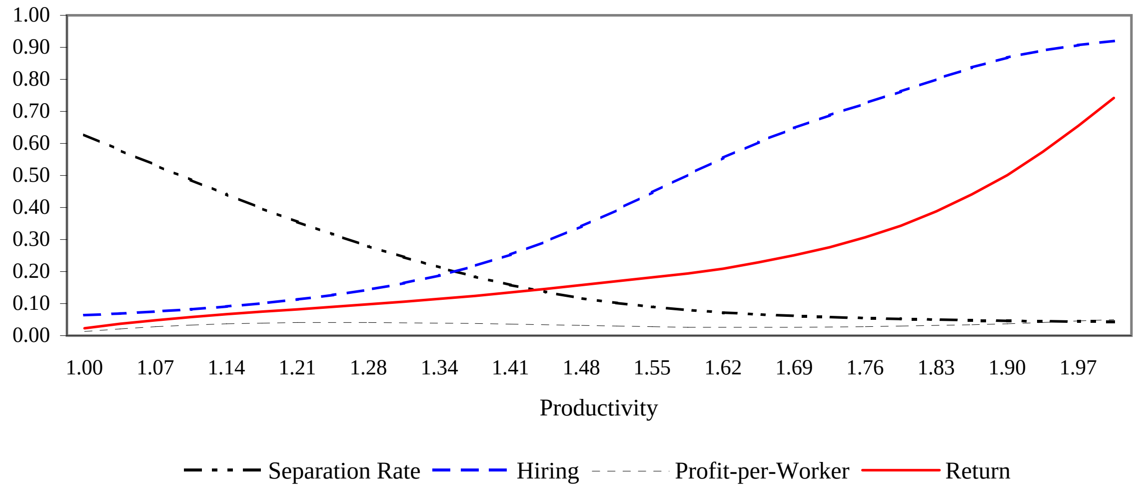


Figure 6: Labor Productivity

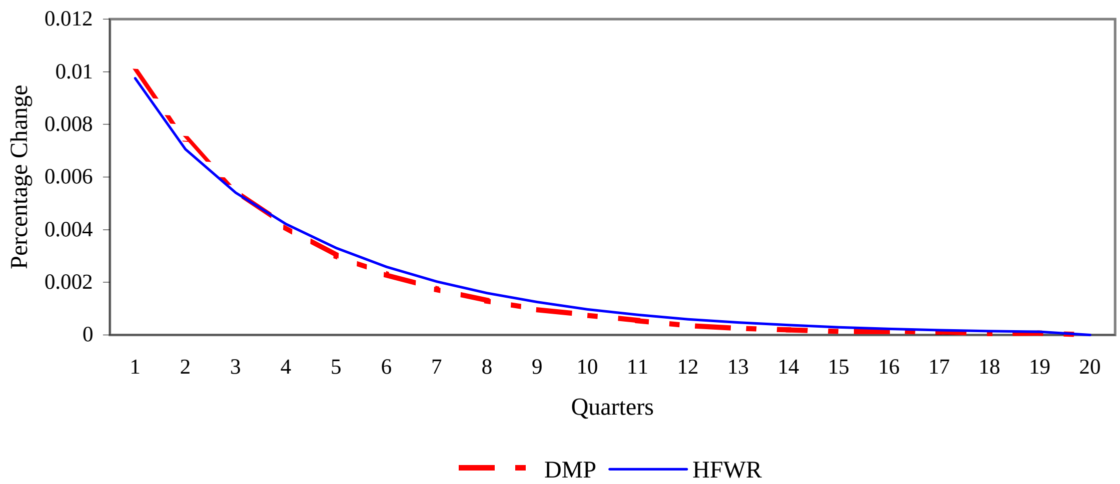


Figure 7: Unemployment Rate

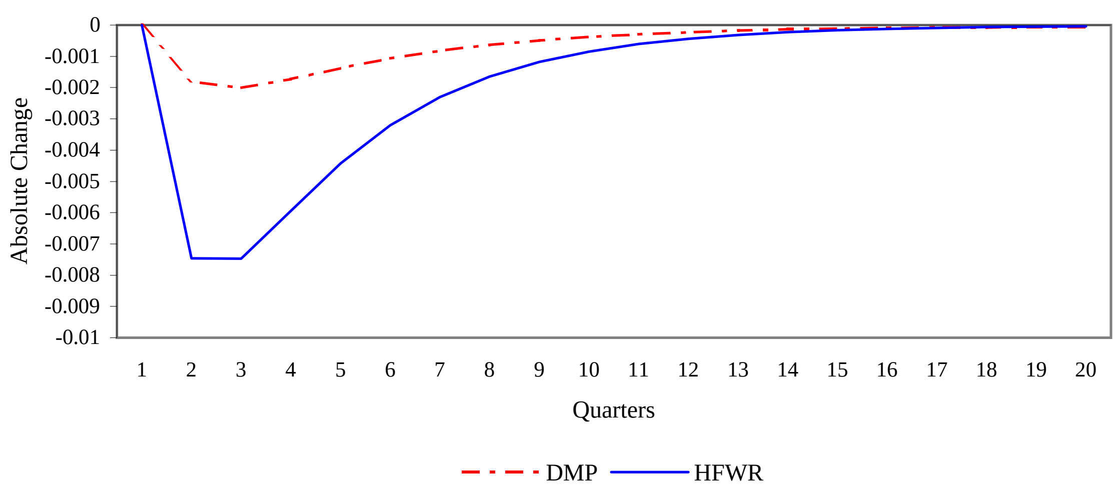


Figure 8: Total Vacancies

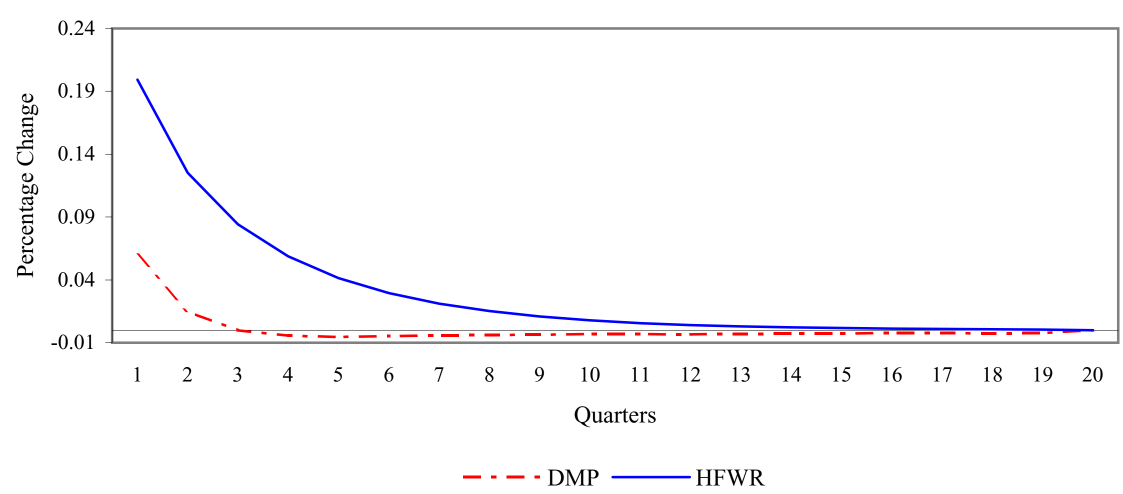


Figure 9: Average Wage

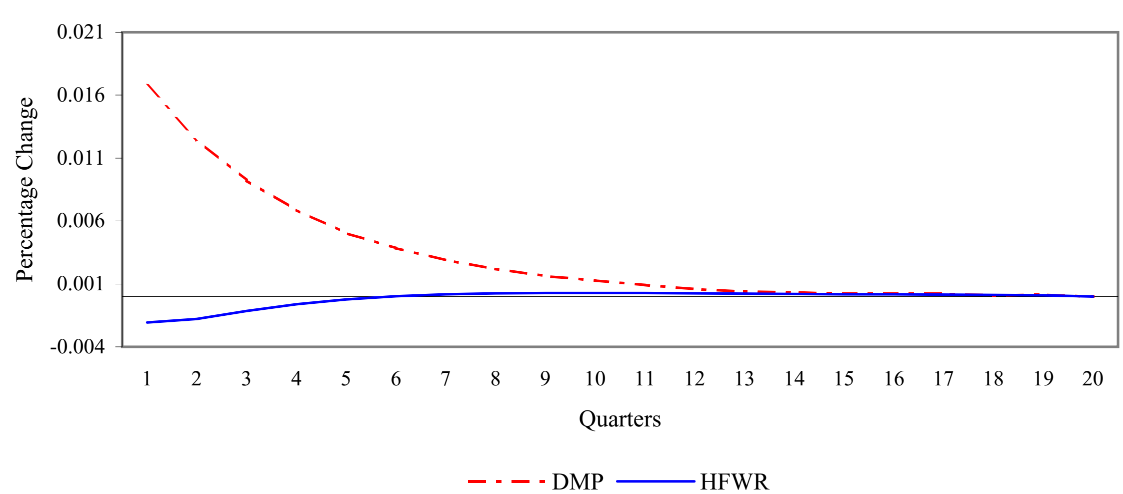


Figure 10: Aggregate Output

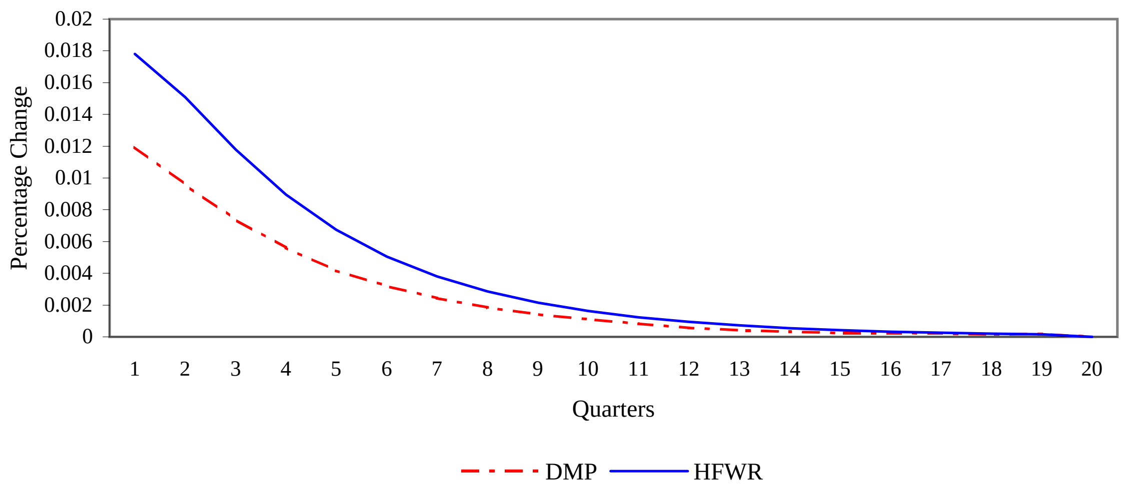
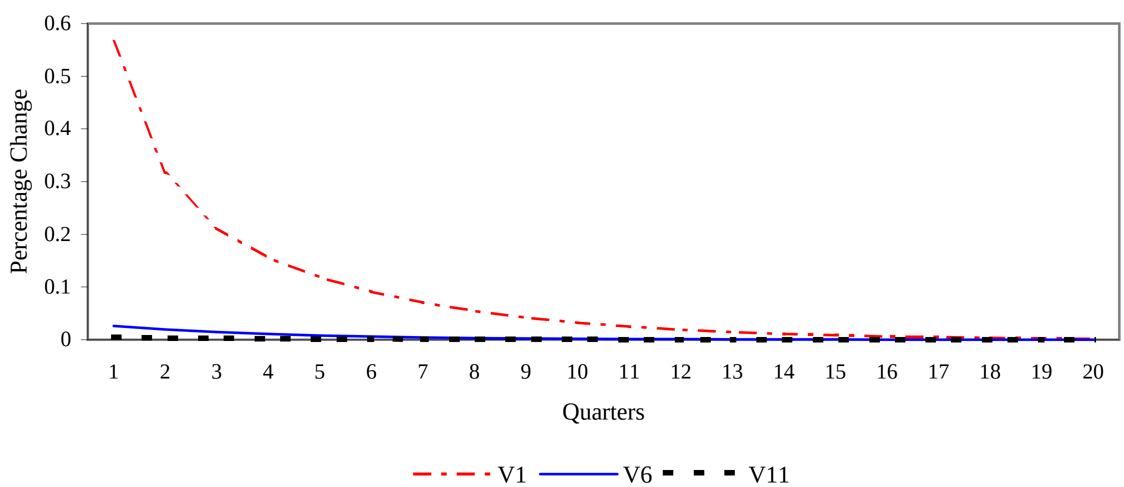


Figure 11: Disaggregated Vacancies



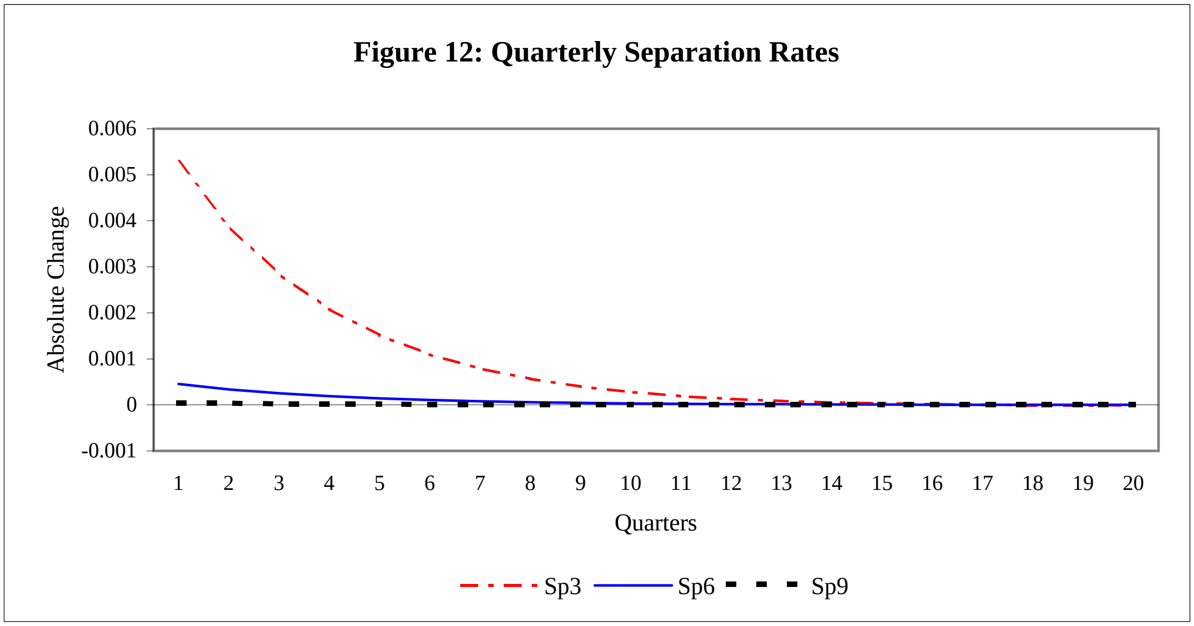


Figure 12: Quarterly Separation Rates
Absolute Change
0.006
0.005
0.004
0.003
0.002
0.001
0
-0.001
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Quarters
Sp3
Sp6
Sp9

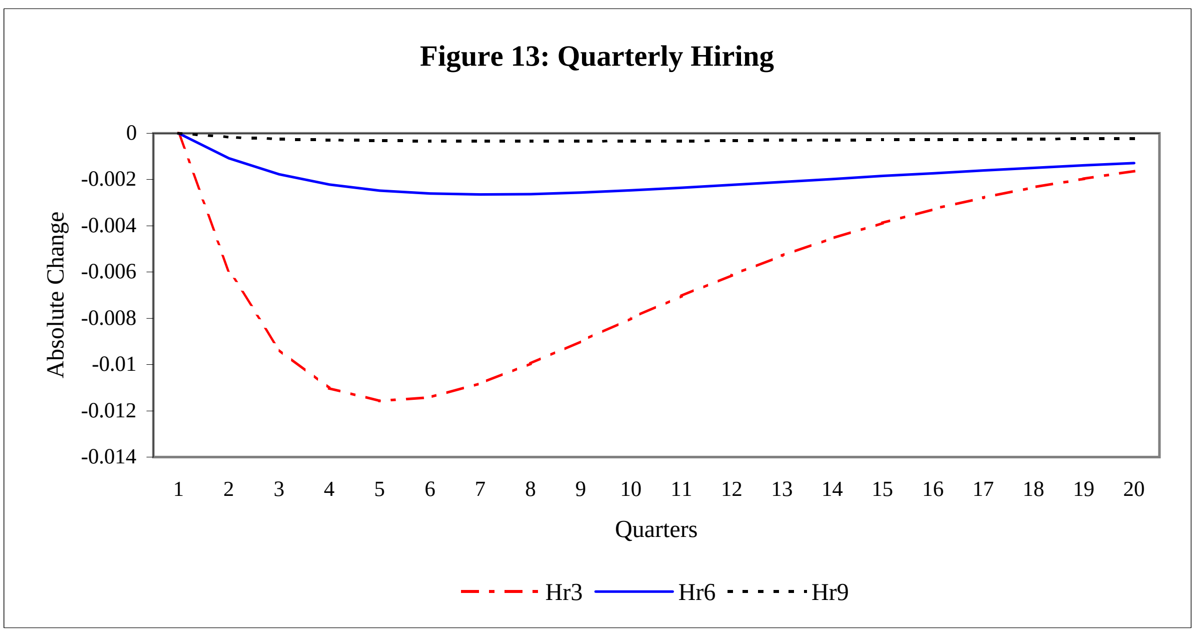


Figure 13: Quarterly Hiring
Absolute Change
0
-0.002
-0.004
-0.006
-0.008
-0.01
-0.012
-0.014
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Quarters
Hr3
Hr6
Hr9

Figure 14: Composition Effects