

STICKY PRICES: A NEW MONETARIST APPROACH

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Abstract

Why do some sellers set nominal prices that apparently do not respond to changes in the aggregate price level? In many models, prices are sticky by assumption; here it is a result. We use search theory, with two consequences: prices are set in dollars, since money is the medium of exchange; and equilibrium implies a nondegenerate price distribution. When the money supply increases, some sellers may keep prices constant, earning less per unit but making it up on volume so profit stays constant. The calibrated model matches price-change data well. But, in contrast to typical sticky-price models, money is neutral. (JEL: E52, E31, E42)

1. Introduction

Arguably the most difficult question in macroeconomics is this: Why do some sellers set prices in nominal terms that apparently do not adjust in response to changes in the aggregate price level? This seems to fly in the face of elementary microeconomic principles. Shouldn't every seller have a target relative price, depending on real factors, and therefore when the aggregate nominal price level increases by some amount, say

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due to an increase in the money supply, shouldn't every seller necessarily adjust his nominal price by the same amount? In many popular macro models, including those used by most policy makers, prices are sticky *by assumption*, in the sense that there are either restrictions on how often they can change, following Taylor (1980) or Calvo (1983), or there are real resource costs to changing them, following Rotemberg (1982) or Mankiw (1985).¹ We deliver stickiness *as a result*, in the sense that sellers set prices in nominal terms, and some may choose not to adjust in response to changes in the aggregate price level, even though we let them change whenever they like and at no cost. Moreover, in contrast with other theories with sticky prices, we construct our model so that money is neutral: the central bank cannot engineer a boom or end a slump simply by issuing currency. Hence, while we in a sense provide microfoundations for the core ingredient in Keynesian economics—sticky prices or nominal rigidities or whatever one likes to call it—our theory has very different policy implications.

We emphasize at the outset that our objective here is not to establish that monetary policy is neutral or nonneutral in the real world. That is beside the point. Our objective is to show formally two results: (1) one does not need to introduce technological restrictions or costs, as in Calvo- or Mankiw-style models, to generate price stickiness; and (2) the appearance of nominal rigidities does not logically imply that policy can exploit these rigidities, as some economists think. To explain our motivation by analogy to a rather famous paper, Lucas (1972) describes a microfounded monetary model consistent with the observation that, in the data, there is a positive correlation between the aggregate price level (or money supply) and output (or employment), but policy cannot systematically exploit the relationship. That is, increasing inflation by printing money at a faster rate will not increase average output or employment. We think this was a good lesson. We similarly want to show that one can write down a microfounded monetary model consistent with some other observations, those concerning nominal price adjustments, but it is not possible for policy to exploit this. Monetary policy is neutral in the model by design—this is how we make the point that price stickiness does not logically imply nonneutrality.²

Not only does our model provide counterexamples to some popular beliefs about monetary theory and policy, we also argue that it is empirically reasonable, in the following sense. We show that our approach to price stickiness is successful, relative to alternative theories, at matching the salient features of the micro data on individual price dynamics.³ We can account for the average duration of prices in the data, for the fact that price changes are large on average, even though many changes are small, and that prices change more frequently (and not just by larger amounts) when inflation

1. We mention some related approaches, including “sticky information” and “rational inattention”, in the conclusion.

2. To be clear, *it is not the case that monetary policy in the model has no real effects*: changing the inflation or nominal interest rate has real consequences, as in any good monetary model, but *this has nothing to do with nominal rigidities*.

3. Empirical work on price stickiness includes, for example, Cecchetti (1985), Carlton (1986), Bills and Klenow (2005), Campbell and Eden (2007), Klenow and Kryvtsov (2008), Nakamura and Steinsson (2008), and Eichenbaum, Jamovich, and Rebelo (2009). See Klenow and Malin (2010) for a survey.

is higher. In contrast, simple menu cost theories cannot easily account for the second fact—that on average price changes are large even though many changes are small—while Calvo theories cannot easily account for the third—that the frequency of price changes increases with inflation. It is not our claim that somehow complicating or integrating existing theories cannot work, and there are some reasonably successful attempts in the literature, including for example Midrigan (2006). Our claim is that even a very basic version of our theory does a good job of matching the facts.

We think these findings are relevant for the following reasons. Despite the successes of, say, the New Classical and Real Business Cycle paradigms, they seem to miss one basic feature of the data: at least some nominal prices seem sticky in the sense defined previously (they do not respond to changes in the aggregate price level). One should want to know if this somehow invalidates these theories or their policy implications, and means the only valid theories and recommendation emanate from a Keynesian approach. It seems clear to us that the observation of price stickiness is one of the main reasons why many Keynesians are Keynesian. Consider Ball and Mankiw (1994), who we think representative, when they say: “We believe that sticky prices provide the most natural explanation of monetary nonneutrality since so many prices are, in fact, sticky.” They go on to claim that “based on microeconomic evidence, we believe that sluggish price adjustment is the best explanation for monetary nonneutrality.” And, “As a matter of logic, nominal stickiness requires a cost of nominal adjustment.” Some others that one might not think of as Keynesian present similar positions, including Golosov and Lucas (2003), who argue that “menu costs are really there: The fact that many individual goods prices remain fixed for weeks or months in the face of continuously changing demand and supply conditions testifies conclusively to the existence of a fixed cost of repricing.”⁴

We interpret these claims as containing three points related, respectively, to empirics, theory, and policy. The first claim is that price stickiness is a fact. The quotations assert this, and it is substantiated by numerous empirical studies, including those cited in footnote 3. We concede the point. We embrace the point! The second claim is that price stickiness implies “as a matter of logic” the existence of some technological constraint to price adjustment. We prove this wrong. We do so by displaying equilibria that match not only the broad observation of price stickiness, but also some of the more detailed empirical findings, with recourse to no technological constraints. The third claim, to which at least Ball and Mankiw seem to subscribe, is that price stickiness implies that money is not neutral and that this rationalizes Keynesian policy advice. We also prove this wrong. Our theory is consistent with the relevant observations, but money is neutral, which means that sticky prices simply do not constitute definitive evidence that money is nonneutral or that particular policy recommendations are warranted. To reiterate, the point here is not about whether money

4. The point here is not to pick on any particular individuals but to pick out some that apparently come from very different macro camps, in order to convey a general feeling in the profession about the implications of price stickiness. It is possible to find many more such quotations from many other economists, but we hope these suffice to make the point.

is neutral is the real world, it is rather about constructing a coherent, and we think compelling, economic environment with two properties: (1) it matches the sticky-price facts; and (2) it nevertheless delivers neutrality.

It is clear that the issues at hand concern monetary phenomena: Why are prices quoted in dollars? Why do they not all adjust to changes in the money supply? What does this imply about central bank policy? To study these questions, naturally, we use a monetary model. We work with a version of the New Monetarist framework, recently surveyed by Williamson and Wright (2010a,b) and Nosal and Rocheteau (2011). This framework tries to be explicit about details of the trading process, so that one can ask, who trades with whom, and how? Thus, specialization and search frictions can limit barter, while commitment and information frictions can limit credit, making money essential for at least some exchanges. Because the points we make are really quite general, we could also make them with other monetary models, including cash-in-advance, money-in-the-utility-function or overlapping-generations models, but we think the search approach is useful for several reasons. First, it is the approach used by most people these days doing monetary theory (if not monetary policy). Also, the framework has proved very tractable and easily generalizable in other applications. And a search-based approach not only can generate a role for money, it can generate endogenous price dispersion, which is an important element of our theory.

To explain this idea, first note that many New Keynesian models, such as those described in Clarida, Gali, and Gertler (1999) or Woodford (2003), generate price dispersion if and only if there is inflation. Suppose that in a stationary real environment a number of sellers set the same p_t at date t . Then, at $t_1 > t$, some seller is the first one allowed to change price and changes it to p_{t_1} , at date $t_2 > t_1$ a second seller is allowed to change, and so on. This induces price dispersion if and only if inflation is not zero. But the data suggest that there is price dispersion even during periods of low or zero inflation (something we first noticed in Campbell and Eden 2007). This suggests that it is important to work with models that can deliver price dispersion even without inflation. There are several candidate models, including Varian (1980), Albrecht and Axel (1984), or Stahl (1989), but we use Burdett and Judd (1983). In Burdett–Judd models, search frictions deliver price dispersion, and since these same frictions help generate a role for money, it is parsimonious in terms of assumptions to use a search-based framework. Burdett–Judd has also proved useful in other applications, including the large literature on labor markets following Burdett and Mortensen (1998); see Mortensen and Pissarides (1999) for a survey.

To understand Burdett–Judd, it helps to give a very brief history of search theory. The earliest models of McCall (1970) and Mortensen (1970) were partial equilibrium models, in the sense that they characterized the optimal search strategy of a searcher taking as given the distribution of prices, or wages in labor applications, posted by firms, and were soundly criticized on this point (for example, Rothschild 1973). Diamond (1971) set out to build a general equilibrium search model in which the price distribution was derived endogenously: first firms post prices, taken as given the prices of others; then individuals search over these firms as in the standard theory. What he found is that there is a unique equilibrium and it entails a degenerate price distribution. The proof is

easy. Given any $F(p)$, individuals use a reservation price R , buying when they sample the first $p \leq R$. But then there is no reason for any firm to set anything other than $p = R$. This proves equilibrium must have a single price. Moreover, the single price turns out to be the pure monopoly price. Since there could not be price dispersion, the result looked bad for search theory, but Diamond's findings also set off a wave of research on search, trying to generate endogenous price or wage dispersion.

The approach in Burdett and Judd (1983) is to make one, ostensibly minimal, change in the standard sequential search model: rather than sampling prices one at a time, suppose there is a positive probability of sampling two or more at once. Then it is not hard to see that equilibrium must entail a nondegenerate price distribution. We are more precise when we present the formal model, but the idea is this. Suppose all sellers in some set (with positive measure) set the same p . A buyer who samples two such sellers has to use some tie-breaking rule to pick one. This gives an individual seller a big incentive to lower price, to get the sale for sure. In fact, in equilibrium, all sellers charge different prices, and one can actually derive the closed-form solution for the distribution $F(p)$. The model captures standard results as special cases: when the probability that a buyer meets two or more sellers approaches 1, we converge to a single price and it is the perfectly competitive price; and when this probability approaches 0, we converge to Diamond's monopoly price.

We embed Burdett–Judd pricing into a dynamic New Monetarist model, where agents alternate between trading in centralized and decentralized markets, and in the latter market buyers use money as a medium of exchange because frictions preclude the use of credit.⁵ In equilibrium, sellers post prices in dollars, naturally, since this is how buyers are paying. As in the baseline Burdett–Judd model, at any date t , there is a continuous distribution of prices $F_t(p)$ with nondegenerate support $[p_t, \bar{p}_t]$. While the equilibrium pins down the distribution, it does not pin down the price of an individual seller: every seller gets the same profit from any $p_t \in [p_t, \bar{p}_t]$, because one that posts a low price earns less per unit but makes it up on the volume. When the money supply increases from M_t to M_{t+1} , the equilibrium distribution shifts to $F_{t+1}(p)$ with support $[p_{t+1}, \bar{p}_{t+1}]$. For this to happen, some sellers must change their prices, but not all of them: if an individual seller's price is $p_t \notin [p_{t+1}, \bar{p}_{t+1}]$ it must adjust; but if $p_t \in [p_{t+1}, \bar{p}_{t+1}]$ it may not.

As regards the question with which we started—Shouldn't every seller have a target real price, and therefore when M_t increases shouldn't every seller adjust his nominal price by the same amount?—the answer is No! Sellers do not have a unique target price. Equilibrium requires a distribution of prices all of which yield the same profit. If you do not change your p_t when M_t increases, you indeed earn less profit per unit, but again you make it up on the volume. Hence, sellers can change prices

5. This alternating-market structure is taken from Lagos and Wright (2005), mainly because it is extremely tractable, but as we have said, any other monetary model could be used. Previous analyses in this framework have used several different pricing mechanisms, including various bargaining solutions, price posting with directed search, Walrasian price taking, and auctions (see the previously mentioned surveys). No one has previously tried Burdett–Judd pricing in the model, although it was used in the related model of Shi (1977) by Head and Kumar (2005) and Head, Kumar, and Lapham (2010).

infrequently in the face of continuous movements in the aggregate price level, even though they are allowed to change whenever they like at no cost. One might say that sellers can be “rationally inattentive” to the aggregate price level and monetary policy, within some range, since as long as $p_t \in [\underline{p}_t, \bar{p}_t]$, their place in this distribution does not matter. But policy cannot exploit this. The distribution of relative prices is pinned down uniquely, and if, say, M_t were to unexpectedly double, $F_t(p)$ must adjust to keep the real distribution the same, even if many individual prices do not adjust. Hence, the level of the money supply M_t or the aggregate price level are irrelevant—they amount to a choice of units—even if inflation, nominal interest or money growth rates in general do matter for real outcomes. This is classical neutrality.⁶

We then show that a calibrated version of the model can match quite well the empirical behavior of prices in the US retail sector. First, the calibrated model predicts an average price duration that is reasonably close to what one sees in the data. Second, our theory generates a price change distribution that has the same shape and the features of the empirical price change distribution—for example, the average price change is large, yet there are many small changes, and even many negative price changes. Third, in the model the probability and magnitude of price adjustments are approximately independent of the time since the last adjustment, as in the data. Fourth, the theory correctly predicts that inflation increases both the frequency and the magnitude of price changes. Overall, our model of price stickiness appears empirically reasonable, even though money is neutral. This demonstrates formally that nominal stickiness neither requires technological restrictions on price adjustment nor justifies particular interventions by central banks.⁷

2. The Model

Time is discrete and continues forever. In every period, two markets open sequentially. We call the first the Burdett–Judd market, or BJ for short, a decentralized market for a consumption good q_t in which buyers and sellers meet through a frictional matching process. Here barter is not feasible since buyers have nothing to offer by way of quid

6. Although we focus in this application on changes in M_t , the same argument applies to real changes—what Golosov and Lucas (2003) call “continuously changing demand and supply conditions”. Any change in utility or cost functions can change the Burdett–Judd price distribution, but this does not imply that all sellers must adjust their individual prices.

7. There are several other interesting models where, despite price stickiness, money may be (sometimes approximately) neutral. These include Caplin and Spulber (1987), Eden (1994), and Golosov and Lucas (2007). Our approach differs in a number of respects. First, we start with a general equilibrium model where money is essential. Second, by design, money is exactly neutral. Third, stickiness arises entirely endogenously and robustly—it does not depend on particular functional forms, timing, the money supply process, and so forth. Fourth, the distribution of prices is endogenous, derived from standard microeconomics (Burdett–Judd), instead of simply assuming, say, that prices are distributed uniformly (as in Caplin–Spulber). Also, we take our model to the data, as do some (e.g., Golosov and Lucas 2007) but not all (e.g., Caplin and Spulber 1987) of the previously mentioned studies. This is not to disparage previous work, upon which we obviously build; we simply want to differentiate our product, even if some of the results look similar (see, for example, Figure 4 in Eden 1994).

pro quo, and credit is not feasible because they are anonymous. Hence, exchange takes place using fiat money, supplied by the government according to the rule $M_{t+1} = \mu_t M_t$, where $\mu_t > \beta$ is the money growth rate at t . After the BJ market closes, there convenes a centralized market where agents trade a different good x_t , as well as labor h_t and money m_t , called the Arrow–Debreu market, or AD for short. In AD households receive a lump sum transfer (or tax) T_t to accommodate increases (or decreases) in M_t . Also, in this market, as in standard general equilibrium theory, we cannot say who trades with whom or how—the approach does not allow one to ask if they use barter, money or credit, only requiring that household satisfy their budget equations and that markets clear.⁸

There is a continuum of households with measure 1. Each household has preferences described by the utility function

$$\sum_{t=0}^{\infty} \beta^t [u(q_t) + v(x_t) - h_t], \quad (1)$$

where $\beta \in (0, 1)$ is the discount factor, while $u(\cdot)$ and $v(\cdot)$ are strictly increasing and concave functions over the BJ good and AD good, respectively. There is a continuum of firms with measure s . Firms operate technologies for producing goods described as follows: producing a unit of x requires $h = x$ hours of labor, and producing a unit of q requires $h = cq$ hours of labor, so that c is the constant marginal cost of BJ goods in terms of ADgoods. As in standard general equilibrium theory, households own the firms, and receive profits as dividends D_t , in dollars, in the AD market.⁹

In the BJ market at t , each firm posts a nominal price p_t , taking as given the distribution of prices posted by all the other firms, described by the CDF $F_t(p)$, as well as the distribution of money across buyers in the market, in general, although in this model that is degenerate—that is, along the equilibrium path, $m_t = M_t$ for each household in the BJ market. Households know the distribution $F_t(p)$, but only contact and hence can only purchase from a random sample of BJ sellers. A household generally contacts k sellers with probability α_k . For simplicity we assume $\alpha_0 \in [0, 1)$, $\alpha_1 \in (0, 1 - \alpha_0)$ and $\alpha_2 = 1 - \alpha_0 - \alpha_1$, so that a household contacts at most two firms. One can easily generalize this, in a variety of ways, without changing the substantive results (e.g., as in Mortensen 2005); one can also allow households to choose endogenously how many sellers they sample at some cost (e.g., as in the

8. As in most New Monetarist models, where agents trade with each other and not merely against their budget lines, the role of money in our BJ market is basically the same as in Kiyotaki and Wright (1989); see Kocherlakota (1998) or Wallace (2010) for rigorous discussions. One should not worry about the assumption that changes in M_t are accomplished via lump sum transfers or taxes. It is equivalent for what we do to have the government use increases or decreases in M_t to buy more or fewer AD goods.

9. The baseline assumption is that q is produced in the AD market, and carried into the next BJ market by firms, who know exactly how much they will sell as a function of their price by the law of large numbers (and we do not dwell here on technicalities regarding the conditions needed for this law to apply). This allows us to interpret firms as simply technologies, owned by households, as in standard general equilibrium theory, but it is usually equivalent, and sometimes more convenient, to alternatively interpret firms as individuals who produce and consume (see Section 3).

original Burdett–Judd 1983 paper) and show that in equilibrium we get $\alpha_1, \alpha_2 \in (0, 1)$ and $\alpha_n = 0 \forall n > 2$. We avoid this by simply assuming the structure on the exogenous α_k , but the results can be generalized. Also, although for ease of notation we assume all trade in the BJ market is monetary, it is easy to allow some credit trades, since for money to be essential we only need to have some BJ trade where credit is unavailable (see Head et al. 2011).

Of course, there are options for the types of mechanisms firms can post. In principle, they could post menus, where buyers can have any q , perhaps in some set Q , for a payment $P(q)$, but here we impose linearity, $P(q) = pq$.¹⁰ We experimented with alternatives, but decided to focus on the linear case for now. We do not know definitively if this is important for the conclusions, but we doubt it. We also studied a version of the model where BJ goods were indivisible (Liu 2010; Head et al. 2011), which avoids the issue, since the only option is to post a p giving the price/payment for an indivisible unit. That version is easier on some dimensions, although it also has some problems. In particular, monetary models with indivisible goods and price posting can admit a multiplicity of equilibria (see Jean, Rabinovich, and Wright 2010 and the references therein). At some level, this multiplicity does not matter, since all the equilibria are qualitatively similar, but it is inconvenient. One can get around this indeterminacy, in principle, using different methods, including a version of the model we analyzed with costly credit. This introduces complications that might distract from the main message, however. So we stick to the divisible goods model and simply impose linear pricing, but more work ought to be done on these issues.

2.1. Households' Problem

Let $W_t(m_t)$ and $V_t(m_t)$ be the value functions for a household with m_t dollars in the AD and BJ market, respectively. Let ϕ_t be the value of money (the inverse of the nominal price level) in AD, where the price of x_t and the real wage are both 1 given our technology. Then the AD problem for a household is

$$\begin{aligned} W_t(m_t) = \max_{h_t, x_t, \hat{m}_t} \{ & v(x_t) - h_t + \beta V_{t+1}(\hat{m}_t) \} \\ \text{subj. to: } & x_t = h_t + \phi_t(m_t - \hat{m}_t + D_t + T_t), \end{aligned} \quad (2)$$

with nonnegativity constraints implicit. Eliminating h_t using the budget equation, we can reduce this to

$$W_t(m_t) = \phi_t(m_t + D_t + T_t) + \max_{x_t, \hat{m}_t} \{ v(x_t) - x_t + \phi_t \hat{m}_t + \beta V_{t+1}(\hat{m}_t) \}. \quad (3)$$

The solution satisfies the FOC

$$v'(x_t) = 1 \quad \text{and} \quad \beta V'_{t+1}(\hat{m}_t) = \phi_t, \quad (4)$$

10. Ennis (2008) and Dong and Jiang (2011), for example, study related monetary models where nonlinear pricing is used by sellers to elicit private information about buyers' preferences.

plus the budget constraint, $h_t = x_t + \phi_t(\hat{m}_t - m_t - D_t - T_t)$. This implies that (1) \hat{m}_t and x_t are independent of m_t , so that in particular the equilibrium distribution money is degenerate across households entering the BJ market, and (2) W_t is linear with slope ϕ_t .

For a household in the BJ market with m_t dollars, conditional on sampling at least one price and the lowest price sampled being p , we define

$$U_t(p, m_t) = \max_{q_t} \{u(q_t) + W_t(m_t - pq_t)\} \quad (5)$$

subj. to: $pq_t \leq m_t$.

Thus, $q_t = q_t(p, m_t)$ solves an elementary demand problem with liquidity constraint $pq_t \leq m_t$. It is easy to show that the difference between the slopes in (q, p) space of the unconstrained demand curve and the constraint at equality has the same sign as $1 - \gamma(q)$ when the curves cross, where $\gamma(q) = qu''(q)/u'(q)$ is the coefficient of relative risk aversion. It is convenient to have a single crossing, so that the constraint binds either for high p or for low p , and a sufficient condition for this is either $\gamma(q) > 1 \forall q$ or $\gamma(q) < 1 \forall q$. We assume constant relative risk aversion, $u(q) = q^{1-\gamma}/(1-\gamma)$, and assume $\gamma \in (0, 1)$, so that demand is constrained at low p (see Liu 2010 for the case $\gamma > 1$, and for results with a general function u).

With this specification, the conditional BJ problem is

$$U_t(p, m_t) = \max_{q_t} \left\{ \frac{q_t^{1-\gamma}}{1-\gamma} + W_t(m_t - pq_t) \right\} \quad (6)$$

subj. to: $pq_t \leq m_t$.

This is easily solved to get

$$q_t(p, m_t) = \begin{cases} m_t/p & \text{if } p \leq \hat{p}_t, \\ (p\phi_t)^{-\frac{1}{\gamma}} & \text{if } p > \hat{p}_t; \end{cases} \quad (7)$$

where $\hat{p}_t = \phi_t^{1/(\gamma-1)} m_t^{\gamma/(\gamma-1)}$. If $p < \hat{p}_t$ households cash out; otherwise $pq_t(p, m_t) < m_t$, so they have money to spare and demand is unconstrained. This is shown in Figure 1, where constrained demand is given by the lower envelope of the two curves representing unconstrained demand and the constraint at equality.

The unconditional value function entering the BJ market, before potentially contacting sellers and observing prices, is

$$V_t(m_t) = \alpha_0 W_t(m_t) + \alpha_1 \int U_t(p, m_t) dF_t(p) \quad (8)$$

$$+ \alpha_2 \int U_t(p, m_t) d\{1 - [1 - F_t(p)]^2\}.$$

Thus, with probability α_0 the household contacts no seller and enters the next AD market with m_t unchanged; with probability α_1 the household contacts one seller

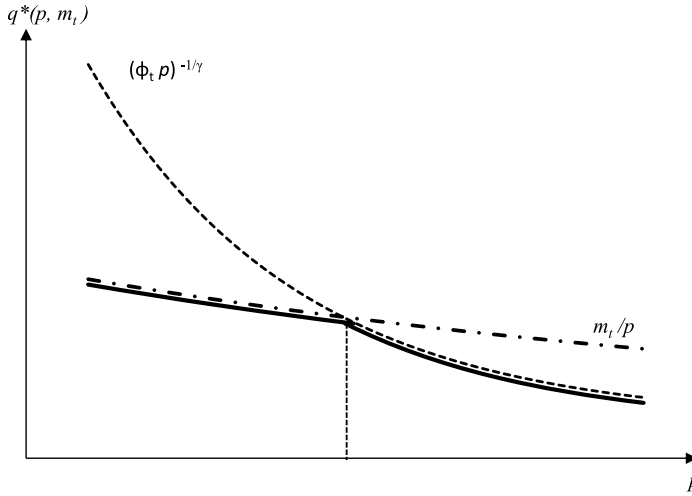


FIGURE 1. Household's demand for the BJ good.

posting a draw from $F_t(p)$; and with probability α_2 the household contacts two firms and the lower of the two prices is a random draw from $1 - [1 - F_t(p)]^2$. Algebra reduces this to

$$V_t(m_t) = \alpha_0 W_t(m_t) + \int [\alpha_1 + 2\alpha_2 - 2\alpha_2 F_t(p)] U_t(p, m_t) dF_t(p). \quad (9)$$

Differentiating $V_t(m_t)$, the FOC, $\phi_t = \beta V'_{t+1}(\hat{m}_t)$, becomes

$$\phi_t = \beta \phi_{t+1} \left\{ 1 + \int_0^{\hat{p}_{t+1}} [\alpha_1 + 2\alpha_2 - 2\alpha_2 F_{t+1}(p)] \left[\frac{1}{p\phi_{t+1}} \left(\frac{\hat{m}_t}{p} \right)^{-\gamma} - 1 \right] dF_{t+1}(p) \right\}. \quad (10)$$

Although we focus on stationary equilibria in what follows, for now we do not impose this restriction. In general, the inflation rate is $\pi_t = \phi_t/\phi_{t+1}$, and the Fisher equation gives the nominal interest rate by $1 + i_t = (1 + r_t)(1 + \pi_t)$, where $1 + r_t = 1/\beta$ is the real interest rate, which is time invariant here due to quasi-linear utility. To be clear, as is standard, we can obviously price any asset in equilibrium, including real or nominal claims between the AD market at t and the AD market at $t + 1$, even if these do not circulate in the BJ market (say, because they are not tangible assets, simply claims on numeraire goods or money in AD). This defines the previously mentioned interest rates, and allows us to rewrite condition (10) as

$$i_t = \int_0^{\hat{p}_{t+1}} [\alpha_1 + 2\alpha_2 - 2\alpha_2 F_{t+1}(p)] \left[\frac{1}{p\phi_{t+1}} \left(\frac{\hat{m}_t}{p} \right)^{-\gamma} - 1 \right] dF_{t+1}(p). \quad (11)$$

Heuristically, the LHS of (11) is the marginal cost of carrying cash between t and $t + 1$ (the nominal interest rate), and the RHS is the marginal benefit—the expected value of relaxing the liquidity constraint in the next BJ market which binds when $p < \hat{p}_{t+1}$.

2.2. Firms' Problem

If a firm posts p in the BJ market, profit is

$$\Pi_t(p) = \frac{1}{s}[\alpha_1 + 2\alpha_2 - 2\alpha_2 F_t(p) + \alpha_2 \xi_t(p)]R_t(p), \tag{12}$$

where $\xi_t(p) = \lim_{\varepsilon \rightarrow 0^+} F_t(p) - F_t(p - \varepsilon)$, and $R_t(p)$ is profit per buyer served, given that in equilibrium all buyers have M_t :

$$R_t(p) = q_t(p, M_t)(p\phi_t - c). \tag{13}$$

The term in brackets in (12) is the number of customers served: α_1/s households purchase from the firm because this is their only contact; $2\alpha_2[1 - F_t(p)]/s$ households purchase from the firm because they contact another seller who has a price above p ; and there are $2\alpha_2 \xi_t(p)/s$ households that contact the firm plus another with the same p , and in this case we can assume they randomize, although as we shall see this term vanishes in equilibrium because the probability two sellers set the same price is 0.

Figures 2 and 3 show two curves. One is $(M_t/p)(p\phi_t - c)$, which is profit in units of numeraire from selling to a buyer that is constrained. The other is $(p\phi_t)^{-1/\gamma}(p\phi_t - c)$, which is profit from selling to a buyer that is not liquidity constrained. Actual profit per customer is the lower envelope of these curves. Figure 2 illustrates the case in which the constraint $p q_t \leq m_t$ is not very tight, and the price that maximizes profit per customer is $p = c/\phi_t(1 - \gamma)$. Figure 3 illustrates the case in which the constraint is tighter, and the price that maximizes profit per customer is $p = \hat{p}_t$. The profit-maximizing price in general is $p_t^m = \max\{c/\phi_t(1 - \gamma), \hat{p}_t\}$, which we call the monopoly price.

Each firm chooses p to maximize $\Pi_t(p)$. Therefore, a price distribution $F_t(p)$ is consistent with profit maximization by all firms when $\Pi_t(p)$ is maximized by every p

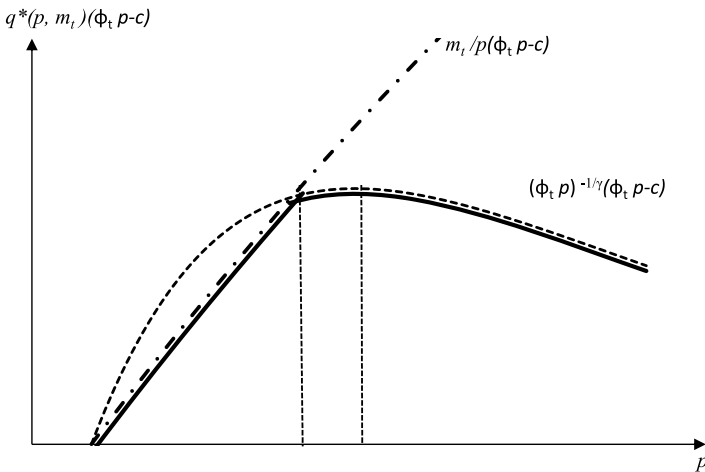


FIGURE 2. Firm's profit per customer in the BJ market

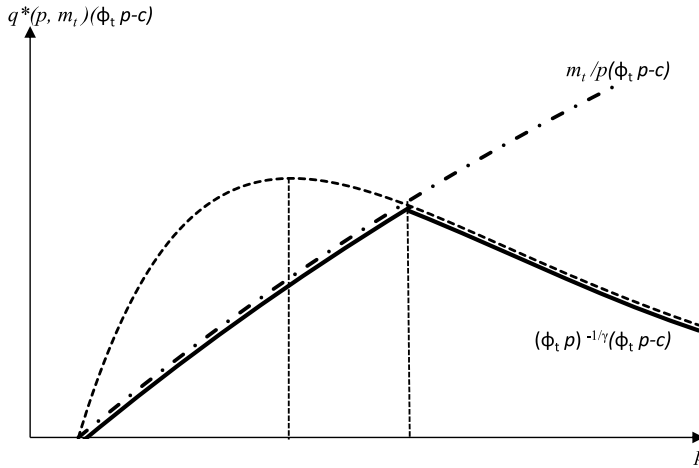


FIGURE 3. Firm's profit per customer in the BJ market

on the support of F_t , denoted \mathcal{F}_t . In other words, profit maximization means

$$\Pi_t(p) = \Pi_t^* \equiv \max_p \Pi_t(p) \forall p \in \mathcal{F}_t. \quad (14)$$

The following result characterizes F_t by adapting the arguments in Burdett and Judd (1983), generalized because we assume the BJ good is divisible and because buyers can be liquidity constrained. The proof is in Appendix A.

PROPOSITION 1. *The unique price distribution consistent with profit maximization by all firms at t is*

$$F_t(p) = \frac{\alpha_1 + 2\alpha_2}{2\alpha_2} - \frac{\alpha_1}{2\alpha_2} \frac{R_t(p_t^m)}{R_t(p)}, \quad (15)$$

with support $\mathcal{F}_t = [p_t, \bar{p}_t]$, where the bounds are given by

$$R_t(p_t) = \frac{\alpha_1}{\alpha_1 + 2\alpha_2} R_t(p_t^m) \text{ and } \bar{p}_t = p_t^m. \quad (16)$$

The price distribution is continuous, intuitively, because if it had a mass point at some p_0 , say, a firm posting p_0 could increase profit by changing to $p_0 - \varepsilon$, as this leaves profit per customer approximately constant and increases sales by a discrete amount. The support \mathcal{F}_t is connected, intuitively, because if it had a gap between p_0 and p_1 , say, a firm posting p_0 could increase profits by changing to $p_1 - \varepsilon$, as this does not reduce the number of sales and increases profit per sale. Since $F_t(p)$ has no mass points (12) reduces to

$$\Pi_t(p) = \frac{1}{s} [\alpha_1 + 2\alpha_2 - 2\alpha_2 F_t(p)] R_t(p). \quad (17)$$

The closed form in (15) is derived as follows: $\Pi_t^* = (\alpha_1/s) R_t(p_t^m)$ since $p_t^m \in \mathcal{F}_t$; equating this to (17), we solve for $F_t(p)$.

2.3. Equilibrium

We are now in the position to define an equilibrium.

DEFINITION 1. Given a process $\{M_t\}$, an equilibrium Σ^* is a (bounded and nonnegative) sequence of AD quantities $\{h_t^*, x_t^*, \hat{m}_t^*\}$, BJ decision rules $\{q_t^*(p, \hat{m})\}$ and prices $\{\phi_t^*, F_t^*(p)\}$ satisfying the following conditions for all t :

1. $(h_t^*, x_t^*, \hat{m}_t^*)$ solves the household's AD problem, and in particular \hat{m}_t^* satisfies (11);
2. $q_t^*(p, \hat{m})$ solves the household's BJ problem as described in (7);
3. $F_t^*(p)$ solves the firm's BJ problem as described in Proposition 1 with support $\mathcal{F}_t^* = (\underline{p}_t^*, \bar{p}_t^*)$;
4. ϕ_t implies market clearing, $m_t^* = M_t$.

As mentioned previously, we are mostly interested here in stationary outcomes, which makes sense when policy is stationary, $M_{t+1} = \mu M_t$, $\forall t$ for some constant μ . Assuming this is the case, we have the following definition.

DEFINITION 2. A stationary monetary equilibrium is an equilibrium where all nominal variables grow at rate μ , all real variables at rate 0, and $\phi_t^* > 0$.

Stationarity implies $F_{t+1}^*(\mu p) = F_t^*(p)$ and $q_{t+1}^*(\mu p) = q_t^*(p)$, which means that the real distribution of BJ prices and the BJ decision rule are time invariant. It also implies a constant inflation rate $\pi = \mu$ and nominal interest rate $1 + i = \mu/\beta$.

To define some terminology, classical neutrality means the following: suppose we have an equilibrium Σ , and we change M_t to $M'_t = \Theta M_t$ $\forall t$ for some $\Theta > 0$. Then there exists an equilibrium Σ' where all nominal variables increase by a factor Θ —for example, $p'_t = \Theta p_t$, $\phi'_t = \phi_t/\Theta$, and so on—while all real variables are the same—for example, $q'_t = q_t$, and so forth. Clearly, in this model, equilibria (stationary or otherwise) display neutrality in this sense. This merely says that units do not matter. Later we consider another notion of neutrality, given an unexpected change in M_t . In any case, we emphasize that neutrality does not imply superneutrality: changing the growth rate μ in the rule $M_{t+1} = \mu M_t$ does have real effects. Also note that in a stationary monetary equilibrium it is equivalent to choose the money growth rate μ , the inflation rate π , or the nominal interest rate i as a policy instrument.

We establish the existence of a stationary monetary equilibrium formally in Appendix B, but here we give the basic idea behind the argument. First, we show that prices posted in the BJ market are decreasing, in the sense of first-order stochastic dominance, with respect to the amount of money firms expect households to carry. Intuitively, if households have more money the liquidity constraint is relaxed, which increases profit at low-price firms relative to high-price firms, because the former are where the constraint binds; so, to keep firms indifferent between low and high prices, the distribution must shift to reduce the number of customers served by low- relative to high-price firms. Then we prove that the amount of money carried by households is decreasing with respect to prices in the BJ market. Intuitively, if prices are higher, in

the sense of first-order stochastic dominance, a household has a lower probability of meeting a low-price seller and hence a lower probability of being liquidity constrained, so the value of money in the BJ market falls. It follows that the amount of money households carry is a monotone function of the amount firms expect them to carry. Moreover, the amount of money households carry is bounded. Hence, from a fixed point theorem of Tarski (1955), there exists an m_t^* such that (1) m_t^* solves the households' problem given F_t^* , and (2) F_t^* is the BJ price distribution given m_t^* . Given m_t^* and F_t^* we easily get all the other endogenous variables.

PROPOSITION 2. *A stationary monetary equilibrium exists.*

3. Sticky Prices

Equilibrium uniquely pins down the aggregate BJ price distributions for all t —both real and nominal—but not the price of any individual firm, since by definition equilibrium implies the same profit from any $p \in \mathcal{F}_t$. Figure 4 illustrates the implications for the dynamics of the distribution and individual prices when $\mu > 1$, by showing the densities associated with F_t^* and F_{t+1}^* . All firms with p in the vertically shaded area must change between t and $t + 1$, since such a p price does not maximize $\Pi_{t+1}(p)$, even though it did maximize $\Pi_t(p)$. The firms in the horizontally shaded area, however, are indifferent between keeping p constant and posting a new $p \in [p_{t+1}, \bar{p}_{t+1}]$. The only equilibrium restriction on the individual price dynamics between t and $t + 1$ is that the aggregate distribution at $t + 1$ has to be F_{t+1}^* .

DEFINITION 3. In a stationary monetary equilibrium, a repricing policy $p_{t+1}^*(p)$ is admissible if, when the distribution at t is $F_t^*(p)$ and all firms follow policy $p_{t+1}^*(p)$, the distribution at $t + 1$ is $F_{t+1}^*(p)$.

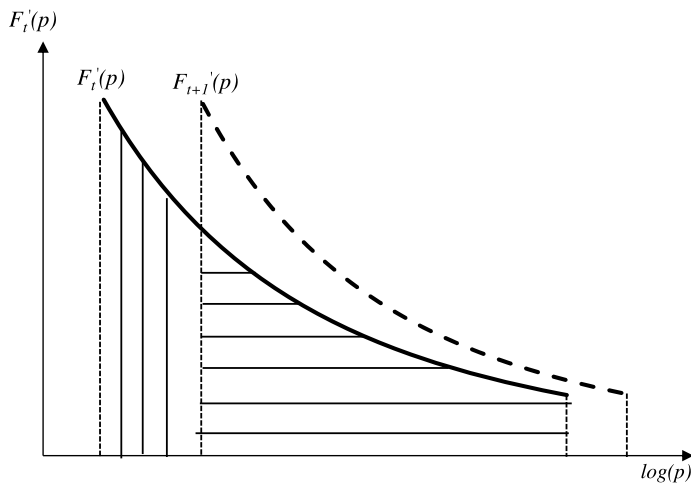


FIGURE 4. Equilibrium price distribution

In the remainder of the paper, we restrict attention to stationary outcomes, positive inflation $\mu \geq 1$, and repricing policies of the form

$$\begin{aligned} &\text{if } p_t \notin \mathcal{F}_{t+1} \text{ then } p_{t+1}^*(p_t) = p', \\ &\text{if } p_t \in \mathcal{F}_{t+1} \text{ then } p_{t+1}^*(p_t) = \begin{cases} p_t & \text{with probability } \rho, \\ p' & \text{with probability } 1 - \rho, \end{cases} \end{aligned} \quad (18)$$

where $p' \in \mathcal{F}_{t+1}$ is a profit-maximizing price at $t + 1$, determined as discussed in what follows. The parameter ρ is a probability used as a tie-breaking rule: if you are indifferent between changing and not changing your price, you randomize. Although this may bear a superficial resemblance to Calvo pricing, we cannot emphasize strongly enough that *it could not be more different*. With Calvo pricing, firms may be *desperate* to change p , but are only allowed to do so with some probability each period. Here, any firm that wants to change p can and will; only those who are genuinely indifferent may randomize.

The only additional structure we impose on repricing is symmetry. This means that, first, all sellers use the same ρ , and second, those who reprice between t and $t + 1$ all draw a new p' from the same distribution, say $G_{t+1}(p')$. We now show that once ρ is specified $G_{t+1}(p')$ is pinned down uniquely. To begin, note that in stationary equilibrium $F_{t+1}(\mu p) = F_t(p)$, which says that when inflation is μ the probability of finding a price below p today is the same as the probability of finding price below μp tomorrow. What kind of repricing distribution makes this happen? We now derive the unique repricing distribution that does the trick.

Given $F_t(\cdot)$, and any $G_{t+1}(\cdot)$, we compute $F_{t+1}(\cdot)$ as follows: first, for $p \in (\mu \underline{p}_t, \bar{p}_t)$,

$$\begin{aligned} F_{t+1}(p) &= F_t(\mu \underline{p}_t) G_{t+1}(p) \\ &\quad + [1 - F_t(\mu \underline{p}_t)](1 - \rho) G_{t+1}(p) + [F_t(p) - F_t(\mu \underline{p}_t)] \rho. \end{aligned} \quad (19)$$

This says that the measure of sellers below p evolves as follows: a measure $F_t(\mu \underline{p}_t)$ fall off the support between t and $t + 1$ and they all reprice using $G_{t+1}(p)$; a measure $1 - F_t(\mu \underline{p}_t)$ do not fall off the support and do not have to reprice, but do so anyway with probability $1 - \rho$; and a measure $F_t(p) - F_t(\mu \underline{p}_t)$ with price below p do not have to reprice and choose not to with probability ρ . Algebra yields

$$F_{t+1}(p) = [1 - \rho + \rho F_t(\mu \underline{p}_t)] G_{t+1}(p) + [F_t(p) - F_t(\mu \underline{p}_t)] \rho. \quad (20)$$

Similarly, for $p > \bar{p}_t$ we have

$$F_{t+1}(p) = [1 - \rho + \rho F_t(\mu \underline{p}_t)] G_{t+1}(p) + [1 - F_t(\mu \underline{p}_t)] \rho. \quad (21)$$

We now impose stationarity, $F_{t+1}(\mu p) = F_t(p)$, and solve for the repricing distribution:

$$G_{t+1}^*(p) = \begin{cases} \frac{F_t^*(p/\mu) - [F_t(p) - F_t(\mu \underline{p}_t)] \rho}{1 - \rho + \rho F_t^*(\mu \underline{p}_t)} & \text{if } p \in (\underline{p}_t, \bar{p}_t/\mu), \\ \frac{F_t^*(p/\mu) - [1 - F_t(\mu \underline{p}_t)] \rho}{1 - \rho + \rho F_t^*(\mu \underline{p}_t)} & \text{if } p \in (\bar{p}_t/\mu, \bar{p}_t). \end{cases} \quad (22)$$

Given inflation μ , which is a policy variable, and any tie-breaking rule ρ , the unique repricing distribution that keeps the real price distribution constant is (22). The equilibrium law of motion for the nominal price distribution is

$$F_{t+1}(p) = \begin{cases} [1 - \rho + \rho F_t^*(\mu \underline{p}_t)] G_{t+1}^*(p) & \text{if } p < \mu \underline{p}_t, \\ [F_t^*(p) - F_t^*(\mu \underline{p}_t)] \rho + [1 - \rho + \rho F_t^*(\mu \underline{p}_t)] G_{t+1}^*(p) & \text{if } p \geq \mu \underline{p}_t. \end{cases} \quad (23)$$

We have established the following result.

PROPOSITION 3. *The pricing policy (18), with all new prices drawn from $G_{t+1}^*(p')$ as given in (22), is consistent with stationary monetary equilibrium $\forall \rho \in [0, 1]$.*

The class of repricing policies (18) is not exhaustive, but it captures a wide range of behavior in a parsimonious way. For $\rho = 1$, (18) describes an extreme case in which firms only change p when it is no longer profit maximizing, giving the smallest fraction of price changes and highest average price duration consistent with equilibrium. For $\rho = 0$, we have the opposite extreme in which firms change p in every period, giving the largest fraction of changes and the lowest average duration consistent with equilibrium. As ρ increases from 0 to 1, the frequency of changes and the average price duration move from one extreme to the other. For any ρ , we now compute this frequency and average price duration.

The distribution of new prices in period t is $G_t^*(p)$. Let N be the largest integer such that $\mu^N \underline{p}_t \leq \bar{p}_t$. For $n = 1, 2, \dots, N$, a fraction $G_t^*(\mu^n \underline{p}_t) - G_t^*(\mu^{n-1} \underline{p}_t)$ of new prices are in $[\mu^{n-1} \underline{p}_t, \mu^n \underline{p}_t]$, and a fraction $1 - G_t^*(\mu^N \underline{p}_t)$ are in $[\mu^N \underline{p}_t, \bar{p}_t]$. Each $p \in [\mu^{n-1} \underline{p}_t, \mu^n \bar{p}_t]$ changes at $t + i$ and not before with probability $\rho^{i-1}(1 - \rho)$, and will change in period $t + n$ with probability ρ^{n-1} . So the average duration of prices in $[\mu^{n-1} \underline{p}_t, \mu^n \bar{p}_t]$ is $(1 - \rho) + 2\rho(1 - \rho) + \dots + n\rho^{n-1} = (1 - \rho^n)/(1 - \rho)$. Each $p \in [\mu^N \underline{p}_t, \bar{p}_t]$ will change at $t + i$ with probability $\rho^{i-1}(1 - \rho)$, $i = 1, 2, \dots, N$, and at $t + N + 1$ with probability ρ^N . So the average duration in the interval $[\mu^N \underline{p}_t, \bar{p}_t]$ is $(1 - \rho^{N+1})/(1 - \rho)$. The overall average duration of a new price is thus

$$A(\rho) = \left\{ \sum_{n=1}^N [G_t^*(\mu^n \underline{p}_t) - G_t^*(\mu^{n-1} \underline{p}_t)] \frac{1 - \rho^n}{1 - \rho} \right\} + [1 - G_t^*(\mu^N \underline{p}_t)] \frac{1 - \rho^{N+1}}{1 - \rho}. \quad (24)$$

Since $(1 - \rho^n)/(1 - \rho)$ is increasing in ρ and n , and G_t^* is increasing in ρ in the first-order stochastic dominance sense, $A(\rho)$ is increasing in ρ .

We now compute the fraction of prices that change between t and $t + 1$, starting from $F_t^*(p)$. A fraction $F_t^*(\mu \underline{p}_t)$ of prices are in $[\underline{p}_t, \mu \underline{p}_t]$, and each of these change with probability 1. A fraction $1 - F_t^*(\mu \underline{p}_t)$ are in $[\mu \underline{p}_t, \bar{p}_t]$, and each of these change with probability $1 - \rho$. The overall fraction of prices that change between t and $t + 1$ is therefore

$$\Phi(\rho) = 1 - \rho + \rho F_t^*(\mu \underline{p}_t), \quad (25)$$

with $\Phi(\rho)$ decreasing in ρ . Finally, we compute the distribution of the magnitude of price changes. The density of firms that post p at t and a different price at $t + 1$ is $F_t^*(p)/\Phi(\rho)$ if $p < \mu \underline{p}_t$ and $(1 - \rho)F_t^*(p)/\Phi(\rho)$ if $p > \mu \underline{p}_t$. Among the firms that post a new p , a fraction $G_{t+1}^*[p(1 + \delta)]$ increase p by $\delta\%$ or less. The distribution for the magnitude of price changes is thus

$$H_t(\delta, \rho) = \frac{1}{\Phi(\rho)} \int G_{t+1}^*[\rho(1 + \delta)](1 - \rho \mathbf{1}\{p \geq \mu \underline{p}_t\}) dF_t^*(p). \quad (26)$$

From (22) and (26), it is immediate that $H_t(0, \rho) > 0$ for all $\rho < 1$.

PROPOSITION 4. *A stationary monetary equilibrium Σ^* together with a repricing policy p_{t+1}^* yields an average price duration $A(\rho)$ and a frequency of price changes $\Phi(\rho)$, with $A(\rho)$ increasing and $\Phi(\rho)$ decreasing in ρ . There is a $\mu^* > 1$ such that $\mu \in (1, \mu^*)$ and $\rho \in (0, 1]$, implies $A(\rho) > 1$ and $\Phi(\rho) < 1$. For all $\mu \in (1, \mu^*)$ and $\rho \in [0, 1)$, the fraction of negative price changes, is $H(0, \rho) > 0$.*

Proof. See Appendix C.

The result tells us that, unless the inflation rate μ is too high, the model is consistent with the observation that some firms stick to their prices for some time despite a constantly changing aggregate price level.¹¹ Our model delivers this result not because there are technological restrictions on price adjustment, but because standard search frictions imply an interval of prices all of which maximize profit. It is also consistent with the observation that some firms lower their price despite a constantly increasing aggregate price level. It also delivers this result because of search frictions, and not because of idiosyncratic shocks. More broadly, the results show that one should be cautious about making inferences concerning the existence or degree of menu costs and related restrictions on the timing of price changes from the observed stickiness of individual prices. Similarly, one should be cautious about making inferences concerning idiosyncratic productivity shocks from observed price changes.

Perhaps most importantly, one should be very cautious about making policy recommendations based on these observations. Some firms may well stick to the same nominal p for many periods, but this cannot be exploited by policy in our model economy. Government cannot, for example, increase short-run production or consumption through an unexpected increase in M . If we were to unexpectedly double the stock of money at the opening of the AD market, the m that each household carries into BJ would double, and so would the distribution of nominal prices in that market. Theory—that is, utility maximization, profit maximization, and equilibrium taken together—pins down uniquely the distribution of real prices here, and doubling M does not affect this. Similarly, the amount of money agents bring back to the AD market doubles, but the value of this money ϕ is cut in half. This is classical neutrality.

11. Obviously, if inflation is too high, all firm must reprice every period. If, for example, we start at t with prices in $\mathcal{F}_t = [1, 2]$, and double the money supply between t and $t + 1$, the support moves to $\mathcal{F}_{t+1} = [2, 4]$, and the set of agents with $p_t \in \mathcal{F}_t \cup \mathcal{F}_{t+1}$ has measure 0.

Expanding M is neutral, intuitively, because while the price posted by some sellers can be rigid in the short run, the aggregate distribution F_t is perfectly flexible. This contrasts sharply with what would happen if there were positive menu costs or if sellers were only allowed to change with probability less than 1. In these cases, if we unexpectedly double M , it is not possible in general to keep the distribution of real prices constant—for example, suppose the support goes from $\mathcal{F}_t = [1, 2]$ to $[2, 4]$ after M doubles. This requires firms to change their prices with probability 1, and in our model they do. But if a fraction of sellers are not allowed to change p after a shock to M , as in Calvo-style models, or if some sellers have a high enough cost to changing p , as in Mankiw-style models, they are stuck with prices that are too low and do not maximize profit. This obviously does affect the real outcome and welfare. Without working through the details, it is clear that many households are going to find BJ goods going at bargain-basement prices and, in general will demand more, which might force the firms to supply more, depending on how one specifies the details.¹²

Although the exact outcome may depend on details, the general conclusions are very robust. In Head et al. (2011), for example, we present an indivisible goods version of the model, where there is no scope for changes in money to affect production or consumption on the intensive margin, but introduce a participation decision: households must pay a fixed cost to enter the BJ market, analogous to the free-entry condition for firms to enter the labor market in Pissarides (2000). With Calvo- or Mankiw-style pricing, an increase in M that catches sellers by surprise means many real prices are too low from a profit-maximizing perspective, and generally we expect this to increase entry of households into the BJ market. That is, when sellers cannot change their prices, even though they would like to, monetary policy can instigate a shopping spree by households in search of bargains, and this sets off a production boom when sellers are obliged to meet demand, as in most sticky-price models. Symmetrically, a fall in M can lead to a slump in Calvo- or Mankiw-style models. Neither a boom nor a slump occurs in our setup under these policy scenarios, where prices are reset quickly, even though in normal times many prices may be reset only gradually.

4. Quantitative Evaluation

We have a theory of nominal rigidities that relies on search frictions in product markets, not on the existence of technological frictions to repricing. In this section, we ask if the theory can account for the empirical evidence. While our model delivers equilibrium

12. A detail we mention here is that, in the previous description of the environment, we said sellers buy inventories in AD and bring them to BJ, with expectations about how much they will sell, that are correct with probability 1. This cannot happen if M doubles and the nominal distribution does not—but whether this results in firms stocking out, or somehow producing additional output, is something we do not go into here. The point is simply that something other than the expected equilibrium has to happen. If we assume that sellers can produce q (as opposed to selling out of inventory), and that they are obliged to do so for everyone that pays the posted price, as in most Keynesian models, then a surprise increase (decrease) in M can raise (lower) consumption and output with Calvo- or Mankiw-style models. By contrast, in our model, the real allocation is not affected by the policy under consideration.

price distributions, we choose to look at equilibrium price-change distributions instead, since many macroeconomists have been focusing on the latter of late. Still it is worth mentioning that future work could analyze price distributions. The labor-market version of Burdett–Judd, the Burdett–Mortensen (1998) model, for example, has been applied to study wage (not wage-change) distributions. While the simplest Burdett–Mortensen models do not fit the data very well, much has been learned from adapting and extending the model to do better. Something similar could help us learn about product markets. But for this project, we instead look at the evidence on price changes, as described in a representative study by Klenow and Kryvtsov (2008) (again, see Klenow and Malin 2010 for a survey of related empirical work).

For our purposes, in terms of preferences and technology, we need to specify the discount factor β , the utility function for the BJ good, $u(q) = q^{1-\gamma}/(1-\gamma)$, and the marginal cost, which we normalize to $c = 1$. We do not need to specify utility for the AD good $v(x)$, although it may be needed to see how well the model fits observations other than those on which we focus. It can, for example, affect the model-generated money demand curve—the relationship between i and real balances—which one can compare to the data. This is studied in an extension of the framework by Wang (2011), where the model does reasonably well on this dimension, so here we concentrate on other issues. In particular, we concentrate on repricing behavior, as described by a function $p_{t+1}^*(p, \rho)$ with parameter ρ . We also need to parameterize search frictions, as described by α_k , where α_k is the probability that a household contacts k firms, $k = 0, 1, 2$. We restrict attention to the case where each household attempts to solicit two price quotes from the BJ market, and each attempt succeeds independently with probability λ . Thus, $\alpha_0 = (1-\lambda)^2$, $\alpha_1 = 2(1-\lambda)\lambda$, and $\alpha_2 = \lambda^2$. Finally, monetary policy is described by the growth rate of the money supply μ , although as we have said, this is equivalent to targeting inflation or nominal interest rates.

We calibrate the model to the US economy over the period 1988–2004. We choose the model period to be one month, and set β so that the annual real interest rate matches the average in the data, 1.035. We set μ so that the annual inflation rate in the model matches that in the data, 1.03. We interpret the BJ market as a retail sector and choose λ so that the average markup in the BJ market is 30%, which is an average across retailers in the survey data discussed in Faig and Jerez (2005). We then choose γ and ρ to minimize the distance between the model-generated distribution of price changes in the BJ market $H_t(\delta, \rho)$ defined in (26) and its empirical counterpart for the retail sector, as described by Klenow–Kryvtsov. After calibrating the parameters, the predictions of the model regarding price-changes in the BJ market are uniquely pinned down. There is a simple intuition behind our calibration strategy for γ and ρ . The parameter γ determines the elasticity of profit per customer $R_t(p)$. Hence, γ affects the distribution $F_t^*(p)$, and therefore the price-change distribution $H_t(\delta, \rho)$. Similarly, ρ determines the probability that a firm does not adjust its price when indifferent, and so affects the distribution of prices among firms that do not change, and hence the distribution among those that do, $G_t^*(p)$, and thus the distribution of price changes $H_t(\delta, \rho)$.

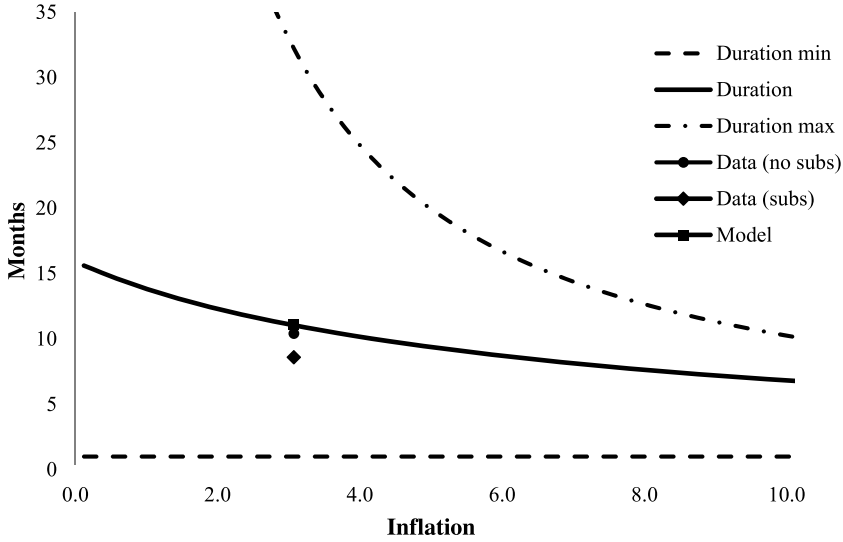


FIGURE 5. Average price duration

4.1. Results

The bottom line is that our theory of price rigidity can account quite well for the empirical behavior of prices. According to the data analyzed by Klenow–Kryvtsov, the average duration of a price in the retail sector is between 6.8 and 10.4 months, depending on whether temporary sales and product substitutions are interpreted as price changes: if both are interpreted as price changes, the average duration of a price is 6.8; if product substitutions are interpreted as price changes but temporary sales are not, the average duration is 8.6; and if neither are interpreted as price changes, the average duration is 10.4. The average duration of a price predicted by the model, given an inflation rate of 3% and a calibrated value of $\rho = 0.93$, is 11.6 months. We did not calibrate to this number. Heuristically, the two parameters γ and ρ are set to try to match the price-change distribution, and the predicted duration happens to come out at 11.6, which is on the high end of the range given by Klenow–Kryvtsov, but still very reasonable. Obviously, for higher values of ρ average duration increases, up to a maximum of 34 months, and for lower values of ρ average price duration falls, down to a minimum of 1. With $\rho = 0.91$ average duration is at the low end of the range, 6.8 months. Also, note that average duration is decreasing with respect to inflation, as inflation increases the fraction of prices that exit the support \mathcal{F}_t each period. See Figure 5.¹³

13. At zero inflation, the model has an indeterminacy depending on ρ (although, again, once ρ is specified there is a unique symmetric equilibrium repricing distribution G_t). Thus, with no inflation sellers can reprice each period or never. But as inflation increases the indeterminacy diminishes quickly, as can be seen in Figure 5 from the minimum price duration curve becoming fairly low even for moderate inflation rates.

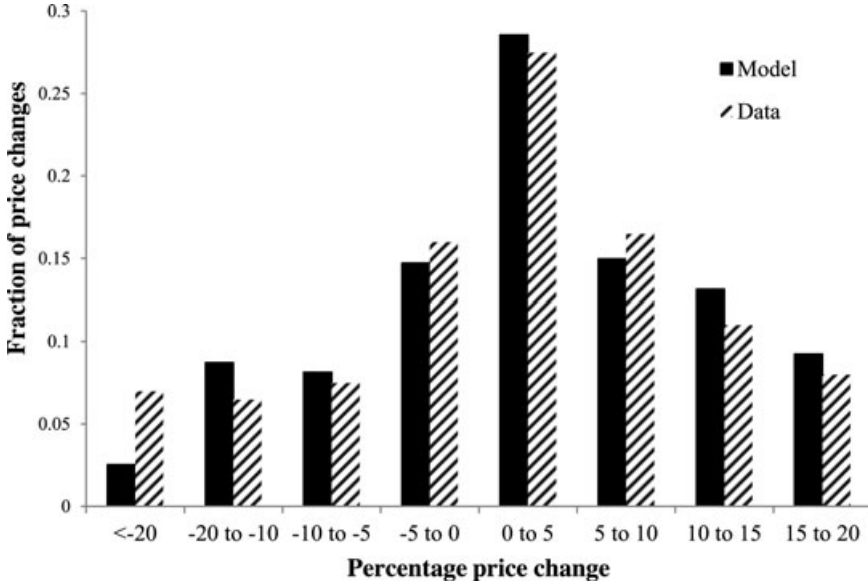


FIGURE 6. Price change distribution

One can consider the ability of the model to match the average duration of prices an independent check on the calibration, which was targeted to the histogram of price changes. One can also ask how well calibration matches this target. The striped histogram in Figure 6 is the empirical price-change distribution from Klenow–Kryvtsov, while the solid histogram is the distribution predicted by the model. One can immediately see they are very close. Three features of the empirical distribution are worth emphasizing. First, the average price change is large, around 11%. Second, despite the large average, many price changes are small, with 44% smaller than 5% in absolute value. Third, many price changes are negative, around 45%. This is problematic for a simple menu cost story, since the large average change suggests large menu costs, but that is inconsistent with so many small and negative changes. Klenow and Krystov (2008), Golosov and Lucas (2007), and Midrigan (2006) interpret the existence of many small and negative price changes as evidence of large and frequent shocks to individual seller’s idiosyncratic productivity.¹⁴

For our model-generated price-change distribution, the average absolute value is 9%, the fraction of changes between -5% and $+5\%$ is 43%, and the fraction of negative price changes is 35%. We capture the empirical distribution quite well with no seller-specific productivity shocks. According to our theory, average price changes are large because search frictions create a lot of dispersion in the equilibrium distribution.

14. We are sympathetic to the idea that one may be able to account for some of these observations by sellers sometimes moving prices around to uncover information about demand, costs, etc. This would seem to have little to do with monetary neutrality, however, since presumably what they care about is real demand, costs, etc.

The price posted by a firm at the 90th percentile of the distribution, for example, is approximately double that posted at the 10th percentile. Hence, when p exits the support \mathcal{F}_t , on average firms make a large adjustment. Many price changes are small, however, because there are many firms that change p before it exits \mathcal{F}_t , and for the same reason many changes are negative. One can describe several other features of the data that the model matches well, including the fact that when two firms reprice at the same t they typically do not both adjust to the same p' , as predicted by at least simple menu cost models. Of course one may be able to get a less-simple menu cost model to do better; we only mention that we do not need any bells or whistles here, as the most basic version of the theory does fairly well.

Klenow–Kryvtsov estimate the relationship between the probability that a firm adjusts its price for a given item—that is, the price-change hazard—and the time since the previous adjustment—that is, the age of the price. Moreover, they estimate the relationship between the absolute value of price adjustments—that is, the price-change size—and age. After controlling for unobserved heterogeneity across items, they find the price-change hazard remains approximately constant during the first eleven months and increases significantly during month 12, and the price-change size is approximately independent of age. We do not think these observations are especially puzzling since, for example, perhaps at least some price changes are discussed before implementation at annual meetings. Nonetheless, in Figure 7 the histogram shows the price-change hazard predicted by the model. As in the data it is approximately constant for the first eleven months in the life of a price, although it does not increase in month 12. This is because F_t in the model has a wide support. Therefore, during the twelve months after a change, few firms need to readjust, so the majority change only with probability ρ , independent of p 's age. We do not predict a spike after twelve months because our firms have no seasonal reason to adjust, like an annual meeting.

We now turn to the effects of inflation. Using time variation over the period 1988–2005, Klenow–Kryvtsov measure the effect of inflation on the frequency of price adjustments (the extensive margin) and on their magnitude (the intensive margin). They accomplish this by estimating the coefficient on inflation in a regression of the frequency of price adjustments and in a regression of the magnitude. Their main finding is that inflation has a positive effect on both the frequency and the magnitude of price adjustments. More specifically, they find that a one percentage point increase in inflation increases the frequency of price adjustments by 2.38% and the magnitude of price adjustments by 3.55%. Figure 8 illustrates the predictions of our model. According to the model, an increase in inflation increases both the frequency and the magnitude of price adjustments. This is easy to explain. First, an increase in inflation leads to a decline in the real balances carried by the households in the BJ market and, in turn, to a compression of the support \mathcal{F}_t . Second, given the support, an increase in inflation reduces the time it takes for a price to exit \mathcal{F}_t . For both reasons, an increase in inflation increases the fraction of prices that adjust every month. For similar reasons, an increase in inflation leads to a greater average price adjustment.

It is obvious that at least the standard Calvo-style model cannot match these observations: the magnitude of price changes may be endogenous but the frequency

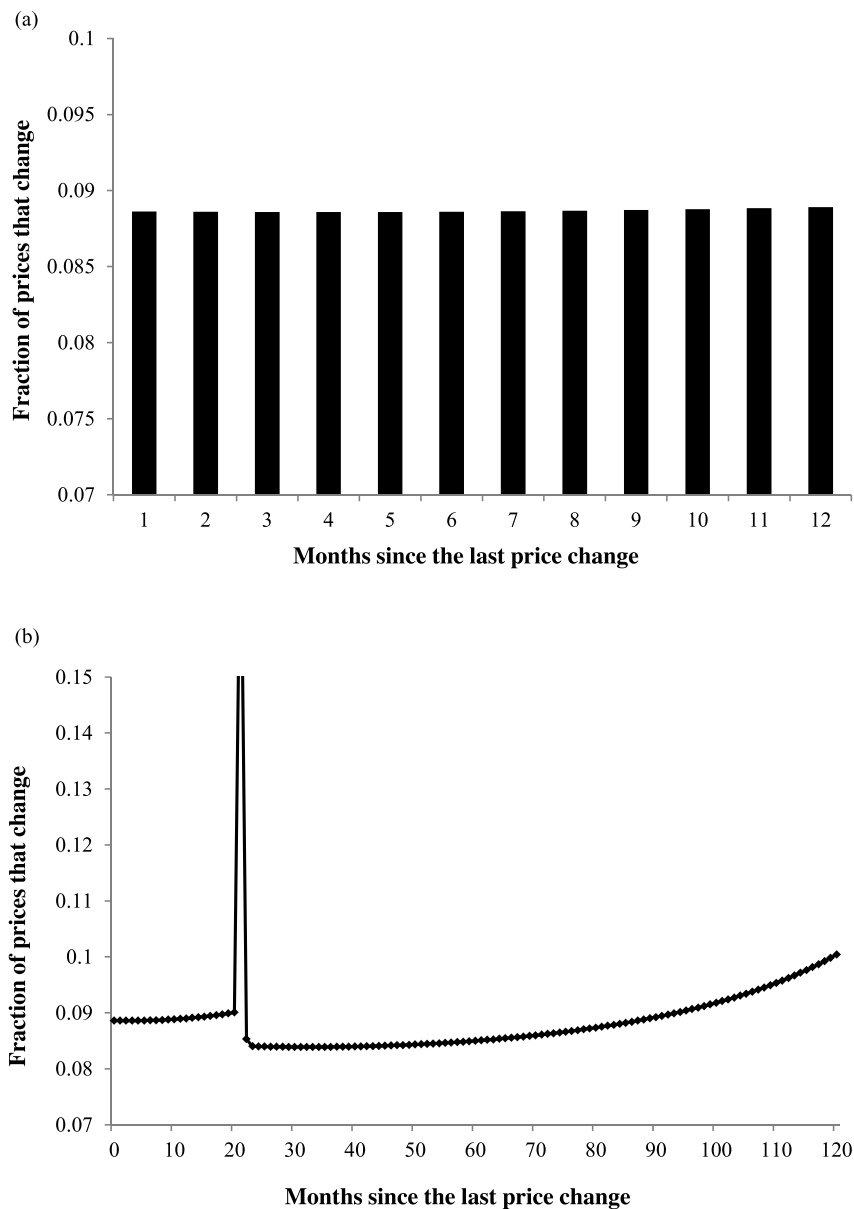


FIGURE 7. (a): Hazard rate of a price change, (b): Hazard rate of a price change

is exogenous and as such cannot depend on inflation. Our model matches the stylized facts about the extensive and intensive margins qualitatively, but does not nail them quantitatively. An increase in inflation from 3% to 4%, for example, increases the frequency of price adjustment by approximately nine percentage points and the magnitude by approximately five percentage points. This discrepancy between the

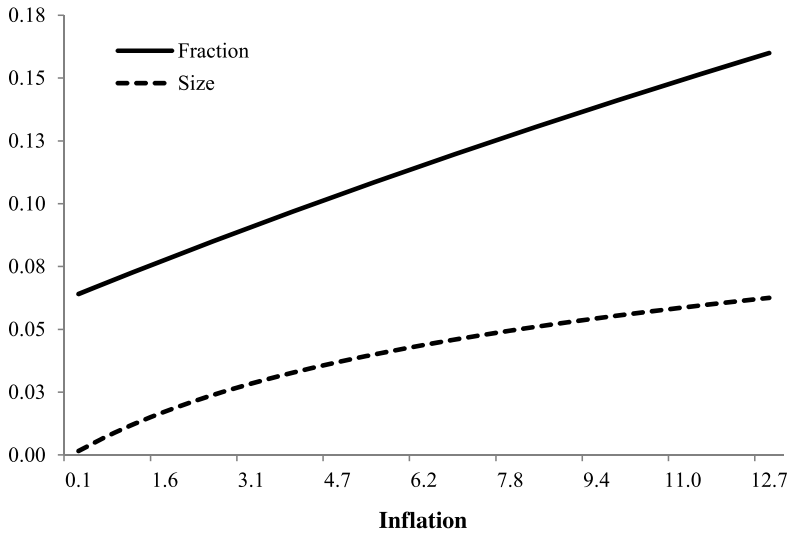


FIGURE 8. Fraction and size of price changes

predictions of the model and the results of the regression analysis should be neither too surprising nor much of a concern. In reality, fluctuations in the inflation rate may be correlated with other shocks that are not in the regression. There is still some work to do on both measuring the impact of inflation on the two margins, including the study of other episodes and countries, as well as modeling in more detail price-setting behavior. Our framework, however, gives one potentially interesting laboratory in which to explore these issues.

Finally, Klenow–Kryvtsov measure the effect of inflation on the fraction of prices that increase and the fraction of prices that decrease. Again, they accomplish this by estimating the coefficient on inflation in regressions of the fraction of prices that increase and on the fraction that decrease. Their main finding is that inflation has a positive effect on the fraction of price increases and a negative effect on the fraction of prices that decrease. This was not a foregone conclusion, since it could be, for example, that inflation induces more positive changes but has little impact on negative changes, or vice versa. They find that a 1% increase in inflation raises the fraction of positive price changes by 5.48% and decreases the fraction of negative changes by 3.10%. Figure 9 illustrates the predictions of our model. As in the data, the model predicts that increases in inflation raise the fraction of positive and lower the fraction of negative adjustments. Again, however, the magnitude of the effect is different than in the regression analysis. According to the model, an increase in inflation from 3% to 4% increases the fraction of positive changes by approximately 10% and decreases the fraction of negative changes by approximately 2.5%. Although we do not match this exactly, we are encouraged by the ability of the model to get the facts qualitatively correct, and think that it provides an avenue for further research.

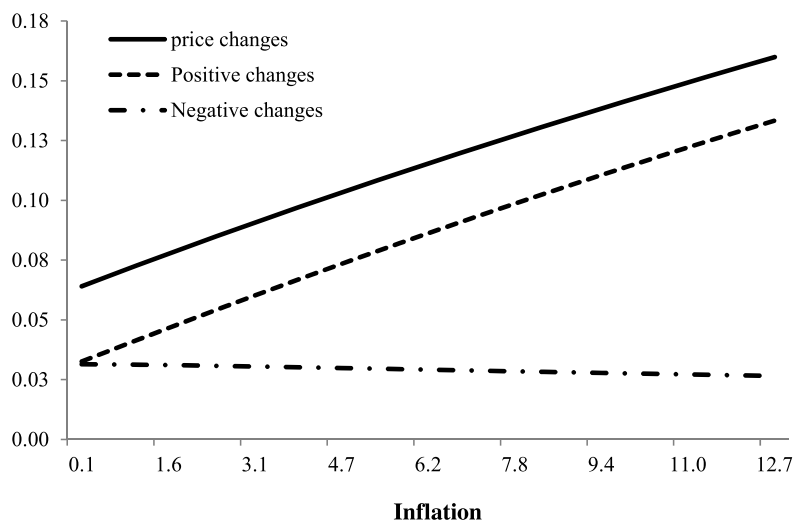


FIGURE 9. Positive and negative price changes

4.2. Summary of Quantitative Findings

Our theory can account well for the empirical behavior of nominal price changes. First, it predicts an average price duration of 11.6 months, which is at the high end of what one sees in the data, but still quite reasonable. Second, our price-change distribution matches the salient features of the empirical distribution: the average magnitude is large, yet there are many small price changes, and so forth. Third, as is observed in the data, in the model the probability and magnitude of price adjustments are approximately independent of the age of a price. Fourth, the model correctly predicts that inflation increases both the frequency and the magnitude of price changes. Finally, the model correctly predicts that inflation increases the fraction of positive price changes and reduces the fraction of negative price changes. We do not say that these are the key features of the empirical price-change distribution because the model does well on these dimensions—these are what are reported to be the key features in the empirical papers mentioned previously. Our model also makes predictions about the data not emphasized in the existing literature, including the functional form of the price (as opposed to the price-change) distribution. We have not studied these predictions in detail, but in principle one can try to fit actual distributions for different products, since the BJ distribution depends on micro parameters like utility, cost, and search frictions in particular markets, as well as macro variables like inflation.¹⁵

15. It has been suggested that our model makes the following testable prediction: while p may not increase with inflation, for an individual firm, if p does not increase then for sure q must go up. But this is also true of many New Keynesian theories, and is really nothing more than the law of demand. A stronger test would be to see if q goes up by exactly enough to keep profit constant, which is the essence of BJ models.

Existing theories cannot account for all these features of the behavior of prices. On the one hand, menu cost theories of price rigidity (e.g., Golosov and Lucas 2007) cannot simultaneously account for the average duration of prices and size of changes, which suggest that menu costs are large, and the large fraction of price changes that are small, which suggests that menu costs are not large. On the other hand, time-dependent theories of price rigidity (e.g., Calvo 1983 or Taylor 1980) cannot account for the effect of inflation on the frequency of price adjustment, because this is a technological parameter. One theory that matches the empirical behavior of prices reasonably well is the one by Midrigan (2006), which combines elements of state-dependent and time-dependent theories. However, in his model money is not neutral. Based on Midrigan's results, one might conclude that one needs a model where money is not neutral to account for the data, and that certain policy prescriptions are therefore warranted. We show this is not correct, by providing a model consistent with the facts and with exact neutrality.

5. Conclusion

This paper provides a theory of sticky prices that does not impose ad hoc restrictions on repricing: sellers are free to change prices whenever they like at no cost. Yet the model is consistent with the sticky-price facts. It relies on standard search frictions, which deliver equilibrium price dispersion, plus (combined with some other assumptions) a genuine role for money. Hence, it is natural that firms set prices in dollars, and it is permissible for some of them to not change their individual prices when the aggregate money supply and price level change, or when real factors change. Stickiness is thus a corollary of dispersion. This is true of relative prices in nonmonetary economies, and nominal prices in monetary economies. Contrary to claims that one sees in the literature, price rigidity does *not* require technological restrictions on repricing, and does *not* imply we can exploit particular policy options. Money is neutral in the model, although not superneutral, since real effects result from changes in money growth, inflation, or nominal interest rates. This is important because it makes it hard to rule out classical neutrality in the data—how can one be sure that any real effects we see result from changes in M , not changes in μ , π , or i ?¹⁶

We understand that our results may be controversial. As requested by a referee, we acknowledge the following points, quoting liberally from his or her report. This referee asks how one should interpret the results. Three possibilities are suggested. If one believes in neutrality, then “this is a model that rationalizes price stickiness pretty nicely. There are alternative stories of course, but this model does a decent job at fitting the micro data and should be taken seriously. End of the story.” However, if one “instead believes that money is not neutral . . . there are two options: The first

16. We did not expand on exactly how changes in the inflation (or money growth or nominal interest) rate may affect equilibrium allocations, prices, or welfare in the model, mostly in the interests of space. Again, see Wang (2011) for a start on analyzing this important issue.

possibility is to interpret this paper as a first step of an ambitious research agenda that ultimately aims at providing a unified explanation for both price rigidity and monetary nonneutrality, based on search theory . . . The second option is instead interpreting the result of the paper as suggesting that economists who believe in the non-neutrality of money should not focus on price stickiness as a source of this non-neutrality, but on something else.” The referee asks us to “present these three possible interpretations more explicitly, as guidance to the reader,” and we agree.

The report goes on to say that, in addition to comparing the results to those from Calvo and Mankiw models, we ought to provide a discussion of and comparison to imperfect-information models of price rigidities, which are “similar in spirit to the model of this paper . . . [and] becoming increasingly popular in the literature.” We are not as sure about this point. Of course we support the idea that information processing costs are important in theory and in the real world—this is what search theorists have been on about for the last 50 years! For some reason, it seems that the “sticky information” and “rational inattention” literatures do not recognize this. But we are happy to acknowledge any similarities one sees between the approaches. It is true that in earlier versions of the paper, except for the title, we ignored “rational inattention,” but one can hardly expect the people who work on that topic to find that surprising. Still, we mention research in this area going back to Mankiw and Reis (2002), Woodford (2002), Sims (2003), and others; see Mackowiak and Wiederhold (2009) for a recent contribution with more references.

Much more can be done with this type of model. For instance, we only consider homogeneous sellers. It is well known that in BJ models, if every seller has a different marginal cost, one still gets price dispersion, but firms are not indifferent: in equilibrium, each seller does have a unique target real price. In this case, a change in M leads to a proportional change in p for every firm that preserves their ranking in the distribution. But if there are only a finite number of seller types, there will be a finite number of BJ distributions, and in the support of each distribution firms are indifferent, so we still get nominal rigidity. We think it is reasonable to assume a finite number of firm types. In the model, as in the real world, marginal cost for a retailer is the wholesale price, and each bottle of shampoo you sell (at least over a big range) is replaced by the supplier at the same price. Even though our retailers sell goods in frictional markets, they buy their inventories in centralized markets, where the wholesale price is unique, since the law of one price holds in frictionless markets. There may be some deviations from this law—for example, Walmart may get shampoo at a lower cost due to quantity discounts—but we do not think that every single seller in the economy has a distinct marginal cost.¹⁷

It would be interesting to also analyze models with a finite number of sellers, not only a finite number of types. Would this also generate nominal rigidities? It also seems important to consider versions of the model where sellers set potentially nonlinear price-quantity schedules, which were not considered here. We do not know

17. Even if all sellers pay different rents or wages, have different opportunity costs, etc., this obviously does not mean that their *marginal* costs are all different.

what these extensions might deliver. Finally, we took the extreme position that menu costs are literally zero. One could in principle study models with $\varepsilon > 0$ menu costs, and imagine a refinement that selects a particular outcome of our model as the result of taking $\varepsilon \rightarrow 0$, but, as far as we can see, this is not a trivial extension. Of course menu costs that are literally zero are unrealistic. As Peter Diamond pointed out after the Marshall Lecture, our model misses one obvious aspect of the data: it actually *does* cost something to change your price. Sure. It also costs something to change your quantity, change your shoes, change your mind, or change your password. It is not as clear, however, that economists ought to base policy recommendations on theories that rely critically on one such cost to the exclusion of all others. At the very least, we hope that this paper can be appreciated as a cautionary tale about drawing conclusions, concerning either theory or policy, from the observation that some prices are sticky.

Appendix A. Proof of Proposition 1

We prove the result in the following five steps.

CLAIM A.1. $\Pi_t^* > 0$.

Proof. For any $\tau > 1$, the profit from posting $p = \tau c/\phi_t$ is

$$\begin{aligned}\Pi_t(\tau c/\phi_t) &= \frac{1}{s}\{\alpha_1 + 2\alpha_2[1 - F_t(\tau c/\phi_t)] + \alpha_2\xi_t(\tau c/\phi_t)\}q_t^*(\tau c/\phi_t)(\tau - 1)c/\phi_t \\ &> \frac{\alpha_1}{s}q_t^*(\tau c/\phi_t)(\tau - 1)c/\phi_t,\end{aligned}\quad (\text{A.1})$$

where $\xi_t(\tau c/\phi_t)$ was defined immediately after (12). Since $q_t^*(\tau c/\phi_t) > 0$ and $\tau > 1$, $\Pi_t(\tau c/\phi_t) > 0$. Hence, $\Pi_t^* \geq \Pi_t(\tau c/\phi_t) > 0$. \square

CLAIM A.2. F_t is continuous.

Proof. Suppose $\exists p_0 \in \mathcal{F}_t$ such that $\xi_t(p_0) > 0$, and

$$\Pi_t(p_0) = \frac{1}{s}\{\alpha_1 + 2\alpha_2[1 - F_t(p_0)] + \alpha_2\xi_t(p_0)\}R_t(p_0).\quad (\text{A.2})$$

Given $R_t(p)$ is continuous in p , there is a $p_1 < p_0$ such that $R_t(p_1) > 0$ and $\Lambda \equiv R_t(p_0) - R_t(p_1) < \alpha_2\xi_t(p_0)R_t(p_0)/(\alpha_1 + 2\alpha_2)$. Then

$$\begin{aligned}\Pi_t(p_1) &= \frac{1}{s}\{\alpha_1 + 2\alpha_2[1 - F_t(p_1)] + \alpha_2\xi_t(p_1)\}R_t(p_1) \\ &\geq \frac{1}{s}\{\alpha_1 + 2\alpha_2[1 - F_t(p_0)] + 2\alpha_2\xi_t(p_0)\}[R_t(p_0) - \Lambda] \\ &\geq \Pi_t(p_0) + \alpha_2\xi_t(p_0)[R_t(p_0) - \Lambda] - (\alpha_1 + 2\alpha_2)\Lambda,\end{aligned}\quad (\text{A.3})$$

where the second line follows from $F(p_0) - F(p_1) \geq \xi_t(p_0)$. With $R_t(p_0) > \Lambda$ and $\Lambda < \alpha_2\xi_t(p_0)R_t(p_0)/(\alpha_1 + 2\alpha_2)$, then Equation (A.3) implies $\Pi_t(p_1) > \Pi_t(p_0)$. This contradicts $p_0 \in \mathcal{F}_t$. \square

CLAIM A.3. p_t^m (monopoly price) is the highest price in \mathcal{F}_t .

Proof. Suppose that $\bar{p}_t \neq p_t^m$ is the highest price in \mathcal{F}_t . Then

$$\Pi_t(\bar{p}_t) = \frac{\alpha_1}{s} R_t(\bar{p}_t). \quad (\text{A.4})$$

With $F_t(p_t^m) \geq 0$, profits at p_t^m satisfy

$$\Pi_t(p_t^m) = \frac{1}{s} \{ \alpha_1 + 2\alpha_2[1 - F_t(p_t^m)] \} R_t(p_t^m) \geq \frac{\alpha_1}{s} R_t(p_t^m) > \frac{\alpha_1}{s} R_t(\bar{p}_t). \quad (\text{A.5})$$

Now Equations (A.4) and (A.5) imply $\Pi_t(p_t^m) > \Pi_t(\bar{p}_t)$. However, by the equal profit condition, $\Pi_t(\bar{p}_t) = \Pi_t^* \geq \Pi_t(p_t^m)$. This establishes the claim. \square

CLAIM A.4. \mathcal{F}_t is connected.

Proof. Suppose $p_0, p_1 \in \mathcal{F}_t$ with $p_0 < p_1$ and $F_t(p_0) = F_t(p_1)$. Then

$$\Pi_t(p_0) = \frac{1}{s} \{ \alpha_1 + 2\alpha_2[1 - F_t(p_0)] \} R_t(p_0), \quad (\text{A.6})$$

$$\Pi_t(p_1) = \frac{1}{s} \{ \alpha_1 + 2\alpha_2[1 - F_t(p_1)] \} R_t(p_1). \quad (\text{A.7})$$

Since $F_t(p_1) = F_t(p_0)$, we have $\alpha_1 + 2\alpha_2[1 - F_t(p_1)] = \alpha_1 + 2\alpha_2[1 - F_t(p_0)]$. Since $p_0, p_1 \in \mathcal{F}_t$, we have $c/\phi_t < p_0 < p_1 \leq p_t^m$. Given $R_t(p)$ strictly increasing $\forall p \in [c/\phi_t, p_t^m]$, $R_t(p_1) > R_t(p_0)$. Combining these results, $\Pi_t(p_1) > \Pi_t(p_0)$, which contradicts $\Pi_t(p_0) = \Pi_t(p_1) = \Pi_t^*$. \square

CLAIM A.5. F_t is given by

$$F_t(p) = 1 - \frac{\alpha_1}{2\alpha_2} \left[\frac{R_t(p_t^m)}{R_t(p)} - 1 \right]. \quad (\text{A.8})$$

Proof. Since F_t has no mass points,

$$\Pi_t(p) = \frac{1}{s} \{ \alpha_1 + 2\alpha_2[1 - F_t(p)] \} R_t(p). \quad (\text{A.9})$$

At p_t^m , profit is maximized at $\Pi_t^* = \frac{\alpha_1}{s} R_t(p_t^m)$. By equal profit, we get

$$\frac{1}{s} \{ \alpha_1 + 2\alpha_2[1 - F_t(p)] \} R_t(p) = \frac{\alpha_1}{s} R_t(p_t^m) \quad \forall p \in [p_t, \bar{p}_t]. \quad (\text{A.10})$$

Solving (A.10) for F_t leads to (A.8). \square

Appendix B. Proof of Proposition 2

It is convenient to rewrite the model in real terms: $n_t = \phi_t \hat{m}_t$ is real money taken out of AD and put into BJ; $z_t = \phi_t p$ is the real price associated with nominal price p in BJ and $J_t(z, n)$ is the distribution of real prices in BJ, now written explicitly as depending on real balances in equilibrium. The following definition is equivalent to the concepts presented in the main text.

DEFINITION B.1. A stationary monetary equilibrium in real variables is a list of AD quantities (h^*, x^*, n^*) , a BJ decision rule $q^*(z, n^*)$, and a BJ distribution $J^*(z, n^*)$ satisfying the following conditions.

1. (h^*, x^*, n^*) solves the household's AD problem, including the real analog of (11):

$$i = \int_0^{\hat{z}(n^*)} [\alpha_1 + 2\alpha_2 - 2\alpha_2 J^*(z, n^*)] \left[\frac{1}{z} \left(\frac{n^*}{z} \right)^{-\gamma} - 1 \right] dJ^*(z, n^*), \quad (\text{B.1})$$

where $\hat{z}(n^*)$ is the real price below which households cash out in BJ.

2. $q^*(z, n^*)$ solves the household's BJ problem as described by the real analog of (7):

$$q^*(z, n^*) = \begin{cases} n^*/z & \text{if } z \leq \hat{z}(n^*), \\ z^{-\frac{1}{\gamma}} & \text{if } z > \hat{z}(n^*). \end{cases} \quad (\text{B.2})$$

3. $J^*(z, n^*)$ solves the firm's BJ problem as described by the analog to Proposition 1:

$$J^*(z, n^*) = 1 - \frac{\alpha_1}{2\alpha_2} \left[\frac{R(z^m, n^*)}{R(z, n^*)} - 1 \right], \quad (\text{B.3})$$

where $R(z, n^*) = q^*(z, n^*)(z - c)$, and the support of J^* is $\mathcal{J}^* = [\underline{z}(n^*), \bar{z}(n^*)]$.

There is no relative price of n (analogous to ϕ) since the relative price of real balances is 1 by construction.

We now show there exists n^* such that households choose $n = n^*$ given the distribution $J^*(z, n^*)$. This is equivalent to showing there exists n^* such that

$$i = \int_0^{\hat{z}(n)} \left[\frac{1}{z} \left(\frac{n}{z} \right)^{-\gamma} - 1 \right] [\alpha_1 + 2\alpha_2 - 2\alpha_2 J^*(z, n^*)] dJ^*(z, n^*) \quad (\text{B.4})$$

is solved by $n = n^*$. We proceed in three steps.

CLAIM B.1. Let \underline{n}^* and \bar{n}^* be defined by

$$\hat{z}(\underline{n}^*) = c(1 - \gamma)^{-1} \text{ and } \hat{z}(\bar{n}^*) = c \left[1 - \frac{\alpha_1}{(\alpha_1 + 2\alpha_2)} \frac{1}{n^*} \left(\frac{c}{1 - \gamma} \right)^{-1/\gamma} \left(\frac{\gamma c}{1 - \gamma} \right) \right]^{-1}. \quad (\text{B.5})$$

For n_0^* and n_1^* such that $0 < n_0^* < n_1^* \leq \bar{n}^*$, then $J(z, n_0^*)$ first-order stochastically dominates $J(z, n_1^*)$. For n_0^* and n_1^* such that $\bar{n}^* \leq n_0^* < n_1^*$, then $J(z, n_0^*) = J(z, n_1^*)$.

Proof. For $n^* \in (0, \underline{n}^*]$,

$$J(z, n^*) = 1 - \frac{\alpha_1}{2\alpha_2} \left\{ \frac{\bar{z}(n^*)^{-1} n^* [\bar{z}(n^*) - c]}{z^{-1} n^* (z - c)} - 1 \right\} \quad (\text{B.6})$$

with support $[\underline{z}(n^*), \bar{z}(n^*)]$, where

$$\bar{z}(n^*) = (n^*)^{-\gamma/(1-\gamma)} \text{ and } \underline{z}(n^*) = c \left\{ 1 - \frac{\alpha_1}{\alpha_1 + 2\alpha_2} \left[\frac{\bar{z}(n^*) - c}{\bar{z}(n^*)} \right] \right\}^{-1}. \quad (\text{B.7})$$

Consider any n_0^* and n_1^* such that $0 < n_0^* < n_1^* \leq \bar{n}^*$. Clearly, $\bar{z}(n_0^*) > \bar{z}(n_1^*)$ and $\underline{z}(n_0^*) > \underline{z}(n_1^*)$. For $z \geq \bar{z}(n_0^*)$, we have $J(z, n_0^*) = J(z, n_1^*) = 1$. For $z \in (\underline{z}(n_1^*), \bar{z}(n_0^*))$, then $J(z, n_0^*) < J(z, n_1^*)$ because $\bar{z}(n_0^*) > \bar{z}(n_1^*)$. For $z \leq \underline{z}(n_1^*)$, then $J(z, n_0^*) = J(z, n_1^*) = 0$. Hence, $J(z, n_0^*)$ first-order stochastically dominates $J(z, n_1^*)$.

For $n^* \in [\underline{n}^*, \bar{n}^*]$,

$$J(z, n^*) = \begin{cases} 1 - \frac{\alpha_1}{2\alpha_2} \left\{ \frac{\bar{z}(n^*)^{-1/\gamma} [\bar{z}(n^*) - c]}{z^{-1/\gamma}(z - c)} - 1 \right\} & \text{if } z \in [\hat{z}(n^*), \bar{z}(n^*)], \\ 1 - \frac{\alpha_1}{2\alpha_2} \left\{ \frac{\bar{z}(n^*)^{-1/\gamma} [\bar{z}(n^*) - c]}{z^{-1}n^*(z - c)} - 1 \right\} & \text{if } z \in [\underline{z}(n^*), \hat{z}(n^*)], \end{cases} \quad (\text{B.8})$$

with support $[\underline{z}(n^*), \bar{z}(n^*)]$, where

$$\bar{z}(n^*) = \frac{c}{1 - \gamma} \text{ and } \underline{z}(n^*) = c \left\{ 1 - \frac{\alpha_1}{\alpha_1 + 2\alpha_2} \frac{\bar{z}(n^*)^{-1/\gamma} [\bar{z}(n^*) - c]}{n^*} \right\}^{-1}. \quad (\text{B.9})$$

Consider any n_0^* and n_1^* such that $\underline{n} \leq n_0^* < n_1^* \leq \bar{n}$. Clearly, $\bar{z}(n_0^*) = \bar{z}(n_1^*)$, $\hat{z}(n_0^*) > \hat{z}(n_1^*)$, and $\underline{z}(n_0^*) > \underline{z}(n_1^*)$. For $z \geq \hat{z}(n_0^*)$, then $J(z, n_0^*) = J(z, n_1^*)$. For $z \in [\hat{z}(n_1^*), \hat{z}(n_0^*)]$, then $J(z, n_0^*) < J(z, n_1^*)$ because $z^{-1}n_0^* < z^{-1/\gamma}$. For $z \in [\underline{z}(n_0^*), \hat{z}(n_1^*)]$, then $J(z, n_0^*) < J(z, n_1^*)$ because $n_0^* < n_1^*$. For $z \leq \underline{z}(n_0^*)$, then $J(z, n_0^*) \leq J(z, n_1^*)$. Hence, $J(z, n_0^*)$ first-order stochastically dominates $J(z, n_1^*)$.

For $n^* \geq \bar{n}^*$,

$$J(z, n^*) = 1 - \frac{\alpha_1}{2\alpha_2} \left\{ \frac{\bar{z}(n^*)^{-1/\gamma} [\bar{z}(n^*) - c]}{z^{-1/\gamma}(z - c)} - 1 \right\}, \quad (\text{B.10})$$

with support $[\underline{z}(n^*), \bar{z}(n^*)]$, where

$$\bar{z}(n^*) = \frac{c}{1 - \gamma} \text{ and } \underline{z}(n^*) = c + \frac{\alpha_1}{\alpha_1 + 2\alpha_2} \left[\frac{\underline{z}(n^*)}{\bar{z}(n^*)} \right]^{-1/\gamma} [\bar{z}(n^*) - c]. \quad (\text{B.11})$$

In this case, $J(z, n_0^*) = J(z, n_1^*)$ for all n_0^* and n_1^* such that $\bar{n}^* \leq n_0^* < n_1^*$. \square

CLAIM B.2. *Given $J(z, n^*)$, let the unique solution for n in the household's AD problem be $n = \psi(n^*)$. Then $\psi(n^*)$ has the following properties:*

- $\forall n_0^*, n_1^*$ such that $0 < n_0^* < n_1^* \leq \bar{n}^*$, $\psi(n_0^*) \leq \psi(n_1^*)$,
- $\forall n_0^*, n_1^*$ such that $\bar{n}^* \leq n_0^* < n_1^*$, $\psi(n_0^*) = \psi(n_1^*)$,
- $\forall n^* > 0$, $\psi(n^*) \in [\underline{\psi}, \bar{\psi}]$, where $\underline{\psi} > 0$ and $\bar{\psi} = \bar{n}^*$.

Proof. Given the distribution $J(z, n^*)$, the equilibrium condition for n is

$$i = \alpha_1 \int_0^{\hat{z}(n)} \left[\left(\frac{z}{n} \right)^\gamma \frac{1}{z} - 1 \right] dJ(z, n^*) + \alpha_2 \int_0^{\hat{z}(n)} \left[\left(\frac{z}{n} \right)^\gamma \frac{1}{z} - 1 \right] d\{1 - [1 - J(z, n^*)]^2\}. \quad (\text{B.12})$$

Let $\chi(n, n^*)$ denote the RHS. Notice $\lim_{n \rightarrow 0} \chi(n, n^*) = \infty$ and $\chi(n, n^*)$ is strictly decreasing in $n \forall n \in (0, \hat{z}^{-1}[\underline{z}(n^*)])$. Also, $\chi(n, n^*) = 0 \forall n \geq \hat{z}^{-1}[\underline{z}(n^*)] =$

$\bar{z}(n^*)^{-(1-\gamma)/\gamma}$. Hence there is a unique solution $n = \psi(n^*)$ to $i = \chi(n, n^*)$, and $0 < \psi(n^*) < \hat{z}^{-1}[\bar{z}(n^*)]$.

Consider any n_0^*, n_1^* such that $0 < n_0^* < n_1^* \leq \bar{n}^*$. Claim B.1 implies that $J(z, n_0^*)$ first-order stochastically dominates $J(z, n_1^*)$ and, consequently, $1 - [1 - J(z, n_0^*)]^2$ first-order stochastically dominates $1 - [1 - J(z, n_1^*)]^2$. From this and the fact that $(z/n)^\gamma/z - 1$ is decreasing in z , it follows that $\chi(n, n_0^*) \leq \chi(n, n_1^*) \forall n$. Therefore, $\psi(n_0^*) \leq \psi(n_1^*)$. Moreover, it is straightforward to verify that $\psi(n_0^*) \geq \underline{\psi}$ for some $\underline{\psi} > 0$.

Now consider any n_0^*, n_1^* such that $\bar{n}^* \leq n_0^* < n_1^*$. Claim B.1 implies that $J(z, n_0^*) = J(z, n_1^*)$ and $1 - [1 - J(z, n_0^*)]^2 = 1 - [1 - J(z, n_1^*)]^2$. It follows that $\chi(n, n_0^*) = \chi(n, n_1^*)$, and hence $\psi(n_0^*) = \psi(n_1^*)$. It is straightforward to verify $\psi(n_1^*) < \bar{n}^*$. \square

CLAIM B.3. $\exists n^* \in [\underline{\psi}, \bar{\psi}]$ such that $\psi(n^*) = n^*$.

Proof. Claim B.2 implies that $\psi(n^*)$ is increasing and $\psi(n^*) \in [\underline{\psi}, \bar{\psi}] \forall n^* \in [\underline{\psi}, \bar{\psi}]$. By Tarski's theorem, $\exists n^* \in [\underline{\psi}, \bar{\psi}]$ such that $\psi(n^*) = n^*$. This establishes existence. Claim B.2 implies $\psi(n^*) \geq \underline{\psi} \forall n^* < \underline{\psi}$ and $\psi(n^*) < \bar{\psi} \forall n^* \geq \bar{\psi}$. Hence, $\nexists n^* \notin [\underline{\psi}, \bar{\psi}]$ such that $\psi(n^*) = n^*$. \square

Appendix C. Proof of Proposition 4

We take two steps to prove the results.

CLAIM C.1. *In any stationary monetary equilibrium*

$$\frac{\bar{p}_t - p_t}{\bar{p}_t} \geq \Lambda > 0, \quad (\text{C.1})$$

where $\Lambda = 2\alpha_2\gamma/[2\alpha_2 + \alpha_1(1 - \gamma)]$.

Proof. Households are either constrained in all BJ transactions, or constrained in some but not others. First suppose they are constrained in all transactions. Then

$$\phi_t \bar{p}_t = (\phi_t \hat{m}_t^*)^{\gamma/(\gamma-1)} \geq \frac{c}{1-\gamma}, \quad (\text{C.2})$$

$$\phi_t p_t = c \left(1 - \frac{\alpha_1}{\alpha_1 + 2\alpha_2} \frac{\phi_t \bar{p}_t - c}{\phi_t \bar{p}_t} \right)^{-1}. \quad (\text{C.3})$$

From Equations (C.2) and (C.3), it follows that

$$\frac{\bar{p}_t - p_t}{\bar{p}_t} = \frac{2\alpha_2(\phi_t \bar{p}_t - cw)}{2\alpha_2\phi_t \bar{p}_t + \alpha_1 c} \geq \frac{2\alpha_2\gamma}{2\alpha_2 + \alpha_1(1 - \gamma)} = \Lambda. \quad (\text{C.4})$$

Now suppose households are constrained in only some transactions. Then

$$\phi_t \bar{p}_t = \frac{c}{1-\gamma} \geq (\phi_t \hat{m}_t^*)^{\gamma/(\gamma-1)}, \quad (\text{C.5})$$

$$\phi_t \underline{p}_t = c \left[1 - \frac{\alpha_1}{\alpha_1 + 2\alpha_2} (\phi_t \bar{p}_t)^{-1/\gamma} \left(\frac{\phi_t \bar{p}_t - c}{\phi_t \hat{m}_t^*} \right) \right]^{-1}. \quad (\text{C.6})$$

From Equations (C.5) and (C.6),

$$\frac{\bar{p}_t - \underline{p}_t}{\bar{p}_t} = \frac{(\alpha_1 + 2\alpha_2)\gamma\phi_t\hat{m}_t^* - \alpha_1\gamma \left(\frac{c}{1-\gamma} \right)^{(\gamma-1)/\gamma}}{(\alpha_1 + 2\alpha_2)\phi_t\hat{m}_t^* - \alpha_1\gamma \left(\frac{c}{1-\gamma} \right)^{(\gamma-1)/\gamma}} \geq \frac{2\alpha_2\gamma}{2\alpha_2 + \alpha_1(1-\gamma)} = \Lambda, \quad (\text{C.7})$$

where the inequality uses $\phi_t\hat{m}_t^* \geq [(1-\gamma)/c]^{(1-\gamma)/\gamma}$. Then (C.4) and (C.7) imply (C.1). \square

CLAIM C.2. Let $\mu \in (1, \mu^*)$, where $\mu^* = (1 - \Lambda)^{-1}$. Then, given repricing policy $p_{t+1}^*(p, \rho)$, we have: $\Phi(\rho) < 1$ and $A(\rho) > 0 \forall \rho \in (0, 1]$; and $H_t(0, \rho) > 0 \forall \rho \in [0, 1)$.

Proof. From Claim C.1 and $\mu \in (1, \mu^*)$, we have

$$\mu \underline{p}_t < \frac{\underline{p}_t}{1 - \Lambda} \leq \bar{p}_t. \quad (\text{C.8})$$

For any $\rho \in (0, 1]$, the fraction of prices that adjust is

$$\begin{aligned} \Phi(\rho) &= F_t^*(\mu \underline{p}_t) + (1 - \rho)[1 - F_t^*(\mu \underline{p}_t)] \\ &< F_t^*((1 - \Lambda)^{-1} \underline{p}_t) + (1 - \rho)[1 - F_t^*((1 - \Lambda)^{-1} \underline{p}_t)] < 1, \end{aligned} \quad (\text{C.9})$$

using (C.8) and $F_t^*[(1 - \Lambda)^{-1} \underline{p}_t] < F_t^*(\bar{p}_t) = 1$. Since the fraction of prices that adjust is less than 1, $A(\rho) > 1$. Finally, it is easy to verify $J_t(0, \rho) > 0 \forall \rho \in [0, 1)$. \square

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