Equilibrium Price Dispersion Across and Within Stores*

Guido Menzio
University of Pennsylvania and NBER

Nicholas Trachter
Federal Reserve Bank of Richmond

August 2017

Abstract

We develop a search-theoretic model of the product market that generates price dispersion across and within stores. Buyers differ with respect to their ability to shop around, both at different stores and at different times. The fact that some buyers can shop from only one seller while others can shop from multiple sellers causes price dispersion across stores. The fact that the buyers who can shop from multiple sellers are more likely to be able to shop at multiple times causes price dispersion within stores. Specifically, it causes sellers to post different prices for the same good at different times in order to discriminate between different types of buyers.

JEL Codes: D43.

Keywords: Search, Multidimensional price dispersion, Price discrimination, Bargain hunting.

*Menzio: Department of Economics, University of Pennsylvania, 3718 Locust Walk, Philadelphia, PA 19013 (email: gmenzio@sas.upenn.edu). Trachter: Federal Reserve Bank of Richmond, 701 E. Byrd Street, Richmond, VA 23219 (email: nicholas.trachter@rich.frb.org). We are grateful to an Associate Editor and two anonymous referees for their fruitful suggestions. We also wish to thank Ken Burdett, Greg Kaplan, Agata Menzio, Leena Rudanko and Ali Shourideh for their comments. The views expressed in this paper are those of the authors and do not necessarily represent the views of the Federal Reserve Bank of Richmond or the Federal Reserve System.

1 Introduction

This paper is a contribution to the theory of equilibrium price dispersion and it is motivated by three facts. First, it is well-known that the same product is sold at very different prices, even when one restricts attention to the same geographical area and the same period of time. In his pioneering article on price dispersion, Stigler (1961) finds that the same car was sold at very different prices by different dealerships in Chicago. Sorensen (2000) finds that the average standard deviation of the price posted by different pharmacies for the same drug in the same town in upstate New York is 22%. In a systematic study of price dispersion that covers 1.4 million goods in 54 geographical markets within the US, Kaplan and Menzio (2015) find that the average standard deviation of the price at which the same product is sold within the same geographical area and during the same quarter is 19%. Second, it is well-known that price dispersion is caused both by differences in prices across different stores and by differences in prices within each store. For instance, Kaplan and Menzio (2015) find that approximately half of the variance of prices for the same good in the same area and in the same quarter is due to the fact that different stores sell the good at a different price on average, while the remaining half is due to the fact that the same store sells the same good at different prices during the same quarter.[1] Third, as documented by Nakamura and Steinsson (2008) and Klenow and Kryvtsov (2008), a large fraction of all the changes in the price of a particular good at a particular store are due to temporary sales, defined as temporary price reductions.[2]

Motivated by the above observations, we develop a theory of price dispersion across and within stores by building a model that combines the search theory of price dispersion across sellers (see, e.g., Burdett and Judd 1983) with the intertemporal price discrimination theory of temporary sales (see, e.g., Conslik, Gerstner and Sobel 1984). To this aim, we build a model of the market for an indivisible good. On the demand side, there are buyers who differ with respect to their ability to shop at different stores, as well as with respect to their ability to shop at different times: Some buyers can shop from only one seller while others can shop from multiple sellers, and some buyers can shop only during

[1] These decompositions follow immediately from Table 3 in Kaplan and Menzio (2015). For a particular good, the fraction of the price variance coming from across-store variation is given by the sum of the variance of the store component and the store-good component. The fraction of the price variance coming from within store variation is the variance of the transaction component.

[2] Temporary sales account for approximately half of all the changes in the price of a particular good at a particular store. They account for a larger fraction of the within-seller price variance because, on average, price changes due to temporary sales are three times larger than regular price changes (see Nakamura and Steinsson 2008).

the day while others can shop both during the day and during the night. On the supply side, there are identical sellers and each seller posts a (potentially different) price for the good during the day and during the night.

We prove the existence and uniqueness of equilibrium. We find that the equilibrium features dispersion in the average price posted by different sellers (i.e., price dispersion across stores). Moreover, if the type of buyers who are more likely to be able to shop from multiple sellers are also more likely to be able to shop at different times, the equilibrium also features dispersion in the price posted by the same seller during the day and during the night (i.e., price dispersion within stores). In contrast, if the type of buyers who are more likely to be able to shop from multiple sellers are less likely to be able to shop at different times, the equilibrium does not feature any variation in the price posted by the same seller during the day and during the night (i.e., the equilibrium displays no price dispersion within stores).

The properties of equilibrium are intuitive. Price dispersion across sellers arises for essentially the same reason as in Burdett and Judd (1983). That is, whenever sellers face a population of potential customers that includes both captive buyers (i.e. buyers who cannot shop from any other seller) and non-captive buyers (i.e., buyers who can shop elsewhere), the equilibrium must feature price dispersion. Price dispersion within sellers arises if and only if the buyers who are more likely to be non-captive are also more likely to be able to shop at any time. In fact, a seller would like to charge higher prices to captive buyers than to non-captive buyers, because—given price dispersion across stores—captive buyers have on average a higher reservation price. If the non-captive buyers are more likely to be able to shop at any time while captive buyers are more likely to be able to only shop during the day, a seller can successfully price discriminate by posting a higher price during the day and a lower price during the night. If, on the other hand, non-captive buyers are less likely to be able to shop at any time, a seller cannot successfully price discriminate. Indeed, if the seller posted a lower price at night, he would end up offering a better deal to the captive buyers. And if the seller posted a lower price during the day, he would end up offering a better deal to both the non-captive and the captive buyers.

Our theory of price dispersion across and within sellers requires the same assumptions as the search theory of price dispersion and the intertemporal price discrimination theory of sales. As in the search theory of price dispersion (see, e.g., Butters 1977, Varian 1980, Burdett and Judd 1983), we assume that some buyers can shop from multiple sellers while others can shop from only one seller. The assumption can be seen as a by-product of heterogeneity in the buyers' opportunity cost of shopping at different locations. As in the intertemporal price discrimination theory of sales (see, e.g., Conslik, Gerstner and Sobel 1984, Sobel 1984, Albrecht, Postel-Vinay and Vroman 2013), we assume that some buyers need to shop at a particular time while others can shop at any time. The assumption can be seen as a by-product of heterogeneity in the buyers' opportunity cost of shopping at different times. Additionally, the theory of intertemporal price discrimination assumes that the buyers who must shop at a particular time have a higher willingness to pay for the good. In this paper, we make the closely related assumption that buyers who must shop at a particular time are less likely to be able to shop from multiple sellers which, in the presence of price dispersion, implies that these buyers have a higher reservation price. Basically, we endogenize the difference in the willingness to pay between buyers who are flexible and those who are not flexible with respect to the shopping schedule, and we do so in a natural way, as buyers who have a low opportunity cost of shopping at different locations presumably also have a low opportunity cost of shopping at different points in time. Even though there is not much direct evidence on the assumptions underlying the search theory of price dispersion and the intertemporal price discrimination theory of sales[3], these theories are commonly viewed as the leading explanations for, respectively, dispersion in the price of a particular good across different stores and temporary sales.

The assumptions and properties of our model are a simple combination of the properties of models of price dispersion and intertemporal price discrimination. Yet, the characterization of the equilibrium of our model presents novel challenges and calls for a novel solution strategy. As this is the main technical contribution of the paper, let us expand on this point. In Burdett and Judd (1983), each seller posts one price. In equilibrium, each seller must attain the same profit by posting any price on the support of the price distribution. Then, characterizing the equilibrium amounts to solving one equation—the seller's equal profit condition—for one unknown—the price distribution. In our model, each seller posts two prices, a day and a night price, and the two prices interact in the

[3]Evidence on the assumptions of the model is hard to come by because the assumptions involve heterogeneity across buyers in the number of their shopping option, not in the number of their choices. If one is willing to extrapolate the assumptions outside of the model, one can find some evidence that there is heterogeneity across buyers in their willingness to shop at different location, in their willingness of shopping at different times, and that the two traits are positively correlated. For instance, Aguiar and Hurst (2007) find that retirement-age people spend more time on each shopping trip (perhaps to reach less convenient locations and, if so, revealing a greater willingness to shop around), take more shopping trips (perhaps to take advantage of asynchronized sales and, if so, revealing a greater willingness to shop at different times), and end up paying lower prices than younger people. Similarly, Kaplan and Menzio (2015, 2016) show that non-employed, working-age people spend more time shopping, take more shopping trips and pay lower prices than employed people.

seller's profit as some of the customers can shop at any time and pick the lowest price. The interaction between the prices results in the constraint that the seller's night price be non-greater than his day price. As in Burdett and Judd (1983), each seller must attain the same profit by posting any pair of prices on the support of the equilibrium price distribution. However, this observation is no longer enough to characterize the equilibrium, as it provides one equation—the seller's equal profit condition—for two unknowns—the distribution of day prices and the distribution of night prices. More is needed to solve for the equilibrium.

The key step to solve for the equilibrium of our model is showing that—depending on parameter values—one of two cases may arise. Either the seller's profit from day and night trades are both constant on the support of the respective marginal price distributions, or the seller's profit from night trades is strictly increasing in the night price. In the first case, we have two equations—the seller's equal profit condition from day and night trades—for two unknowns—the marginal distributions of day and night prices. After solving these equations, we find the equilibrium joint price distribution as one (of many) distribution that is consistent with the marginals and such that every seller posts a lower price at night than during the day. In the second case, every seller wants to post as high a price as possible during the night, subject to the constraint that the night price is non-greater than the day price. In this case, the equilibrium is such that every seller posts the same price in both periods and solving for the common marginal distribution boils down to solving one equation—the seller's equal profit condition given that the day and night prices are equal—in one unknown—the common marginal distribution of day and night prices.

The solution technique developed in this paper may be useful to solve other versions of Burdett and Judd (1983) where sellers set multiple prices. Indeed, Kaplan et al. (2016) borrow it to solve for the equilibrium of a version of Burdett and Judd (1983) in which sellers carry multiple goods and buyers seek multiple goods. Lester et al. (2016) independently developed an argument to characterize the equilibrium of a version of Burdett and Judd (1983) in which sellers post multiple contracts to screen different types of buyers.[4] It is worth noting that these are the very first successful attempts at solving

[4] The characterization strategy in Lester et al. (2016) is different than ours, as their model is different than ours. They show that the seller's profit function is supermodular in the utilities associated with the contracts offered to the two types of buyers in the market. They then use the supermodularity of the profit function to show that any equilibrium is such that the two contracts offered by a seller have the same rank in the marginal distributions of utility offered by sellers to each type of buyer. Garrett,

for multidimensional price distributions in search models of imperfect competition.[5]

2 Environment

We consider the market for an indivisible good that operates in two subperiods: *day* and *night*. On one side of the market, there is a measure 1 of identical sellers who can produce the good on demand at a constant marginal cost, which we normalize to zero. Each seller simultaneously and independently posts a pair of prices (p_d, p_n), where $p_d \in [0, u]$ is the price of the good during the day, $p_n \in [0, u]$ is the price of the good during the night, and $u > 0$ is the buyers' valuation of the good. We denote as $G(p_d, p_n)$ the cumulative distribution function of prices across sellers. We denote as $F_d(p_d)$ the marginal cumulative distribution function of day prices across sellers and with $\phi_d(p_d)$ the measure of sellers with a day price equal to p_d (i.e., $\phi_d(p_d) = F_d(p_d) - F_d(p_d-)$ denotes a mass point in the marginal distribution F_d). Similarly, we denote as $F_n(p_n)$ the marginal distribution of night prices and with $\phi_n(p_n) = F_n(p_n) - F_n(p_n-)$ the measure of sellers with a night price of p_n. We also find it useful to denote as $F_m(p)$ the measure of sellers whose lowest price (between day and night) is non-greater than p and with $\phi_m(p)$ the measure of sellers whose lowest price is equal to p.

On the other side of the market, there is a measure $\theta_x > 0$ of buyers of type x, and a measure $\theta_y > 0$ of buyers of type y. The two types of buyers differ with respect of their ability to shop from different sellers, as well as with respect of their ability to shop at different times. In particular, a buyer of type x is in contact with only one seller with probability $\alpha_x \in (0, 1)$ and with multiple (for the sake of simplicity, two) sellers with probability $1 - \alpha_x$. The buyer observes both the day and night price of the sellers with whom he is contact. The buyer is able to shop from these sellers during both day and night with probability $1 - \beta_x$. With probability $\beta_x \in (0, 1)$, the buyer is able to shop only during the day. Similarly, a buyer of type y is in contact with one seller with probability α_y, and with multiple (two) sellers with probability $1 - \alpha_y$. A buyer of type y is able to shop from the contacted sellers during both day and night with probability $1 - \beta_y$, and

Gomes and Maestri (2016) use a very similar characterization strategy as Lester et al. (2016) in a very related model.

[5] Zhou (2013) and Rhodes (2015) are also search-theoretic models in which stores sell multiple goods and, hence, post multiple prices. Yet, solving for the equilibrium of these models does not present the technical challenges that we face. This is because, in the models by Zhou (2013) and Rhodes (2015), the seller's optimal pricing strategy is generally unique and, hence, solving for the equilibrium does not require solving for a distribution of prices.

only during the day with probability β_y. Both types of buyers enjoy a utility of $u - p$ if they purchase the good at the price p, and a utility of zero if they do not purchase the good. Without loss in generality, we assume that buyers of type x are on average in contact with fewer sellers than buyers of type y, i.e. $\alpha_x \geq \alpha_y$.

The definition of equilibrium for this model is standard (see, e.g., Burdett and Judd 1983 or Head et al. 2012).

Definition 1: *An equilibrium is a price distribution G such that the seller's profit is maximized everywhere on the support of G.*

Before turning to the characterization of equilibrium, a few comments about the interpretation of the environment are in order. We assume that some buyers are in contact with one seller and others are in contact with multiple sellers. We think of the contacts as the network of sellers that the buyer can easily access (e.g., the sellers on the way between the buyer's home and his workplace, the sellers that are close to his children's preschool, etc...), rather than the set of sellers whose price is known to the buyer. In this sense, we think of our model as a version of Hotelling (1929) where the buyer's transportation cost to different sellers is either zero or infinity, rather than as a model in which buyers discover sellers through a search process. Yet, just as in the search models of Butters (1977) and Burdett and Judd (1983), the fraction of buyers in contact with multiple sellers is the parameter that controls the degree of competitiveness of the product market.

We assume that some buyers can shop only during the "day," while others can shop both during the day and during the "night." We do not think of day and night literally. Instead, we interpret the "day" as the time when all buyers are able to shop and the "night" as the time when only a subset of buyers is able to shop. For instance, the "day" might be the time when both employed and non-employed people can shop (i.e., after 5 PM), and the "night" might be the time when only the non-employed people can shop (i.e., before 5 PM). Alternatively, the "night" might be the days of the week when only people with a flexible schedule can shop (i.e., Monday through Friday), and the "day" might be the days when everyone can shop (i.e., the week-end). As in Conslik, Gerstner and Sobel (1984) or Albrecht, Postel-Vinay and Vroman (2013), it is the fact that some buyers can strategically time their purchases to take advantage of low prices while others cannot that gives sellers the opportunity to carry out some intertemporal price discrimination. Notice that the opportunity for intertemporal price discrimination also arises if we assume that a group of buyers needs to shop at a particular time that is idiosyncratic to each buyer (rather than being the day for all buyers), while another group of buyers can shop at any time. Under this alternative assumption, all times are equally convenient for shopping and, thus, "night" and "day" have no special meaning besides being two different periods. In either case, a seller who posts a lower price during the night than during the day runs something that looks very much like a temporary sale, i.e. a price reduction that lasts for some period of time (the night) and then disappears (when the next day comes).

3 Characterization of Equilibrium

This section contains a complete characterization of equilibrium. Section 3.1 shows that, without loss in generality, we can restrict attention to equilibria in which every seller posts a night price non-greater than the day price. Section 3.2 builds on this insight to show that depending on parameter values only one of two cases can emerge in equilibrium. Either the seller's profit from night trades is constant with respect to the night price, or the seller's profit from night trade is strictly increasing in the night price. Section 3.3 focuses on the first case, which occurs when buyers of type x are not only more likely to be captive to one seller but are also more likely to have to shop during the day. We establish the existence and uniqueness of equilibrium, characterize the equilibrium marginal distribution of night and day prices, and the equilibrium joint price distribution. We show that the equilibrium features dispersion across and within sellers. Section 3.4 focuses on the case in which night profits are strictly increasing in the night price. This case occurs when buyers of type x are more likely to be captive to one seller but less likely to have to shop during the day. We establish the existence and uniqueness of equilibrium, characterize the equilibrium marginal distribution of night and day prices, and the equilibrium joint price distribution. We show that the equilibrium features dispersion across but not price dispersion within sellers. Section 3.5 derives comparative statics for the equilibrium with both types of price dispersion.

3.1 General property of equilibrium

In this subsection, we show that we can restrict attention to equilibria G in which sellers post prices (p_d, p_n) such that $p_n \leq p_d$. This is the case because—by assumption—all of the buyers who can shop at night can also shop during the day and, hence, a seller posting a higher price during the night than during the day enjoys the same profit and exerts the same competition on other sellers as if he were to post the day price in both periods.

To formalize the above argument, we need to write down the profit function $V(p_d, p_n)$ for a seller posting arbitrary prices $(p_d, p_n) \in [0, u]^2$. The profit function is

$$
\begin{aligned}
V(p_d, p_n) = & \left[\mu_{1d} + \mu_{2d}(1 - F_d(p_d) + \phi_d(p_d)/2)\right] p_d \\
& + \left[\mu_{1n} + \mu_{2n}(1 - F_m(\min\{p_d, p_n\}) + \phi_m(\min\{p_d, p_n\})/2)\right] \min\{p_d, p_n\},
\end{aligned}
\tag{1}
$$

where the constants μ_{1d} and μ_{1n} are defined as

$$
\begin{aligned}
\mu_{1d} &= \theta_x \alpha_x \beta_x + \theta_y \alpha_y \beta_y, \\
\mu_{1n} &= \theta_x \alpha_x (1 - \beta_x) + \theta_y \alpha_y (1 - \beta_y),
\end{aligned}
\tag{2}
$$

and the constants μ_{2d} and μ_{2n} are defined as

$$
\begin{aligned}
\mu_{2d} &= 2\theta_x (1 - \alpha_x) \beta_x + 2\theta_y (1 - \alpha_y) \beta_y, \\
\mu_{2n} &= 2\theta_x (1 - \alpha_x)(1 - \beta_x) + 2\theta_y (1 - \alpha_y)(1 - \beta_y).
\end{aligned}
\tag{3}
$$

Let us carefully explain the expressions (1)-(3). First, consider the constants μ_{1d}, μ_{1n}, μ_{2d}, μ_{2n}. The constant μ_{1d} denotes the average number of buyers who contact a particular seller and who can shop only from that seller and who can only shop during the day. This constant is given by the total measure of contacts for buyers who can only shop from only one seller and only during the day divided by the total measure of sellers. The constant μ_{2d} denotes the average number of buyers who contact a particular seller and who can shop from multiple sellers and at both times of day. This constant is given by the total measure of contacts for buyers who can shop from multiple stores and at both times of day divided by the total measure of sellers. Similarly, the constant μ_{1n} denotes the average number of buyers who contact a particular seller and who can shop only from that seller but both during the day and during the night. The constant μ_{2n} denotes the average number of buyers who contact a particular seller and who can shop from multiple sellers and at multiple times.

Now, consider the first line in the profit function (1). The seller contacts μ_{1d} buyers who are not in touch with any other vendor and who can only shop during the day. These buyers are captive and they purchase the good from the seller at the price p_d with probability 1. The seller contacts μ_{2d} buyers who can shop from another vendor and can only shop during the day. These buyers are not captive and they purchase the good from the seller at the price p_d only if the price offered by their second contact is strictly greater than p_d, an event that occurs with probability $1 - F_d(p_d)$, or if the price offered by their second contact is equal to p_d and they break the tie in favor of the seller, an event that occurs with probability $\phi_d(p_d)/2$.

Next, consider the second line in the profit function (1). The seller contacts μ_{1n} buyers who are not in touch with any other vendor but who can only shop both during the day and during the night. These buyers purchase the good from the seller at the lowest price between p_d and p_n with probability 1. The seller contacts μ_{2n} buyers who are in touch with a second vendor and who can shop both during the day and during the night. These buyers purchase the good from the seller at the lowest price between p_d and p_n with probability $1 - F_m(\min\{p_d, p_n\}) + \phi_m(\min\{p_d, p_n\})/2$.

Notice that if a seller posts a day price greater than the night price, he attains the same profit as if he posted the day price in both periods. Formally,

$$V(p_d, p_n) = V(p_d, p_d), \forall (p_d, p_n) \in [0, u]^2 \text{ and } p_n > p_d. \tag{4}$$

Indeed, if $p_n > p_d$, the seller never makes a sale at night. The customers who can only shop during the day will purchase at the price p_d. The customers who can shop at both times will choose to purchase during the day at the price p_d. Therefore, the seller enjoys the same profit if he were to post the prices (p_d, p_d) rather than (p_d, p_n).

Now, consider an equilibrium G in which some sellers may post prices (p_d, p_n) with $p_n > p_d$. Consider an alternative putative equilibrium G^* in which every seller posting (p_d, p_n) changes his prices to $(p_d, \min\{p_d, p_n\})$. That is, every seller posting (p_d, p_n) with $p_n \leq p_d$ keeps his prices unchanges, while every seller posting (p_d, p_n) with $p_n > p_d$ changes his prices to (p_d, p_d). Clearly, the marginal price distributions F_d^* and F_m^* associated with G^* are the same as the marginal price distributions F_d and F_m associated with G. Take any seller who, in the original equilibrium, posts prices (p_d, p_n) with $p_n \leq p_d$. Given G, the prices (p_d, p_n) maximize the seller's profit. Given G^*, the prices (p_d, p_n) also maximize the seller's profit because the profit function (1) only depends on the marginal distribution of day prices and on the marginal distribution of lowest prices which are the same under G and G^*. Now, take any seller who, in the original equilibrium, posts prices (p_d, p_n) with $p_n > p_d$. Given G, the prices (p_d, p_n) maximize the seller's profit. In the alternative equilibrium, the seller posts the prices (p_d, p_d) instead. Given G^*, these prices also maximize the seller's profit because the profit function (1) is unchanged and—as shown in (4)—the seller is indifferent between posting a higher price during the night than during the day and charging the same price during the night as during the day. Therefore, G^* is an equilibrium. Moreover, $G^*$is equivalent to G, in the sense that every seller makes the same number of sales and enjoys the same profit under G^* and G, and every buyer purchases the good at a price drawn from the same distribution under G^* and G.

We have therefore established the following proposition.

Proposition 1: *Let G be an equilibrium. Consider the distribution G^* derived from G by replacing every seller posting (p_d, p_n) with a seller posting $(p_d, \min\{p_d, p_n\})$. (i) The distribution G^* is an equilibrium. (ii) The equilibrium G^* is equivalent to G: the sales and profit of a seller posting $(p_d, \min\{p_d, p_n\})$ in G^* are the same as those of a seller posting (p_d, p_n) in G; the distribution of prices at which a buyer of type $i \in \{x, y\}$ transacts is the same in G^* and G.*

Proposition 1 implies that we can, without any loss in generality, restrict attention to equilibria G in which every seller posts prices $(p_d, p_n) \in [0, u]^2$ with $p_n \leq p_d$, and the marginal distribution of lowest prices, F_m, is equal to the marginal distribution of night prices, F_n.

3.2 Equilibrium properties of the profit function

In light of Proposition 1, we can replace F_m with F_n in (1). Hence, the profit function $V(p_d, p_n)$ for a seller posting prices $(p_d, p_n) \in [0, u]^2$ with $p_n \leq p_d$ can be written as

$$V(p_d, p_n) = V_d(p_d) + V_n(p_n), \tag{5}$$

where V_d and V_n are respectively defined as

$$V_d(p_d) = [\mu_{1d} + \mu_{2d} (1 - F_d(p_d) + \phi_d(p_d)/2)]\, p_d, \tag{6}$$

and

$$V_n(p_n) = [\mu_{1n} + \mu_{2n} (1 - F_n(p_n) + \phi_n(p_n)/2)]\, p_n. \tag{7}$$

In words, $V_d(p_d)$ denotes the seller's profit from trades that take place during the day. During the day, the seller meets μ_{1d} captive buyers and μ_{2d} non-captive buyers. A captive buyer purchases the good from the seller with probability 1. A non-captive buyer purchases the good from the seller with probability $1 - F_d(p_d) + \phi_d(p_d)/2$. Similarly, $V_n(p_n)$ denotes the seller's profit from trades that take place during the night. It is important to keep in mind that this representation of the profit function is valid only for prices (p_d, p_n) with $p_n \leq p_d$. However, when evaluating the seller's equilibrium profit, we can restrict attention to prices (p_d, p_n) with $p_n \leq p_d$ as established in Proposition 1. Moreover, when evaluating an individual seller's deviations from equilibrium, we can restrict attention to prices (p_d, p_n) with $p_n \leq p_d$ as established in (4).

We now want to characterize the equilibrium marginal distributions F_d and F_n in (6) and (7). Unfortunately, we cannot simply apply the same arguments as in Burdett and Judd (1983). In Burdett and Judd, every seller posts one price and must attain the same profit everywhere on the support of the price distribution. Therefore, solving for the equilibrium price distribution only requires solving one equation (i.e., the seller's equal profit condition) for one unknown (i.e., the price distribution). In our model, every seller posts two prices and must attain the same profit everywhere on the support of the joint price distribution G. This observation is not enough to solve for the equilibrium price distributions F_d and F_n, as it only gives us one equation (i.e., the seller's equal profit condition) for two unknowns (i.e., the marginal distributions of day and night prices).

In order to solve for the equilibrium marginal price distributions, we need more. In what follows, we are going to prove that in equilibrium only one of two cases are possible. In the first case, the night profit function V_n is constant over the support of the marginal distribution of night prices F_n, and the day profit function V_d is constant over the support of the marginal distribution of day prices F_d. In this case, we have two equations for two unknowns and we can solve for the equilibrium price distributions. In the second case, the night profit function V_n is strictly increasing in the night price. In this case, every seller is going to set the same price during the day and during the night and, hence, $F_n = F_d$. Thus, we have one equation for one unknown and we can solve for the equilibrium price distributions. As we shall see, the relevant case depends on parameter values.

The first lemma establishes that, in equilibrium, the marginal price distributions F_d and F_n do not have any mass points. The intuition behind this result is the same as in Burdett and Judd (1983). Intuitively, if the equilibrium had a mass point at some daytime price $p_0 > 0$, a seller posting p_0 could attain strictly higher profits by lowering its price by some arbitrarily small amount. In doing so, the seller would enjoy approximately the same profit per sale but it would increase the number of sales, as it would undercut the mass of competitors who post the daytime price p_d. This process of undercutting cannot drive all daytime prices to zero and create an equilibrium with a mass point at $p_0 = 0$ because the seller can always attain a strictly positive profit by setting a price of u and by selling only to captive buyers. Since the marginal distributions F_d and F_n do not have mass points, they are continuous and this property is inherited by the profit functions V_d

and V_n.

Lemma 1: *The marginal price distributions F_d and F_n have no mass points, i.e. $\phi_d(p) = \phi_n(p) = 0$ for all $p \in [0, u]$.*

Proof: In Appendix A. ■

The second lemma establishes that, in equilibrium, the support of the marginal distribution of daytime prices F_d is an interval $[p_{d\ell}, p_{dh}]$, where $p_{dh} = u$. The intuition behind this result is again the same as in Burdett and Judd (1983). If the support of F_d has a gap between the prices p_0 and p_1, then a seller posting a daytime price of p_0 could attain strictly greater profits by raising its price to p_1. In doing so, the seller would enjoy a higher profit per sale without losing any sales.

Lemma 2: *The support of F_d is an interval $[p_{d\ell}, p_{dh}]$ with $p_{dh} = u$.*

Proof: In Appendix A. ■

The third lemma proves that, in equilibrium, the profit function V_n is weakly increasing over the interval $[p_{n\ell}, u]$, where $p_{n\ell}$ denotes the lower bound on the support of the marginal distribution of night prices F_n. There is a simple intuition behind this finding. If V_n is strictly decreasing over some interval $[p_1, p_2]$, sellers never find it optimal to post night prices between p_1 and p_2. As a result, the marginal distribution of night prices F_n would have a gap between p_1 and p_2. But, if F_n has a gap between p_1 and p_2, the profit function V_n in (7) is clearly strictly increasing.

Lemma 3: *The profit function V_n is weakly increasing over the interval $[p_{n\ell}, u]$.*

Proof: On the way to a contradiction, suppose that there is an equilibrium G such that $V_n(p_n)$ is not weakly increasing over the interval $[p_{n\ell}, u]$. Then, there must exist prices p_0 and p_2 such that $p_{n\ell} \leq p_0 < p_2 \leq u$ and $V_n(p_0) > V_n(p_2)$. If $V_n(p_0) > V_n(p_n)$ for all $p_n \in (p_0, p_2]$, let $p_1 = p_0$. If $V_n(p_0)$ is not strictly greater than $V_n(p_n)$ for all $p_n \in (p_0, p_2]$, continuity of V_n guarantees existence of at least one $\hat{p} \in (p_0, p_2]$ and $V_n(\hat{p}) = V_n(p_0)$. Let p_1 denote the largest of such $\hat{p}$'s. By continuity of V_n, $p_1 < p_2$. By construction, $V_n(p_1) = V_n(p_0) > V_n(p_2)$ and $V_n(p_1) > V_n(p)$ for all $p \in (p_1, p_2]$.

In equilibrium, every seller maximizes profits and sets a night price p_n non-greater than the day price p_d. From the properties of V_n, it follows that a seller posting a day price $p_d \geq p_2$ must set a night price $p_n \notin (p_1, p_2)$. A seller posting a day price $p_d \in (p_1, p_2)$ must set a night price $p_n \leq p_1$. A seller with a day price $p_d \leq p_1$ sets a night price $p_n \leq p_1$. These observations imply that the marginal distribution F_n has a gap between p_1 and

p_2, i.e. $F_n(p_1) = F_n(p_2)$. From (7), it follows that $V_n(p_1) < V_n(p_2)$, which contradicts $V_n(p_1) = V_n(p_0) > V_n(p_2)$. ■

The next two propositions show that depending on parameter values, the profit function V_n is either constant over the interval $[p_{n\ell}, u]$ or it is strictly increasing over the interval $[p_{n\ell}, u]$. These two propositions are the key to the characterization of equilibrium. Proposition 2 shows that if $\mu_{1n}/\mu_{2n} > \mu_{1d}/\mu_{2d}$, V_n is strictly increasing over $[p_{n\ell}, u]$. The logic behind this result is simple. If V_n is constant over some interval $[p_0, p_1]$, the marginal distribution F_n must be such that, if a seller increases its night price, the number of night trades falls so as to keep the seller's night profit constant. Moreover, if V_n is constant over the interval $[p_0, p_1]$, we can show that V_d must be constant as well over that interval. Thus, the marginal distribution F_d must be such that, if a seller increases its day price, the number of day trades falls so as to keep the seller's day profit constant. In equilibrium, all sellers post prices $p_n \leq p_d$ and, hence, F_d must first order stochastically dominate F_n. Given the shape of F_n and F_d that keep night and day profits constant, this is only possible if $\mu_{1n}/\mu_{2n} < \mu_{1d}/\mu_{2d}$. Hence, if $\mu_{1n}/\mu_{2n} > \mu_{1d}/\mu_{2d}$, V_n must be strictly increasing everywhere.

Proposition 2: *If* $\mu_{1n}/\mu_{2n} > \mu_{1d}/\mu_{2d}$, V_n *is strictly increasing over the interval* $[p_{n\ell}, u]$.

Proof: Suppose that V_n is such that there exist prices $\hat{p}_0$ and $\hat{p}_1$ such that $p_{n\ell} \leq \hat{p}_0 < \hat{p}_1 \leq u$ and $V_n(\hat{p}_0) = V_n(\hat{p}_1) = V_n^*$. From Lemma 3, it follows that $V_n(p_n) = V_n^*$ for all $p_n \in [\hat{p}_0, \hat{p}_1]$. Let $[p_0, p_1]$ denote the largest interval in $[p_{n\ell}, u]$ such that $[\hat{p}_0, \hat{p}_1] \subset [p_0, p_1]$ and $V_n(p_n) = V_n^*$ for all $p_n \in [p_0, p_1]$. By construction of $[p_0, p_1]$, it follows that $V_n(p_n) > V_n^*$ for all $p > p_1$ and that $V_n(p_n) < V_n^*$ for all $p_n < p_0$.

In equilibrium, every seller maximizes profits and sets a night price p_n non-greater than the day price p_d. From this observation and from the properties of V_n, the following holds. A seller posting a day price $p_d < p_0$ must be setting a night price $p_n < p_0$. A seller posting a day price $p_d \in [p_0, p_1]$ must be setting a night price p_n such that $p_0 \leq p_n \leq p_d < p_1$. A seller posting a day price $p_d > p_1$ must be setting a night price $p_n > p_1$. Therefore, $F_n(p_1) = F_d(p_1)$ and $F_n(p) \geq F_d(p)$ for all $p \in [p_0, p_1]$. Moreover, since $p_1 > p_{n\ell}$, $F_n(p_1) > 0$. And, since $F_d(p_1) = F_n(p_1)$, $p_1 > p_{d\ell}$.

By construction, V_n is constant for all $p_n \in [\max\{p_{d\ell}, p_0\}, p_1]$. Since V_n is constant for all $p_n \in [\max\{p_{d\ell}, p_0\}, p_1]$ and all sellers with a day price $p_d \in [p_0, p_1]$ set a night price $p_n \in [p_0, p_1]$ and they all make the same overall profit, it follows that V_d is constant for all $p_d \in [\max\{p_{d\ell}, p_0\}, p_1]$. Solving $V_n(p) = V_n(p_1)$ and $V_d(p) = V_d(p_1)$ for $p \in [\max\{p_{d\ell}, p_0\}, p_1]$, we find

$$F_n(p_n) = F_n(p_1) - \left[\frac{\mu_{1n}}{\mu_{2n}} + (1 - F_n(p_1))\right] \frac{p_1 - p_n}{p_n},$$
$$F_d(p_d) = F_d(p_1) - \left[\frac{\mu_{1d}}{\mu_{2d}} + (1 - F_d(p_1))\right] \frac{p_1 - p_d}{p_d}. \tag{8}$$

Using the fact that $F_n(p_1) = F_d(p_1)$, we can write the difference $F_n(p) - F_d(p)$

$$F_n(p) - F_d(p) = \left(\frac{\mu_{1d}}{\mu_{2d}} - \frac{\mu_{1n}}{\mu_{2n}}\right) \frac{p_1 - p}{p}. \tag{9}$$

In equilibrium, the above difference must be non-negative as all sellers post prices (p_d, p_n) with $p_n \leq p_d$. Clearly, (9) is non-negative only if $\mu_{1n}/\mu_{2n} \leq \mu_{1d}/\mu_{2d}$. If $\mu_{1n}/\mu_{2n} > \mu_{1d}/\mu_{2d}$, there exist no prices $\hat{p}_0$ and $\hat{p}_1$ such that $p_{n\ell} \leq \hat{p}_0 < \hat{p}_1 \leq u$ and $V_n(\hat{p}_0) = V_n(\hat{p}_1)$. Hence, V_n is strictly increasing for all $p_n \in [p_{n\ell}, u]$. ■

The next proposition shows that if $\mu_{1n}/\mu_{2n} \leq \mu_{1d}/\mu_{2d}$, V_n is constant over the interval $[p_{n\ell}, u]$. Let us give the reader the gist of the proof. If V_n is strictly increasing over some interval (p_0, p_1), all sellers with a day price of $p_d \in (p_0, p_1)$ must be setting a night price p_n equal to p_d, while sellers with a day price $p_d > p_1$ must be setting night prices $p_n > p_1$ and sellers with a day price $p_d < p_0$ must be setting a night price $p_n < p_0$. This implies that $F_n(p)$ and $F_d(p)$ are equal for all $p \in [p_0, p_1]$. Moreover, sellers are indifferent between posting prices (p_0, p_0) and (p_1, p_1). Hence, the marginal price distributions F_n and F_d must be such that, if the seller increases its prices from (p_0, p_0) to (p_1, p_1), the loss in trades must exactly make up for the increase in profit per trade. Given such marginal price distributions, we can compute the seller's profit V_n and show that it is increasing if and only if $\mu_{1n}/\mu_{2n} > \mu_{1d}/\mu_{2d}$. Hence, if $\mu_{1n}/\mu_{2n} \leq \mu_{1d}/\mu_{2d}$, V_n must be constant over the interval $[p_{n\ell}, u]$.

Proposition 3: *If $\mu_{1n}/\mu_{2n} \leq \mu_{1d}/\mu_{2d}$, V_n is constant over the interval $[p_{n\ell}, u]$.*

Proof: Suppose V_n is not constant over $[p_{n\ell}, u]$. Since V_n is continuous, weakly increasing and not everywhere constant, there exist prices p_0 and p_1, with $p_{n\ell} < p_0 < p_1 < u$, such that $V_n(p) < V_n(p_0)$ for all $p \in [p_{n\ell}, p_0)$ and $V_n(p) < V_n(p_1)$ for all $p \in [p_{n\ell}, p_1)$. A seller posting a day price $p_d < p_0$ must be setting a night price $p_n \leq p_d < p_0$. A seller posting a day price $p_d \in [p_0, p_1)$ must be setting a night price $p_n \in [p_0, p_1)$. A seller posting a day price $p_d \geq p_1$ must be setting a night price $p_n \in [p_1, p_d]$. From these observations, it follows that $F_n(p_0) = F_d(p_0)$ and $F_n(p_1) = F_d(p_1)$. Since $p_0 > p_{n\ell}$, $F_n(p_0) > 0$. Since $F_d(p_0) = F_n(p_0)$, $p_0 > p_{d\ell}$. Therefore, $p_1 > p_0 > p_{d\ell}$.

Note that, from the properties of V_n, it follows that a seller posting a day price $p_d = p_0$

must be setting a night price $p_n = p_0$. Similarly, a seller posting a day price $p_d = p_1$ must be setting a night price $p_n = p_1$. Since both p_0 and p_1 are on the support of F_d, the price pairs (p_0, p_0) and (p_1, p_1) maximize the seller's profits. That is,

$$\begin{aligned} &[\mu_{1d} + \mu_{2d}(1 - F_d(p_0)) + \mu_{1n} + \mu_{2n}(1 - F_n(p_0))]\, p_0 \\ = &[\mu_{1d} + \mu_{2d}(1 - F_d(p_1)) + \mu_{1n} + \mu_{2n}(1 - F_n(p_1))]\, p_1. \end{aligned} \tag{10}$$

Using the fact that $F_d(p_0) = F_n(p_0)$ and $F_d(p_1) = F_n(p_1)$, we can solve the above equation for $1 - F_n(p_0)$ and find

$$1 - F_n(p_0) = \left[\frac{\mu_{1n} + \mu_{1d}}{\mu_{2n} + \mu_{2d}} + 1 - F_n(p_1)\right] \frac{p_1}{p_0}. \tag{11}$$

Since V_n is such that $V_n(p_0) < V_n(p_1)$, we have

$$[\mu_{1n} + \mu_{2n}(1 - F_n(p_0))]\, p_0 < [\mu_{1n} + \mu_{2n}(1 - F_n(p_1))]\, p_1. \tag{12}$$

Substituting $1 - F_n(p_0)$ with (11), we can rewrite (12) as

$$\left(\mu_{1n} - \mu_{2n}\frac{\mu_{1n} + \mu_{1d}}{\mu_{2n} + \mu_{2d}}\right)(p_1 - p_0) > 0. \tag{13}$$

The above inequality is satisfied if and only if $\mu_{1n}/\mu_{2n} > \mu_{1d}/\mu_{2d}$. Therefore, if $\mu_{1n}/\mu_{2n} \leq \mu_{1d}/\mu_{2d}$, V_n must be constant for all $p \in [p_{n\ell}, u]$. ■

3.3 Equilibrium with price dispersion across and within stores

The properties of equilibrium depend critically on whether the ratio of captive to non-captive buyers who can shop both during the day and during the night, μ_{1n}/μ_{2n}, is smaller or greater than the ratio of captive to non-captive buyers who can shop only during the day, μ_{1d}/μ_{2d}. The relative magnitude of these two ratios is a complicated function of the underlying parameters of the model. However, with a little bit of algebra, it is possible to show[6] that μ_{1n}/μ_{2n} is smaller (greater) than μ_{1d}/μ_{2d} if and only if $(\alpha_x - \alpha_y)(\beta_x - \beta_y)$ is positive (negative). That is, the ratio of captive to non-captive buyers who can shop at all times is smaller than the ratio of captive to non-captive buyers who can only shop during the day if and only if buyers of type x—who by assumption are more likely to be captive—are also more likely to have to shop during the day.

In this subsection, we solve for the equilibrium price distributions when parameters

[6] We are grateful to a referee for pointing this equivalence out.

are such that

$$(\alpha_x - \alpha_y)(\beta_x - \beta_y) > 0. \tag{14}$$

As established in Proposition 3, any equilibrium under condition (14) is such that the night profit function $V_n(p_n)$ is equal to a constant V_n^* for all $p_n \in [p_{n\ell}, u]$. Since in any equilibrium sellers attain the same profit from night trades and the same total profit, sellers must also enjoy the same profit from day trades. Since in any equilibrium there are sellers posting every day price $p_d \in [p_{d\ell}, u]$, as established in Lemma 2, the day profit function $V_d(p_d)$ must be equal to a constant V_d^* for all $p_d \in [p_{d\ell}, u]$.

Using the above observations, we can now find the marginal price distributions of night and day prices in any equilibrium. Since $V_n(u) = V_n^*$ and $V_d(u) = V_d^*$, any equilibrium is such that $V_n^* = \mu_{1n}u$ and $V_d^* = \mu_{1d}u$. Since $V_n(p_n) = \mu_{1n}u$ for all $p_n \in [p_{n\ell}, u]$ and $V_d(p_d) = \mu_{1d}u$ for all $p_d \in [p_{d\ell}, u]$, any equilibrium is such that the marginal price distributions F_n and F_d are respectively given by

$$F_n(p_n) = 1 - \frac{\mu_{1n}}{\mu_{2n}} \frac{u - p_n}{p_n}, \ \forall p_n \in [p_{n\ell}, u], \tag{15}$$

and

$$F_d(p_d) = 1 - \frac{\mu_{1d}}{\mu_{2d}} \frac{u - p_d}{p_d}, \ \forall p_d \in [p_{d\ell}, u]. \tag{16}$$

The lower bound on the support of the distribution of night prices is the solution to $F_n(p_n) = 0$. Similarly, the lower bound on the support of the distribution of day prices is the solution to $F_d(p_d) = 0$. Solving these two equations gives

$$p_{n\ell} = \frac{\mu_{1n}}{\mu_{1n} + \mu_{2n}} u, \ p_{d\ell} = \frac{\mu_{1d}}{\mu_{1d} + \mu_{2d}} u. \tag{17}$$

It is immediate to verify that the marginal price distributions F_n and F_d in (15) and (16) are proper cumulative distribution functions.

We have thus characterized the marginal distribution of night and day prices in any equilibrium given the parametric restriction (14). Now, we have to verify that an equilibrium actually exists. To this aim, we need to show that there exists a joint price distribution G such that: (a) the support of G is in the region $(p_d, p_n) \in [0, u]^2$ with $p_n \leq p_d$; (b) everywhere on the support of G the profits of the seller are maximized; (c) the joint price distribution G generates the marginal price distributions F_n and F_d in (15) and (16).

First, we identify the subset of the region $(p_d, p_n) \in [0, u]^2$ with $p_n \leq p_d$ in which the

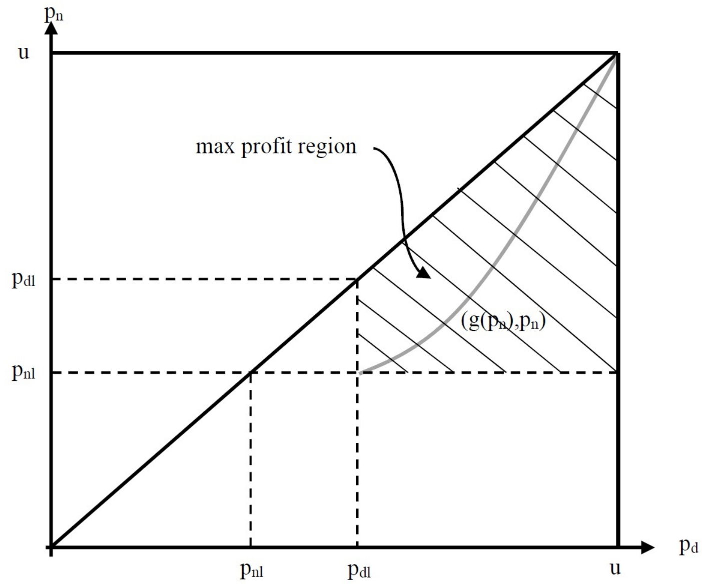

Figure 1: Equilibrium profits

profits of the seller are maximized. To this aim, notice that, if a seller posts any prices $(p_d, p_n) \in [p_{d\ell}, u] \times [p_{n\ell}, u]$ with $p_n \leq p_d$, he attains a profit of $V_d^* + V_n^* = (\mu_{1d} + \mu_{1n})u$. If a seller posts prices (p_d, p_n) such that $p_d \in [p_{d\ell}, u]$, $p_n \in [0, p_{n\ell})$ and $p_n \leq p_d$, he attains a profit of strictly smaller than $V_d^* + V_n^*$. This is because $V_d(p_d)$ is equal to V_d^* and $V_n(p_n)$ is equal to $(\mu_{1n} + \mu_{2n})p_n$ which is strictly smaller than V_n^* for all $p_n < p_{n\ell}$. If a seller posts prices (p_d, p_n) such that $p_d \in [0, p_{d\ell})$, $p_n \in [p_{n\ell}, u]$ and $p_n \leq p_d$, he attains a profit strictly smaller than $V_d^* + V_n^*$. This is because $V_n(p_n)$ is equal to V_n^* and $V_d(p_d)$ is equal to $(\mu_{1d} + \mu_{2d})p_d$ which is strictly smaller than V_d^* for all $p_d < p_{d\ell}$. Similarly, if a seller posts prices (p_d, p_n) such that $p_d \in [0, p_{d\ell})$, $p_n \in [0, p_{n\ell})$ and $p_n \leq p_d$, he attains a profit strictly smaller than $V_d^* + V_n^*$. Moreover, (4) implies that a seller attains the same profit by setting prices $(p_d, p_n) \in [0, u]^2$ with $p_n > p_d$ as by posting the prices (p_d, p_d). These observations imply that the seller's maximum profit is $V_d^* + V_n^*$ and the subset of the region $(p_d, p_n) \in [0, u]^2$ with $p_n \leq p_d$ where the maximum profit is attained is given by $(p_d, p_n) \in [p_{d\ell}, u] \times [p_{n\ell}, u]$ with $p_n \leq p_d$. This subset is illustrated in Figure 1.

Next, we show that there exists a joint price distribution G that has support in the area $(p_d, p_n) \in [p_{d\ell}, u] \times [p_{n\ell}, u]$ with $p_n \leq p_d$ and that generates the marginal distributions F_n and F_d in (15) and (16). To this aim, consider a putative equilibrium in which the fraction $F_n(p_n)$ of sellers posting night prices lower than p_n is given by (15) and in which each seller who posts a night price of p_n sets a day price of $g(p_n)$, with

$$g(p_n) = \left[\frac{\mu_{1n}}{\mu_{2n}}u + \left(\frac{\mu_{1d}}{\mu_{2d}} - \frac{\mu_{1n}}{\mu_{2n}}\right)p_n\right]^{-1}\frac{\mu_{1d}}{\mu_{2d}}up_n. \tag{18}$$

The function $g(p_n)$ is illustrated in Figure 1. The function $g(p_n)$ is strictly increasing in p_n, takes the value $p_{d\ell}$ for $p_n = p_{n\ell}$, and takes the value u for $p_n = u$. Moreover, given the parametric condition (14), $g(p_n) - p_n \geq 0$ for all $p_n \in [p_{n\ell}, u]$. Therefore, the joint price distribution G associated with the putative equ ilibrium has support in the region $(p_d, p_n) \in [p_{d\ell}, u] \times [p_{n\ell}, u]$ with $p_n \leq p_d$. In the putative equilibrium, the marginal distribution of night prices F_n is given by (15). Since $g(p_n)$ is a strictly increasing function, the marginal distribution of day prices $F_d(p_d)$ is given by $F_n(g^{-1}(p_d))$, which is easy to show is exactly the same as (16). This establishes that the joint price distribution G associated with the putative equilibrium generates the marginal price distribution (15) and (16).

We have thus established that there exists an equilibrium and that any equilibrium is such that the marginal distribution of night prices F_n is given by (15), the marginal distribution of day prices F_d is given by (16) and every seller posts lower prices at night than during the day. There may be multiple equilibria. Yet, any equilibrium has two notable features. First, the equilibrium features dispersion in the average price of different sellers. To see this, note that the equilibrium features dispersion in the price of different sellers in any period, as the marginal distributions of day and night prices are non-degenerate. Then, note that the dispersion of prices posted by different sellers in any period does not average out across periods, as a seller's night price is non-greater than his day price. For instance, any seller with a day price of $p_{d\ell}$ has an average price non-greater than $p_{d\ell}$, while any seller with a night price of u has an average price of $u > p_{d\ell}$. Second, the equilibrium features price dispersion within sellers. To see this, notice that the marginal distribution of day prices first order stochastically dominates the distribution of night prices. Hence, there is a positive measure of sellers posting a night price strictly smaller than their day price.

Overall, under condition (14), any equilibrium features price dispersion across and within stores. There is a clear intuition behind this result. The equilibrium features dispersion in the average price of different sellers for the same reason as in Burdett and Judd (1983). In each period, sellers have an incentive to undercut any market-wide price to steal customers from their competitors, but undercutting cannot drive the market-wide price to marginal cost because sellers always have the option of raising the price and sell only to captive customers. Since the dispersion in the price of different sellers cannot perfectly average out across periods, the equilibrium features dispersion in the average price of different sellers. The equilibrium features price dispersion within sellers because sellers have an incentive to post a lower price during the night than during the day in order to discriminate between buyers of type x and buyers of type y. By assumption, buyers of type x are more likely to be captive customers than buyers of type y, i.e. $\alpha_x > \alpha_y$. Hence, a seller faces less competition for buyers of type x than for buyers of type y and has an incentive to charge them different prices. Under condition (14), buyers of type x are also more likely to have to shop during the day, i.e. $\beta_x > \beta_y$. Hence, a seller has the opportunity to charge a higher price to buyers of type x than to buyers of type y by raising his day price above the night price.

The above analysis is summarized in Proposition 4.

Proposition 4: *If* $(\alpha_x - \alpha_y)(\beta_x - \beta_y) > 0$, *there exists at least one equilibrium. In any equilibrium, the joint price distribution* G *is such that each seller posts prices* $(p_d, p_n) \in [0, u]^2$ *with* $p_n \leq p_d$, *the marginal distribution* F_d *of day prices is given by (16), the marginal distribution* F_n *of night prices is given by (15). In any equilibrium, there is price dispersion across and within sellers.*

Under condition (14), all equilibria have the same marginal distributions of day and night prices, but differ with respect to the joint price distributions. For some qualitative questions—such as whether there is price dispersion across sellers and within sellers—the answer is independent of the shape of the joint price distribution and, hence, multiplicity is not an issue. For more quantitative questions—such as breaking down the variance of prices into an across-seller component and a within-seller component—the answer does depend on the shape of the joint price distribution and, hence, will vary depending on equilibrium selection. Luckily, it is easy to show that only one equilibrium is robust to a small perturbation of the environment.

Above, we showed that there always exist an equilibrium in which sellers post prices $(g(p_n), p_n)$, where the fraction $F_n(p_n)$ of sellers with a night price non-greater than p_n is given by (15) and $g(p_n)$ is a strictly increasing function given by (18). This is the only equilibrium where a seller's day price is strictly increasing in his night price and, hence, $F_n(p_n) = F_d(g(p_n))$. In Appendix B, we show that this is the unique equilibrium robust to the introduction of sellers' heterogeneity in the cost of production. Specifically, we consider a version of the model in which the distribution across sellers with respect to the cost of producing the good is uniform with support $[0, \bar{c}]$. For any $\bar{c} \in (0, u)$, the unique equilibrium of this version of the model is such that a seller with a higher cost of production always posts a higher day price and a higher night price, i.e. $F_n(p_n(c); \bar{c}) = F_d(p_d(c); \bar{c}) = c/\bar{c}$. For $\bar{c} \to 0$, the equilibrium marginal price distributions F_n and F_d converge to (15) and (16) and the seller's day price as a function of his night price converges to the $g(p_n)$ in (18).

3.4 Equilibrium without price dispersion within stores

In this subsection, we solve for the equilibrium price distributions when parameters are such that $\mu_{1n}/\mu_{2n} > \mu_{1d}/\mu_{2d}$ or, equivalently,

$$(\alpha_x - \alpha_y)(\beta_x - \beta_y) < 0. \tag{19}$$

As established in Proposition 2, under the parametric restriction (19) any equilibrium is such that the night profit function V_n is strictly increasing for all $p_n \in [p_{n\ell}, u]$. Since sellers maximize profits and post prices (p_d, p_n) with $p_n \leq p_d$, in any equilibrium a seller with a day price of p_d must be setting a night price $p_n = p_d$. First, this observation implies that in any equilibrium the marginal distributions of day and night prices are the same, i.e. $F_d(p) = F_n(p)$ for all $p \in [p_{d\ell}, u]$. Second, the observation implies that in any equilibrium the seller's overall profit $V_d(p) + V_n(p)$ must attain the same value V^* for all $p \in [p_{d\ell}, u]$.

We can now recover the equilibrium marginal distribution of day and night prices. Since $V_d(u) + V_n(u) = V^*$, it follows that in any equilibrium the constant V^* is given by $(\mu_{1d} + \mu_{1n})u$. Since $V_d(p) + V_n(p) = (\mu_{1d} + \mu_{1n})u$ and $F_d(p) = F_n(p)$ for all $p \in [p_{d\ell}, u]$, it follows that in any equilibrium the marginal distributions of day and night prices are given by

$$F_d(p) = F_n(p) = 1 - \frac{\mu_{1d} + \mu_{1n}}{\mu_{2d} + \mu_{2n}} \frac{u - p}{p}, \; \forall p \in [p_{d\ell}, u]. \tag{20}$$

The lower bound on the support of the marginal distribution of day prices is the solutions to the equations $F_d(p_d) = 0$. Similarly, the lower bound on the support of the marginal distribution of night prices is the solution to $F_n(p_n) = 0$, which obviously is the same as the equation $F_d(p_n) = 0$. Solving these equations, we find that $p_{d\ell}$ and $p_{n\ell}$ are given by

$$p_{d\ell} = p_{n\ell} = \frac{\mu_{1d} + \mu_{1n}}{\mu_{1d} + \mu_{2d} + \mu_{1n} + \mu_{2n}} u. \tag{21}$$

It is straightforward to verify that the marginal price distributions F_d and F_n in (20) are proper cumulative distribution functions.

Under the parametric restriction (19), there exists a unique candidate equilibrium. In this candidate equilibrium, every seller posts prices (p_d, p_n) with $p_n = p_d$, the fraction $F_d(p_d)$ of sellers posting a day price non-greater than p_d is given by (20), and the fraction $F_n(p_n)$ of sellers posting a night price non-greater than p_n is also given by (20). Clearly, the candidate equilibrium is such that every seller posts a night price non-greater than his day price. It is also clear that the candidate equilibrium generates the desired marginal distributions of day and night prices. Therefore, to make sure that the candidate equilibrium is indeed an equilibrium, we only have to check that all prices (p, p) with $p \in [p_{d\ell}, u]$ maximize the profit of the seller.

A seller posting prices (p, p) with $p \in [p_{d\ell}, u]$ attains a profit equal to V^*. Now, consider a seller posting prices (p_d, p_n) with $p_{d\ell} < p_n < p_d \leq u$. This seller attains a profit strictly smaller than V^*, as one can easily verify that V_n is strictly increasing in $p_n \in [p_{d\ell}, u]$ and, hence, $V_d(p_d) + V_n(p_n)$ is strictly smaller than $V_d(p_d) + V_n(p_d)$ which, in turn, is equal to V^*. Next, consider a seller posting prices (p_d, p_n) with $0 \leq p_n < p_{d\ell} \leq p_d \leq u$. This seller attains a profit strictly smaller than V^*, as (6) implies that $V_n(p_n) < V_n(p_{d\ell})$ and, hence, $V_d(p_d) + V_n(p_n)$ is strictly smaller than $V_d(p_d) + V_n(p_{d\ell})$ which, in turn, is smaller than V^*. Consider a seller posting prices (p_d, p_n) with $0 \leq p_n \leq p_d < p_{d\ell}$. This seller attains a profit strictly smaller than V^*, as (6) implies that $V_n(p_n) < V_n(p_{d\ell})$ and (5) implies that $V_d(p_d) < V(p_{d\ell})$. Hence, $V_d(p_d) + V_n(p_n)$ is strictly smaller than $V_d(p_{d\ell}) + V_n(p_{d\ell})$ which, in turn, is equal to V^*. Finally, (4) implies that a seller attains the same profit by setting prices $(p_d, p_n) \in [0, u]^2$ with $p_n > p_d$ as by posting the prices (p_d, p_d). These observations allow us to conclude that the maximum profit is V^* and that all prices (p, p) with $p \in [p_{d\ell}, u]$ attains this maximum.

We have thus established that, under condition (19), there exists a unique equilibrium, in which every seller posts prices (p_d, p_n) with $p_n = p_d$, the fraction $F_d(p_d)$ of sellers posting a day price non-greater than p_d is given by (20) and the fraction $F_n(p_n)$ of sellers posting a night price non-greater than p_n is also given by (20). The equilibrium has two notable features. First, the equilibrium features dispersion in the average price of different sellers. To see this, note that the marginal distribution of day prices is non-degenerate. Since every seller posts the same price during the night as during the day, the marginal distribution of average prices is the same as the marginal distribution of day prices and, hence, is also non-degenerate. Second, the equilibrium does not features price dispersion within sellers, as every individual seller sets the same price during the night as during the day. To understand this property of equilibrium, notice that, under condition (19), buyers of type x are more likely to be captive and are more likely to be able to shop both during the day and during the night. Since buyers of type x are more likely to be captive, sellers would like to charge them a higher price. However, since buyers of type x are more likely to be able to shop at any time, sellers cannot successfully use time-variation in prices to discriminate between the two types of buyers. Indeed, if they were to post a higher price during the day than during the night in an attempt to discriminate between the two types of buyers, it would be the buyers of type x who take advantage of the night price not the buyers of type y.

The above analysis is summarized in Proposition 5.

Proposition 5: *If* $(\alpha_x - \alpha_y)(\beta_x - \beta_y) < 0$, *there exists a unique equilibrium. The equilibrium is such that each seller posts prices* $(p_d, p_n) \in [0, u]^2$ *with* $p_n = p_d$, *the marginal distribution* F_d *of day prices is given by (20) and the marginal distribution* F_n *of night prices is given by (20). The equilibrium features price dispersion across sellers, but no dispersion of prices within sellers.*

Propositions 4 and 5 provide a characterization of equilibrium when $(\alpha_x - \alpha_y)(\beta_x - \beta_y) \neq 0$. For completeness, we now consider the case $(\alpha_x - \alpha_y)(\beta_x - \beta_y) = 0$. When $(\alpha_x - \alpha_y)(\beta_x - \beta_y) = 0$, $\mu_{1n}/\mu_{2n} = \mu_{1d}/\mu_{2d}$ and Proposition 3 implies that the profit function V_n is constant. Hence, any equilibrium G is such that the marginal distribution F_n of night prices is given by (15) and the marginal distribution F_d of day prices is given by (16). Moreover, since $\mu_{1n}/\mu_{2n} = \mu_{1d}/\mu_{2d}$, the marginal distribution F_n of night prices is equal to the marginal distribution F_d of day prices. Since every seller posts prices (p_d, p_n) with $p_n \leq p_d$ and $F_n = F_d$, the equilbrium G is such that every seller posts the same price during the day as during the night. Thus, when $(\alpha_x - \alpha_y)(\beta_x - \beta_y) = 0$, the equilibrium features price dispersion across sellers but no price dispersion within sellers, just as in the case of $(\alpha_x - \alpha_y)(\beta_x - \beta_y) < 0$. However, unlike the case of $(\alpha_x - \alpha_y)(\beta_x - \beta_y) < 0$, the reason for the lack of price dispersion within sellers is different. If $\alpha_x = \alpha_y$, there is no price dispersion within sellers because buyers of type x and y are in contact with the same number of sellers and, hence, sellers have no incentive to price discriminate. If $\beta_x = \beta_y$, there is no price dispersion within sellers because buyers of type x and y are equally likely to have a flexible shopping schedule and, hence, there is no way to discriminate them by posting different prices at different times.

3.5 Comparative statics

Proposition 4 and 5 imply that the equilibrium features price dispersion across and within sellers if and only if $(\alpha_x - \alpha_y)(\beta_x - \beta_y) > 0$. It is natural to wonder how the equilibrium outcomes change when we vary the two features of the environment that are necessary and sufficient to the emergence of price dispersion across and within sellers. Namely, the difference $\Delta_\alpha = \alpha_x - \alpha_y > 0$ in the probability that a buyer of type x and a buyer of type y are captive to one seller, and the difference $\Delta_\beta = \beta_x - \beta_y > 0$ in the probability that a buyer of type x and a buyer of type y have to shop during the day.

To carry out comparative statics with respect to Δ_α and Δ_β, it is convenient to express the ratio μ_{1d}/μ_{2d} of captive to non-captive buyers who can shop only during the day and the ratio μ_{1n}/μ_{2n} of captive to non-captive buyers who can shop both during the day and during the night as

$$\frac{\mu_{1d}}{\mu_{2d}} = \frac{\alpha_x \beta_x + (\theta_y/\theta_x)\left(\alpha_x - \Delta_\alpha\right)\left(\beta_x - \Delta_\beta\right)}{2(1-\alpha_x)\beta_x + 2(\theta_y/\theta_x)(1-\alpha_x+\Delta_\alpha)(\beta_x - \Delta_\beta)}, \tag{22}$$

and

$$\frac{\mu_{1n}}{\mu_{2n}} = \frac{\alpha_x(1-\beta_x) + (\theta_y/\theta_x)\left(\alpha_x - \Delta_\alpha\right)\left(1-\beta_x + \Delta_\beta\right)}{2(1-\alpha_x)(1-\beta_x) + 2(\theta_y/\theta_x)(1-\alpha_x+\Delta_\alpha)(1-\beta_x + \Delta_\beta)}. \tag{23}$$

First, consider the effect of an increase in Δ_α. From (22) and (23), it follows that an increase in Δ_α lowers both μ_{1d}/μ_{2d} and μ_{1n}/μ_{2n}. It then follows from (16) that an increase in Δ_α causes a decline in daytime prices, as the equilibrium marginal distribution F_d falls in the sense of first-order stochastic dominance. Similarly, it follows from (15) that an increase in Δ_α causes a decline in nighttime prices, as the equilibrium marginal distribution F_n falls in the sense of first-order stochastic dominance. Moreover, we can show that an increase in Δ_α increases the difference $\mu_{1d}/\mu_{2d} - \mu_{1n}/\mu_{2n}$. It then follows from (15) and (16) that the difference between the marginal distribution of day prices and the marginal distribution of night prices increases. These findings are intuitive. If buyers of type y become better at shopping around, sellers face more competition and prices fall both during the day and during the night. However, since buyers of type y are more likely than buyers of type x to be able to shop at night, night-time prices fall more than day-time prices.

What do the changes to the equilibrium price distributions described above imply about price dispersion? A rudimentary measure of overall price dispersion is the length $u - p_{n\ell}$ of the support of the overall price distribution $F_t(p) = (F_d(p) + F_n(p))/2$. By this measure, an increase in Δ_α increases overall price dispersion as the lowest price $p_{n\ell}$ observed in the market falls. A rudimentary measure of within-seller price dispersion is the length $p_{d\ell} - p_{n\ell}$, which is the difference between the lengths of the support of the distribution of day and night prices (and also the difference between the day and the night price posted by the cheapest seller in the market). By this measure, an increase in Δ_α leads to an increase in within-seller price dispersion, as $p_{n\ell}$ falls more than $p_{d\ell}$.

Now, consider the effect of an increase in Δ_β. From (22) and (23), it follows that an increase in Δ_β increases μ_{1d}/μ_{2d} and lowers μ_{1n}/μ_{2n}. It then follows from (16) that an increase in Δ_β causes an increase in daytime prices, as the equilibrium marginal distribution F_d rises in the sense of first-order stochastic dominance. In contrast, an increase in Δ_β causes a decline in nighttime prices, as the equilibrium marginal distribution F_n falls in the sense of first-order stochastic dominance. From these observations, it follows that an increase in Δ_β increases the difference between the marginal distribution of day prices and the marginal distribution of night prices increases. These findings are also intuitive. If buyers of type y become better at shopping at different times of day, sellers can achieve more precise discrimination between buyers of type y and buyers of type x by posting different prices during the day and during the night. For this reason, sellers lower the night price and raise the day price.

Having characterized the effect of an increase in Δ_β on the equilibrium price distributions, we can now look at its effect on price dispersion. An increase in Δ_β leads to an increase in overall price dispersion measured as the difference $u - p_{n\ell}$ between the highest and the lowest price observed in the market. Moreover, an increase in Δ_β leads to an increase in within seller price dispersion measured as the difference $p_{d\ell} - p_{n\ell}$ between dispersion in night prices and dispersion in day prices.

Finally, it is useful to examine the behavior of the equilibrium when Δ_α and Δ_β go to zero. In the limit for either $\Delta_\alpha \to 0$ or $\Delta_\beta \to 0$, the difference between the ratios μ_{1d}/μ_{2d} and μ_{1n}/μ_{2n} converges to zero. Using this observation, we can then show that the distance between the equilibrium distribution of day prices and the equilibrium distribution of night prices goes to zero, i.e. $\lim F_n(p) - F_d(p) = 0$. Moreover, we can show that overall price dispersion (as measured by $u = p_{n\ell}$) converges to a strictly positive number, while within seller price dispersion (as measured by $p_{d\ell} - p_{n\ell}$) converges to zero, i.e. $\lim u - p_{n\ell} > 0$ and $\lim p_{n\ell} - p_{d\ell} = 0$. These findings reveal that the equilibrium features both price dispersion across and within sellers if and only if $\Delta_\alpha > 0$ and $\Delta_\beta > 0$, and that, when either Δ_α or Δ_β go to zero, the within seller component of price dispersion vanishes.

The model can be easily estimated using data on price dispersion. In fact, the model has a unique equilibrium and, as shown above, its fundamentals affect in a natural way the extent of overall price dispersion and the extent of price dispersion within sellers. The model can be tested using its predictions for the extent of dispersion in prices paid by different households and in the composition of such price dispersion. Indeed, the model has prediction with respect to how much of the variance in prices paid by different households is accounted for by differences in the average price of the store where households shop, how much is accounted for by differences in the discounts enjoyed by households at those stores, and in the covariance between the two components.

While a careful estimation and validation of the model is outside the scope of this paper, it is useful to present a numerical example to show that the model has the ability to quantitatively account for the observed extent of price dispersion across and within sellers. Figure 2 illustrates the properties of the unique robust equilibrium—i.e. the one in which sellers post prices $(g(p_n), p_n)$ with $g(p_n)$ given in (18)—when the parameter are given values $\alpha_x = 0.5$, $\beta_x = 0.9$, $\Delta_\alpha = 0.4$, $\Delta_\beta = 0.8$, $\theta_y/\theta_x = 1$ and $u = 1$. The black dashed line is the marginal distribution of day prices F_d. The gray dashed line is the marginal distribution of night prices F_n. As known from (15) and (16), the marginal distribution of day prices first order stochastically dominates the marginal distribution of night prices. The black solid line is the overall price distribution, F_t. Given the equilibrium joint price distribution, we can decompose the overall variance of prices into an across-seller component and a within-seller component. We find that the overall standard deviation of prices is 23%, the across store variance is 56% of the overall variance, and the within store variance is 44%. These numbers are very close to those found by Kaplan and Menzio (2015), which shows that our simple model has the potential to match some of the key empirical facts about price dispersion.

Naturally, ours is not the only possible theory of price dispersion across and within stores, even though it builds on the leading theory of price dispersion across stores and on the leading theory of temporary sales (which, as documented in Nakamura and Steinsson 2008 and Klenow and Kryvtsov 2008, are the main source of within-seller price dispersion). Price dispersion across stores may arise because each seller is a pure monopolist over some segment of the population and different sellers face different marginal costs or different elasticities of demand. Similarly, price dispersion within stores may arise because each seller faces different marginal costs or different elasticities of demand at different moments in time. Given the lack of data on seller-and-good specific cost and demand functions,

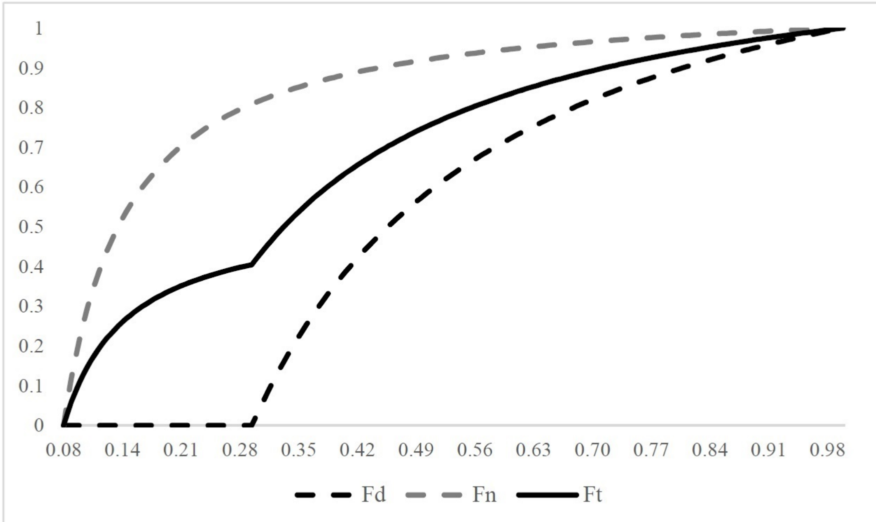

Figure 2: Equilibrium price distributions

this explanation is possible but also non-falsifiable.[7]

Price dispersion across and within stores may also arise because sellers randomly reset the price several times per quarter. In Burdett and Judd (1983), each seller attains the maximum profit by posting any price on the support of the distribution. Then, an individual seller also attains the maximum same profit by posting different prices at different points in time, as long as each one of these prices is on the support of the distribution. We find this explanation unappealing. The introduction of any arbitrarily small amount of heterogeneity in the seller's cost purifies the equilibrium so that, for each type of seller, there is a unique profit-maximizing price.[8]

Another explanation for price dispersion across and within stores is that sellers face a menu cost to change their nominal price (see, e.g., Sheshinski and Weiss 1977, Benabou 1988 or Burdett and Menzio 2017). Because of the menu cost, each seller will occasionally change his nominal price and, thus, there will be price dispersion within stores and, as long as repricing is not perfectly correlated across sellers, there will also be price dispersion

[7] Stigler (1961)—in reference to the dispersion of prices for the same car across different dealerships—writes: "Some automobile dealers might perform more service, or carry a larger range of varieties in stock, and a portion of the observed dispersion is presumably attributable to such differences. But it would be metaphysical, and fruitless, to assert that all dispersion is due to heterogeneity."

[8] Bontemps, Robin and van den Berg (2000) formalize the purification argument in the context of Burdett and Mortensen (1998), which is the labor market analogue of the product market model of Burdett and Judd (1983).

in the average price of different stores. This explanations is unlikely to be relevant, as we observe a great deal of price dispersion across and within stores even when inflation is very low (such as in the US during the 2000s). Moreover, the within-seller price dispersion generated by menu costs does not follow the typical V-pattern of temporary sales.

Price dispersion across and within stores may be due to the management of perishable goods (see, e.g., Lazear 1986). For instance, different sellers may carry on their shelves products with different expiration dates. Sellers whose products are about to expire will post lower prices and, once their products are replaced, they will increase their prices. This explanation is unlikely to account for all of price dispersion. Indeed, Kaplan et al. (2016) document that price dispersion within and across stores is large not only for low-durability but also for high-durability goods. A related explanation is offered by Aguirregabiria (1999) who shows that sellers find it optimal to vary their price as their inventory evolves over time. Yet, it seems diffucult to explain the V-pattern of temporary sales with changes in inventories.

4 Conclusions

Kaplan and Menzio (2015) document that approximately half of the overall price dispersion in the US retail market is due to dispersion across sellers and half to dispersion within sellers. In this paper, we developed a search-theoretic model of the retail market that generates, as equilibrium outcomes, both price dispersion across and within sellers. Price dispersion across sellers obtains because, as in Burdett and Judd (1983), buyers are heterogeneous in their ability to shop at different stores. Price dispersion within sellers obtains because, as in Conlisk, Gerstner and Sobel (1984), buyers are heterogeneous with respect to their ability to shop at different times and those who need to shop at a particular time have a higher reservation price. Overall, our model generates price dispersion across and within stores by combining in a unified and parsimonious framework the insights from the search theory of price dispersion and of the intertemporal price discrimination theory of sales. Yet, the techniques for solving the equilibrium of our model are novel.

The next step in our research agenda is to integrate the dynamic single-product model developed in this paper with the static multi-product model developed in Kaplan et al. (2016) in order to build a unified framework that can simultaneously generate dispersion in the average price of different stores (the store component of price dispersion), the average price of the good at a particular store relative to the average price of the store (the store-good component of price dispersion), and the price of the good at a particular store relative to its average price (the transaction component of price dispersion). Our single-product model does not distinguish between the store and the store-good components of price dispersion. The static model of Kaplan et al. does not generate any price dispersion through the transaction component. Yet, Kaplan and Menzio (2015) convincingly document that all three components of price dispersion are empirically relevant. After having developed such a unified framework, we will be in the position to estimate the model using the econometric techniques developed by Hong and Shum (2006) and Moraga-Gonzales and Wildenbeest (2009). We would also be able to test the many predictions of the model with respect to the extent and sources of dispersion in price indexes across different households.

References

[1] Aguiar, M., and E. Hurst. 2007. "Life-Cycle Prices and Production." *American Economic Review*, 97: 1533-1559.

[2] Aguirregabiria, V. 1999. "The Dynamics of Markups and Inventories in Retailing Firms." *Review of Economic Studies*, 66: 275-308.

[3] Albrecht, J., Postel-Vinay, F., and S. Vroman. 2013. "An Equilibrium Search Model of Synchronized Sales." *International Economic Review*, 54: 472-493.

[4] Benabou R. 1988. "Search, Price Setting and Inflation." *Review of Economic Studies*, 55: 353-376.

[5] Bontemps, C., J. Robin and G. van den Berg. 2000. "Equilibrium Search with Continuous Productivity Dispersion: Theory and Nonparametric Estimation." *International Economic Review*, 41: 305-358.

[6] Burdett, K., and K. Judd. 1983. "Equilibrium Price Dispersion." *Econometrica* 955-969.

[7] Burdett, K., and G. Menzio. 2017. "The (Q, S, s) Pricing Rule." *Review of Economic Studies*, Forthcoming.

[8] Burdett, K., and D. Mortensen. 1998. "Wage Differentials, Employer Size, and Unemployment." *International Economic Review*, 39: 257–73.

[9] Butters, G. 1977. "Equilibrium Distributions of Sales and Advertising Prices." *Review of Economic Studies*, 44: 465-491.

[10] Conlisk, J, Gerstner, E., and J. Sobel. 1984. "Cyclical Pricing by a Durable Goods Monopolist." *Quarterly Journal of Economics*, 99: 489-505.

[11] Garrett, D., Gomes, R., and L. Maestri. 2016. "Competitive Screening under Heterogeneous Information." Manuscript, University of Toulouse.

[12] Head, A., Liu, L., Menzio, G, and R. Wright. 2012. "Sticky Prices: A New Monetarist Approach." *Journal of the European Economic Association*, 10: 939-973.

[13] Hong, J., and M. Shum. 2006. "Using Price Distributions to Estimate Search Costs." *Rand Journal of Economics*, 37: 257-275.

[14] Hotelling, H. 1929. "Stability in Competition". *Economic Journal*, 39: 41-57.

[15] Kaplan, G., and G. Menzio. 2015. "The Morphology of Price Dispersion." *International Economic Review*, 56: 1165-1206..

[16] Kaplan, G., and G. Menzio. 2016. "Shopping Externalities and Self-Fulfilling Unemployment Fluctuations." *Journal of Political Economy*, 124: 771-825.

[17] Kaplan, G., Menzio, G., Rudanko, L., and N. Trachter. 2016. "Relative Price Dispersion: Evidence and Theory." NBER Working Paper 21931.

[18] Klenow, P., and O. Kryvtsov. 2008. "State-Dependent or Time-Dependent Pricing: Does it Matter for Recent U.S. Inflation?" *Quarterly Journal of Economics* 123(3): 863–904.

[19] Lazear, E. 1986. "Retail Pricing and Clearance Sales." *American Economic Review*, 76: 14-32.

[20] Lester, B., Shourideh, A., Venkateswaran, V., and A. Zeitlin-Jones. 2015. "Screening and Adverse Selection in a Frictional Market." Manuscript, University of Pennsylvania.

[21] Moraga-Gonzales, J., and M. Wildenbeest. 2008. "Maximum Likelihood Estimation of Search Costs." *European Economic Review*, 52: 820-848.

[22] Nakamura, E., and J. Steinsson. 2008. "Five Facts About Prices: A Reevaluation of Menu Cost Models." *Quarterly Journal of Economics*, 123: 1415-1464.

[23] Rhodes, A. 2015. "Multiproduct Retailing." *Review of Economic Studies*, 82: 360-390.

[24] Sheshinski, E., and Y. Weiss. 1977. "Inflation and Costs of Price Adjustment." *Review of Economic Studies*, 44: 287-303.

[25] Sobel, J. 1984. "The Timing of Sales." *Review of Economic Studies*, 51: 353-368.

[26] Sorensen, A. 2000. "Equilibrium Price Dispersion in Retail Markets for Prescription Drugs." *Journal of Political Economy*, 108: 833-850.

[27] Stigler, G. 1961. "The Economics of Information." *Journal of Political Economy*, 69: 213-225.

[28] Varian, H. 1980. "A Model of Sales." *American Economic Review*, 70: 651-659.

[29] Zhou, J. 2014. "Multiproduct Search and the Joint Search Effort." *American Economic Review*, 104: 2918-2939.

Appendix

A Proofs of Lemma 1 and 2

Proof of Lemma 1: We begin by establishing that F_d has no mass points. On the way to a contradiction, suppose that there exists an equilibrium G in which F_d has a mass point at $p_0 \in (0, u]$. Consider any seller posting prices (p_0, p_n) with $p_n < p_0$. From (5), it follows that this seller can attain a strictly higher profit by posting the prices $(p_0 - \epsilon, p_n)$ for some $\epsilon > 0$ sufficiently small. Consider any seller posting prices (p_0, p_n) with $p_n = p_0$. From (5), it follows that this seller can attain a strictly higher profit by choosing the prices $(p_0 - \epsilon, p_0 - \epsilon)$ for some $\epsilon > 0$ sufficiently small. From Proposition 1, it follows that no seller posts prices (p_0, p_n) with $p_n > p_0$. Overall, any seller who in equilibrium posts a day price of p_0 is not maximizing profits. Hence, G is not an equilibrium. Now, suppose that there exists an equilibrium G in which F_d has a mass point at $p_0 = 0$. Consider a seller posting prices (p_0, p_n) with $p_n = p_0 = 0$. The profit of the seller is 0. From (5), it follows that this seller can attain a strictly greater profit by posting prices (u, u) and selling only to captive buyers. From Proposition 1, it follows that there are no sellers posting prices (p_0, p_n) with $p_n > p_0$. Overall, any seller posting a day price of p_0 is not maximizing profits. Hence, G is not an equilibrium. This completes the proof that F_d does not have mass points. The proof that F_n has no mass points is analogous. ■

Proof of Lemma 2: We first establish that the support of F_d is an interval $[p_{d\ell}, p_{dh}]$. On the way to a contradiction, let us suppose that there exists an equilibrium G such that the support of F_d has a gap between p_0 and p_1 where $p_{d\ell} < p_0 < p_1 < u$ and p_0 and p_1 are prices on the support of F_d. Since $F_d(p_1) = F_d(p_0)$, any seller posting prices (p_0, p_n) with $p_n \leq p_0$ can attain a strictly higher profit by choosing the prices (p_1, p_n) instead. From proposition 1, it follows that there are no sellers posting prices (p_0, p_n) with $p_n > p_0$. Overall, any seller posting a day price p_0 is not maximizing profits. By continuity of V_d and V_n, any seller posting a day price in a left neighborhood of p_0 is also not maximizing profits. Therefore, p_0 cannot be on the support of F_d. The proof of $p_{dh} = u$ is analogous. ■

B Equilibrium refinement

In this appendix, we consider a version of the model in which sellers are heterogeneous with respect to the cost c of producing the good. We show that under the parametric restriction (14), this version of the model admits a unique interior equilibrium in which the seller's rank in the cost distribution is the same as the seller's rank in the distribution of day prices and as the seller's rank in the distribution of night prices. We show that, in the limit as cost heterogeneity goes to zero, the unique equilibrium of this version of the model converges to the equilibrium of the model without heterogeneity considered in Section 3.3, where each seller posting a night price of p_n posts a day price of $g(p_n)$, with $g(p_n)$ given by (18), and the fraction of sellers posting a night price non-greater than p_n is $F_n(p_n)$, with $F_n(p_n)$ given by (15).

Let $H(c)$ denote the measure of sellers with a production cost non-greater than c. For the sake of exposition, we assume that H is a uniform with support $[0, \bar{c}]$ and $\bar{c} \in (0, u)$. In light of Proposition 1, the pricing problem for a seller of type c can be written as

$$\max_{(p_d, p_n) \in [0,u]^2} V_d(p_d, c) + V_n(p_n, c), \text{ s.t. } p_n \leq p_d, \tag{24}$$

where $V_d(p_d, c)$ and $V_n(p_n, c)$ are respectively given by

$$\begin{aligned} V_d(p_d, c) &= [\mu_{1d} + \mu_{2d}(1 - F_d(p_d))] (p_d - c), &(25)\\ V_n(p_n, c) &= [\mu_{1n} + \mu_{2n}(1 - F_n(p_n))] (p_n - c). &(26)\end{aligned}$$

Note that we have set mass points $\phi_d(p_d)$ and $\phi_n(p_n)$ to zero, as it is straightforward to generalize the argument of Lemma 1 to the case in which seller's costs are heterogeneous.

We focus on interior equilibria where the constraint $p_n \leq p_d$ is slack. In the case of homogeneous costs, this is a feature of all equilibria under the parametric restrictions (14) as the night profit function V_n is constant. Since the purpose of this appendix is to use cost heterogeneity to refine the equilibrium set under homogeneous costs, it is natural to focus on interior equilibria.

In any interior equilibrium, the first order conditions for the seller's profit maximization problem are

$$\begin{aligned} \mu_{1d} + \mu_{2d}(1 - F_d(p_d)) &= \mu_{2d} F_d'(p_d)(p_d - c), &(27)\\ \mu_{1n} + \mu_{2n}(1 - F_n(p_n)) &= \mu_{2n} F_n'(p_n)(p_n - c). &(28)\end{aligned}$$

The left-hand side of (27) is the seller's marginal benefit of increasing the day price p_d, which is given by the seller's number of trades. The right-hand side of (27) is the seller's marginal cost of increasing p_d, which is given by the decline in the number of trades caused by an increase in p_d multiplied by the seller's profit per trade. Condition (27) states that the seller's day price equates marginal cost and marginal benefit. Condition (28) is the analogous condition for the seller's night price.

Notice that the marginal cost of increasing the price p_d is strictly decreasing with respect to the seller's cost of production c, while the marginal benefit of increasing p_d is independent of c. Therefore, (27) implies that the optimal day price, call it $p_d(c)$, for a seller with cost of production c is a strictly increasing function of c. Similarly, (28) implies that the optimal night price, call it $p_n(c)$, for a seller with cost of production c is a strictly increasing function of c. These observations imply that, in any interior equilibrium, the seller's rank in the cost distribution, i.e. $H(c)$, is equal to the seller's rank in the distribution of day prices, i.e. $F_d(p_d(c))$, and to the seller's rank in the distribution of night prices, i.e. $F_n(p_n(c))$. Moreover, it is straightforward to show that, in any equilibrium, the highest price on the support of the distributions of day and night prices is the buyer's valuation u. This observation implies that $p_d(\overline{c}) = p_n(\overline{c}) = u$.

Using the fact that $F_i(p_i(c)) = H(c)$ for $i = \{d, n\}$, we can write the seller's first order conditions (27) and (28) as differential equations for $p_i(c)$. Specifically, we can write $p_i'(c)$ as

$$p_i'(c) = \frac{\mu_{2i} H'(c)(p_i(c) - c)}{\mu_{1i} + \mu_{2i}(1 - H(c))}. \tag{29}$$

The general solution to the differential equation is given by

$$p_i(c) = \frac{2K - \mu_{2i} c^2}{2(\mu_{1i} + \mu_{2i})\overline{c} - 2\mu_{2i} c}. \tag{30}$$

Using the fact that $p_i(\overline{c}) = u$ to solve for the constant of integration K, we conclude that in any interior equilibrium the price in period i for a seller with a production cost c is given by

$$p_i(c) = \frac{2\mu_{1i}\overline{c}u + \mu_{2i}(\overline{c}^2 - c^2)}{2(\mu_{1i} + \mu_{2i})\overline{c} - 2\mu_{2i} c}. \tag{31}$$

Let $c_d(p)$ denote the inverse of $p_d(c)$ and let $c_n(p)$ denote the inverse of $p_n(c)$. Then, the marginal distribution F_d of day prices is given by

$$F_d(p_d) = H(c_d(p_d)) = \frac{p_d}{\overline{c}} - \frac{1}{\overline{c}}\sqrt{(p_d - \overline{c})^2 + 2\left(\frac{\mu_{1d}}{\mu_{2d}}\right)\overline{c}(u - p_d)}. \tag{32}$$

Similarly, the marginal distribution F_n of night prices is given by

$$F_n(p_n) = H(c_n(p_n)) = \frac{p_n}{\overline{c}} - \frac{1}{\overline{c}}\sqrt{(p_n - \overline{c})^2 + 2\left(\frac{\mu_{1n}}{\mu_{2n}}\right)\overline{c}(u - p_n)}. \tag{33}$$

We have thus completed the characterization of the unique candidate interior equilibrium. To summarize, in the candidate equilibrium, every seller with a strictly higher production cost c posts a strictly higher day price $p_d(c)$ and a strictly higher night price $p_n(c)$. The measure $F_d(p_d)$ of sellers who post a day price non greater than p_d is equal to the measure of sellers with a cost non-greater than $c_d(p)$ and it is given by (32). The measure $F_n(p_n)$ of sellers who post a night price non-greater than p_n is equal to the measure of sellers with a cost non-greater than $c_n(p)$ and it is given by (33). After verifying that the seller's second order conditions are satisfied (this can be done either numerically or analytically as in Burdett and Mortensen 1998 or Bontemps et al. 1997), we can conclude that the candidate equilibrium is indeed an equilibrium. Notice that, in contrast to the case with identical sellers, the equilibrium is unique. Moreover, the unique equilibrium is such that the seller's rank in the distribution of day prices is the same as his rank in the distribution of night prices.

Now, consider the limit of the unique equilibrium for $\overline{c} \to 0$. Using de l'Hopital rule, it is easy to verify that the marginal price distributions are such that

$$\lim_{\overline{c}\to 0} F_d(p_d) = 1 - \frac{\mu_{1d}}{\mu_{2d}}\frac{u - p_d}{p_d}, \tag{34}$$

$$\lim_{\overline{c}\to 0} F_n(p_n) = 1 - \frac{\mu_{1d}}{\mu_{2d}}\frac{u - p_n}{p_n}. \tag{35}$$

Applying again de l'Hopital rule, we find that a seller's day price as a function of the night price, i.e. $g(p_n) = p_d(c_n(p_n))$, is such that

$$\lim_{\overline{c}\to 0} g(p_n) = \left[\frac{\mu_{1n}}{\mu_{2n}}u - \left(\frac{\mu_{1d}}{\mu_{2d}} - \frac{\mu_{1n}}{\mu_{2n}}\right)p_n\right]^{-1}\frac{\mu_{1d}}{\mu_{2d}}up_n. \tag{36}$$

A quick comparison between (34)-(36) and (15), (16) and (18) reveals that the limit for $\overline{c} \to 0$ of the unique equilibrium with cost heterogeneity is exactly the equilibrium described in Section 3.3.