Inflation and Unemployment in the Long Run

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Abstract

We study the long-run relation between money (inflation or interest rates) and unemployment. We document positive relationships between these variables at low frequencies. We develop a framework where money and unemployment are modeled using explicit microfoundations, providing a unified theory to analyze labor and goods markets. We calibrate the model and ask how monetary factors account for labor market behavior. We can account for a sizable fraction of the increase in unemployment rates during the 1970s. We show how it matters whether one uses monetary theory based on the search-and-bargaining approach or on an ad hoc cash-in-advance constraint.

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Introduction

We study the relationship between monetary policy, as measured by inflation or nominal interest rates, and labor market performance, as measured by unemployment. While this is an old issue, our focus differs from the existing literature by concentrating on the longer run – we are less interested in business cycles, and more in relatively slowly moving trends.[1] One reason to focus on the longer run is that it may well be more important from a welfare and policy perspective. While many macroeconomists seem obsessed with increases in unemployment, say, over the business cycle, we want to redirect attention to what happens at lower frequencies, since avoiding a bad decade, like the 1970s, from a labor market perspective, probably matters a lot more than smoothing out a typical recession.

Another reason to focus on the long run is that economic theory has much cleaner implications for what happens at lower frequencies, which are less likely to be clouded by complications such as signal extraction problems and other forms of imperfect information, or nominal stickiness and other rigidities. We abstract from such complications to focus on the effect of inflation on the cost of carrying real balances for transactions purposes. As Milton Friedman (1977) put it: "There is a natural rate of unemployment at any time determined by real factors. This natural rate will tend to be attained when expectations are on average realized. The same real situation is consistent with any absolute level of prices or of price change, *provided allowance is made for the effect of price change on the real cost of holding money balances*"

[1] The standard way to describe business cycle phenomena in modern macroeconomics (see e.g., Thomas F. Cooley 1995) is this: Take a given time series y_t; apply the HP (or some other) filter to get the trend y_t^T; then define the cyclical component by the deviation $y_t^D = y_t - y_t^T$. Rather than y_t^D, the object of interest in this study is y_t^T. This is not to say our model does not make predictions about high-frequency behavior – an equilibrium generates y_t, y_t^D, and y_t^T for all t – but we are more confident about the predictions for y_t^T because we abstract from some effects that may be relevant at higher frequencies, as discussed below.

(emphasis added). This is the effect studied here.

To begin, we want to know the facts about the relation between nominal variables and the labor market. Using quarterly U.S. data from 1955-2005, Figure 1 shows scatter plots between inflation and unemployment, progressively removing more of the higher frequency fluctuations as we move through the panels by applying stronger HP filters. The last panel alternatively filters the data using five-year averages. It is clear that after filtering out the higher frequencies, there is a strong positive relationship between the relatively slowly moving trends in these variables. Figure 2 shows a similar pattern using nominal (Aaa corporate bonds) interest rates instead of inflation.[2] Figure 3 shows the time series instead of scatter plots. We conclude that (i) movements in trend unemployment are large, and (ii) they are positively correlated with the trends in the nominal interest and inflation rates. This is true for the period as a whole, even if the relation sometimes goes the other way in the shorter run, including the 1960s where a downward sloping Phillips curve is evident.[3]

[2] This is perhaps no surprise, given the Fisher equation, which says that nominal interest rates move one-for-one with inflation, *ceterus paribus*. In the working paper version of this project (Berentsen, Menzio and Wright 2008), we argue that the Fisher and quantity equations hold in the long run. The quantity equation suggests we should get similar pictures using money growth instead of inflation or interest rates, and there we show this is true, using $M0$, $M1$ or $M2$. We also make the same point using different interest rates, including the T-Bill rate, using employment rather than unemployment, and using an extended sample.

[3] Our way of isolating longer run behavior follows the study of the Fisher and quantity equations in Robert E. Lucas (1980). Lucas (1980) actually warns against making too much of any pattern between filtered inflation and unemployment, given his faith in the arguments of Friedman (1968) and Edmund S. Phelps (1970) that the long-run Phillips curve must be vertical. But our view, following Friedman (1977), is that a positive relation between inflation and unemployment is as much "an implication of a coherent economic theory" as Lucas (1980) suggested the Fisher and quantity equations are. See Andreas Beyer and Roger E. A. Farmer (2007), or Alfred A. Haug and Ian P. King (2009), and references therein, for more formal analyses of the data. Haug and King (2009) in particular apply band-pass filters to the same data, and also find a positive relationship between unemployment and inflation for bands longer than the typical business cycle. They also tested for multiple structural change at unknown dates. They conclude, "After accounting for breaks, the sub-periods lead us to the same conclusion that the longrun association of unemployment with inflation is positive. Although we used different and more formal methods, our findings support the position in [Berentsen, Menzio and Wright]."

We want to know how much we can account for in these observations using basic economic theory. To this end, we build a general equilibrium model of unemployment and money demand based on frictions in labor and goods markets, abstracting from nominal misperceptions and rigidities. As suggested by Friedman (1977), to understand the impact of monetary policy on the natural rate of unemployment, it is important to incorporate the effect of inflation on the cost of holding real balances, which means we need a theory where the cost of holding money and hence the benefit of holding money are made explicit. Additionally, it would seem good to have a theory of unemployment that has proven successful in other contexts.

In recent years, much progress has been made studying both labor and monetary economics using theories that explicitly incorporate frictions, including search and matching frictions, non-competitive pricing, anonymity or imperfect monitoring, etc. Models with frictions are natural for understanding dynamic labor markets and hence unemployment, as well as goods markets and the role of money. However, existing papers analyze either unemployment *or* money in isolation. One objective here is to provide a framework that allows us to analyze unemployment *and* money in an environment with logically consistent microfoundations. Although there are various ways to proceed, in terms of different approaches in the literature, here we integrate the labor market model in Dale T. Mortensen and Christopher Pissarides (1994) with the goods market model in Ricardo Lagos and Wright (2005). The result is a very tractable framework that makes sharp predictions about many interesting effects, including the impact of inflation or interest rates on employment.

We then consider the issue quantitatively by calibrating the model and asking how it accounts for the above-mentioned observations. Suppose for the sake of a controlled experiment that monetary policy is the only driving force over the period

– i.e., assume counterfactually that demographics, productivity, fiscal policy, etc. were constant. Given monetary policy behaved as it did, how well can we account for movements in trend unemployment? We find that the model accounts for a sizable fraction of the lower-frequency movement in unemployment as a result of observed changes in trend inflation and interest rates. For instance, monetary policy alone can generate around half of the 3-point increase in trend unemployment in the 1970s, and about the same fraction of the decline in the 1980s. Money matters. However, we also ask how this prediction is affected by financial innovations, and conclude that in the future money may matter less for the labor market.

Finally, we argue that it makes a difference that we use search-and-bargaining theory, as opposed to some ad hoc approach to money, as follows. First, we consider a version of our setup where the goods market is frictionless except for a cash-in-advance constraint, and show analytically that the channels through which variables interact are qualitatively different in the two models. Second, we compare calibrated versions of the models and show that they behave differently quantitatively, and that the search and bargaining frictions are key to accounting for the observations of interest. Hence, while we like our framework because labor and commodity markets are modeled using logically consistent principles, this is not just a matter of aesthetics – the substantive predictions of a model with these detailed microfoundations are different from the ad hoc approach.

The rest of the paper is organized as follows. The next two sections describe the baseline model and solve for equilibrium. The section after that presents the quantitative analysis. The penultimate section compares our specification with a cash-in-advance model, and shows that these different approaches do not generate the same predictions. The final section concludes. Some more technical material is relegated to the Appendix.[4]

1 The Basic Model

Time is discrete and continues forever. In each period, there are three distinct markets where economic activity takes place: a labor market in the spirit of Mortensen and Pissarides (1994); a goods market in the spirit of Nobuhiro Kiyotaki and Wright (1993); and a general market in the spirit of Arrow-Debreu. We call these the MP, KW and AD markets. Although it does matter for the results, let us assume MP convenes first, then KW, then AD. As shown in Lagos and Wright (2005), alternating KW and AD markets makes the analysis much more tractable than in a standard search model, and we take advantage of that here. There are two types of agents, firms and households, indexed by f and h. The set of h is $[0, 1]$; the set of f is arbitrarily large, but not all are active at any point in time. Households work, consume, and enjoy utility; firms maximize profits and pay dividends.

As in any MP-type model, h and f can match bilaterally to create a job, and e indexes employment status: $e = 1$ if an agent is matched and $e = 0$ otherwise. We define value functions for the MP, KW and AD markets, $U_e^j(z)$, $V_e^j(z)$ and $W_e^j(z)$, which depend on type $j \in \{h, f\}$, employment status $e \in \{0, 1\}$, real balances $z \in$

[4] Other recent attempts to bring monetary issues to bear on search-based labor models include Farmer and Andrew Hollenhorst (2006), Olivier J. Blanchard and Jordi Gali (2008), and Mark Gertler and Antonella Trigari (2009), but they take a different tack by imposing nominal rigidities, which we do not think are so relevant for longer-run issues. Etienne Lehmann (2007), Shouyong Shi (1998, 1999) and Shi and Weimin Wang (2006) are closer to our approach, although the details are different. Guillaume Rocheteau, Peter Rupert and Wright (2008) and Mei Dong (2007) use similar monetary economics but a different theory of unemployment – Richard D. Rogerson's (1988) indivisible labor model; while that leads to some interesting results, there are reasons to prefer Mortensen and Pissarides (1994). Earlier, Cooley and Gary D. Hansen (1989) stuck a cash-in-advance constraint into Rogerson (1988), as Cooley and Vincenzo Quadrini (2004) and Dave Andolfatto, Scott Hendry and Kevin Moran (2004) do to Mortensen and Pissarides (1994). As mentioned, we will discuss below the relation between our approach and reduced-form monetary economics in detail.

$[0, \infty)$, and, in general, aggregate state variables. But for now fundamentals are constant and we focus on steady states, so aggregate state variables are subsumed in the notation.[5] We adopt the following convention for measuring real balances. When an agent brings in m dollars to the AD market, we let $z = m/p$, where p is the current price level. He then takes $\hat{z} = \hat{m}/p$ out of that market and into the next period. In the next AD market the price level is $\hat{p}$, so the real value of the money is $\hat{z}\hat{\rho}$, where $\hat{\rho} = p/\hat{p}$ converts $\hat{z}$ into units of the numeraire good x in that market.

1.1 Households

A household h in the AD market solves

$$W_e^h(z) = \max_{x,\hat{z}} \left\{x + (1-e)\ell + \beta U_e^h(\hat{z})\right\} \tag{1}$$
$$\text{s.t. } x = ew + (1-e)b + \Delta - T + z - \hat{z}$$

where x is consumption, ℓ the utility of leisure, w the wage, b UI (unemployment insurance) benefits, Δ dividend income, T a lump-sum tax, and β a discount factor. Notice h discounts between periods, but not across markets within a period, without loss in generality. Notice also that w is paid in AD, even though matching occurs in MP. Eliminating x from the budget equation,

$$W_e^h(z) = I_e + z + \max_{\hat{z}} \left\{-\hat{z} + \beta U_e^h(\hat{z})\right\}, \tag{2}$$

where $I_e = ew + (1-e)(b+\ell) + \Delta - T$.

This immediately implies the following: W_e^h is linear in z and I_e, and the choice of $\hat{z}$ is independent of z and I_e. Although it looks as though $\hat{z}$ could depend on e

[5] For matched agents, the wage w is also a state variable, since it is set in MP and carried forward to KW and AD; to reduce clutter, this is also subsumed in the notation. In the Appendix, where policy and productivity follow stochastic processes and unemployment varies endogenously over time, we keep track of these plus w as state variables.

through U_e^h, we will see below that $\partial U_e^h/\partial \hat{z}$ and hence $\hat{z}$ are actually independent of e. This means that every h exits the AD market with the same $\hat{z}$ (at least given an interior solution for x, which holds as long as $b + \ell$ is not too small). These results require quasi-linearity, which is valid here because utility is linear in the numeraire good x.[6]

In the KW market, another good q is traded, which gives utility $\upsilon(q)$, with $\upsilon(0) = 0$, $\upsilon' > 0$ and $\upsilon'' < 0$. In this market, agents trade bilaterally, and to generate a role for a medium of exchange we assume at least some meetings are anonymous. To understand this, suppose h asks f for q in KW and promises to pay later, say in the next AD market. But suppose also that h can renege, without fear of a repercussion, as he naturaly could if the KW meetings are anonymous. Then clearly f will not extend credit, and insists on quid pro quo (see Narayana R. Kocherlakota 1998, Neil Wallace 2001, Luis Araujo 2004, and Charalambos D. Aliprantis, Gabriele Camera, and Daniela Puzzello 2007 for formal discussions). If h cannot store x, money has a role. Actually, to make money essential, we only need *some* anonymous meetings, and we need not rule out *all* credit. Thus, let ω denote the probability that a random match is anonymous. For now, as a benchmark, we set $\omega = 1$ and return to the general case below.[7]

[6] In fact, we get a degenerate distribution of $\hat{z}$ as long as AD utility is $x + \Upsilon_e(\mathbf{x})$, where $\mathbf{x}$ is a vector of other goods. A recent extension of this model by Lucy Qian Liu (2009) allows the employed and unemployed to value KW goods differently, leading to a two-point distribution, without complicating the analysis very much.

[7] The case $\omega = 0$, which allows perfect credit, is also of interest, embedding as it does a genuine retail sector, albeit a cashless one, into the standard MP model. This case can be used to study many interesting interactions between commodity and labor markets, including the effects of goods market regulation, sales taxes, etc., on employment. One can also make ω endogenous, as in related models by Dong (2009), where it is a choice of h, and Benjamin R. Lester, Andrew Postlewaite and Wright (2009), where it is a choice of f.

For h in the KW market,

$$V_e^h(z) = \alpha_h \upsilon(q) + \alpha_h W_e^h [\rho (z - d)] + (1 - \alpha_h) W_e^h(\rho z), \tag{3}$$

where α_h is the probability of trade and (q, d) the terms of trade, to be determined below. Using the linearity of W_e^h, we can simplify this to

$$V_e^h(z) = \alpha_h [\upsilon(q) - \rho d] + W_e^h(0) + \rho z. \tag{4}$$

The probability α_h is given by a CRS matching function $\mathcal{M}$: $\alpha_h = \mathcal{M}(B, S)/B$, where B and S are the measures of buyers and sellers in KW. Letting $Q = B/S$ be the queue length, or market tightness, we can write $\alpha_h = \mathcal{M}(Q, 1)/Q$. We assume that $\mathcal{M}(Q, 1)$ is strictly increasing in Q, with $\mathcal{M}(0, 1) = 0$ and that $\mathcal{M}(\infty, 1) = 1$, and $\mathcal{M}(Q, 1)/Q$ is strictly decreasing with $\mathcal{M}(0, 1)/0 = 1$ and $\mathcal{M}(\infty, 1) = 0$, as is true for most standard matching functions (see e.g. Menzio 2007).

In equilibrium, every h participates in KW, so that $B = 1$, and, moreover, every h is identical from the viewpoint of f since they all have the same amount of money. However, f can only participate in KW if $e = 1$, since an unmatched firm has nothing to sell (given inventories are liquidated in AD as discussed below). Thus, $\alpha_h = \mathcal{M}(1, 1-u)$, where u is unemployment entering KW. This establishes a first connection between the goods and labor markets: Consumers are better off in the goods market when times are better in the labor market, in the sense that u is lower, because when employment is higher the probability of trade in the goods market is higher.

For h in the MP market,

$$U_1^h(z) = V_1^h(z) + \delta \left[V_0^h(z) - V_1^h(z) \right] \tag{5}$$

$$U_0^h(z) = V_0^h(z) + \lambda_h \left[V_1^h(z) - V_0^h(z) \right], \tag{6}$$

where δ is the job destruction rate and λ_h the job creation rate. Job destruction is exogenous, but job creation is determined by another matching function $\mathcal{N}$: $\lambda_h = \mathcal{N}(u,v)/u = \mathcal{N}(1,\tau)$, where $\tau = v/u$ is labor market tightness, with u unemployment and v vacancies (one has to distinguish between 'vee' v for vacancies and 'upsilon' υ for utility, but it should always be clear from the context). We make assumptions on $\mathcal{N}$ similar to $\mathcal{M}$. Wages are determined when f and h meet in MP, although they are paid in the AD market. Also, in on-going matches, we allow w to be renegotiated each period.

It is sometimes convenient to summarize the three markets by one equation. Substituting $V_e^h(z)$ from (4) into (5) and using the linearity of W_e^h,

$$U_1^h(z) = \alpha_h [\upsilon(q) - \rho d] + \rho z + \delta W_0^h(0) + (1-\delta) W_1^h(0)$$

Something similar can be done for U_0^h. Inserting these into (2), in the steady state, the AD problem becomes

$$W_e^h(z) = I_e + z + \max_{\hat{z}} \{-\hat{z} + \beta\alpha_h [\upsilon(q) - \rho d] + \beta\rho\hat{z}\} + \beta \mathbb{E} W_{\hat{e}}^h(0) \tag{7}$$

where the expectation is with respect to next period's employment status e. We claim the KW terms of trade (q,d) may depend on $\hat{z}$ but not on employment status (see below). Hence, from (7), the choice $\hat{z}$ is independent of e, as well as I_e and z, and every h takes the same amount of money to KW.[8]

[8] Recall that KW meetings are anonymous with probability $\omega = 1$ in this benchmark. More generally, the maximand in (7) should be

$$-\hat{z} + \beta\alpha_h\omega [\upsilon(q^m) - \rho d^m] + \beta\alpha_h(1-\omega) [\upsilon(q^c) - \rho d^c] + \beta\rho\hat{z}$$

where (q^m, d^m) and (q^c, d^c) are the terms of trade in money and credit meetings, respectively. The crucial difference is that money trades are constrained by $d^m \leq \hat{z}$ while no such constraint applies to credit trades. This implies that the choice of $\hat{z}$ is actually independent of (q^c, d^c). In fact, most of the predictions are exactly the same for all values of $\omega > 0$ as long as we adjust α_h so that $\alpha_h\omega$ is constant.

1.2 Firms

Firms carry no money out of AD. In the MP market, we have

$$U_1^f = \delta V_0^f + (1-\delta)V_1^f \tag{8}$$

$$U_0^f = \lambda_f V_1^f + (1-\lambda_f)V_0^f, \tag{9}$$

where $\lambda_f = \mathcal{N}(u,v)/v = \mathcal{N}(1,\tau)/\tau$. This much is standard. Where we deviate from textbook MP theory is that, rather than having f and h each consume a share of the output, in our setup f takes it to the goods market and looks to trade with another h. Hence in this model, as in reality, households do not always consume what they make each day at work. Output in a match is denoted y, and measured in units of the AD good. If f sells q units in KW, there is a transformation cost $c(q)$, with $c' > 0$ and $c'' \geq 0$, so that $y - c(q)$ is left over to bring to the next AD market.[9]

For f in KW,

$$V_1^f = \alpha_f W_1^f \left[y - c(q), \rho d\right] + (1-\alpha_f)W_1^f(y,0) \tag{10}$$

where $\alpha_f = \mathcal{M}(B,S)/S$. The AD value of f with inventory x, real balances z, and wage commitment w is $W_1^f(x,z) = x + z - w + \beta U_1^f$. Thus,

$$V_1^f = R - w + \beta\left[\delta V_0^f + (1-\delta)V_1^f\right], \tag{11}$$

where $R = y + \alpha_f[\rho d - c(q)]$ is expected revenue. Obviously, the KW terms of trade (q,d) affect R, and hence, in equilibrium, affect entry and employment, establishing another link between goods and labor markets. And as long as f derives at least

[9] We also solved the model where output is in KW goods, and there is a technology for transforming unsold KW goods into AD goods. The results are essentially the same. One can alternatively assume unsold KW goods are carried forward to the next KW market, but having f liquidate inventory in AD avoids the problem of tracking inventories across f, just as the AD market allows us to avoid tracking the distribution of money across h.

some revenue from cash transactions, this means that monetary factors affect labor market outcomes.

To model entry, as is standard, any f with $e = 0$ can pay k in units of x in the AD market to enter the next MP market with a vacancy. Thus

$$W_0^f = \max\left\{0, -k + \beta\lambda_f V_1^f + \beta(1-\lambda_f)V_0^f\right\},$$

where $V_0^f = W_0^f = 0$ by free entry. Thus $k = \beta\lambda_f V_1^f$, which by (11) implies

$$k = \frac{\beta\lambda_f\,(R-w)}{1-\beta(1-\delta)}. \tag{12}$$

Profit over all firms is $(1-u)(R-w) - vk$, which they pay out as dividends. If the representative h holds the representative portfolio (say, shares in a mutual fund) this gives equilibrium dividend income Δ.

1.3 Government Policy

The government consumes G, pays the UI benefit b, levies the tax T, and prints money at rate π, which means that π equals inflation in the steady state. The budget constraint $G + bu = T + \pi M/p$ holds at every date, without loss of generality (Ricardian equivalence). For steady state analysis, we can equivalently describe monetary policy in terms of setting the nominal interest rate i or π, by virtue of the Fisher equation $1+i = (1+\pi)/\beta$. In the stochastic model in the Appendix we specify policy in terms of interest rate rules. We always assume $i > 0$, although one can take the limit as $i \to 0$, which is the Friedman rule.

2 Equilibrium

We assume that agents are price takers in the AD market, and bargain over the terms of trade in MP and KW.[10] Given this, we determine steady state equilibrium as follows. First, taking unemployment u as given, we solve for the value of money q as in Lagos and Wright (2005). Then, taking q as given, we solve for u as in Mortensen and Pissarides (1994). If we depict these results in (u, q) space as the LW curve and the MP curve, their intersection determines equilibrium unemployment and the value of money, from which all other variables easily follow.

2.1 Goods Market Equilibrium

When f and h meet in KW, the terms of trade (q, d) are determined by the generalized Nash bargaining solution

$$\max_{q,d} \left[\upsilon(q) - \rho d\right]^{\theta} \left[\rho d - c(q)\right]^{1-\theta}, \tag{13}$$

s.t. $d \leq z$ and $c(q) \leq y$, which say the parties cannot leave with negative cash balances or inventories. The first term in (13) is the surplus of h and the second term is the surplus of f, using the linearity of W_e^j, while θ is the bargaining power of h. We assume $c(q) \leq y$ is not binding. As established in Lagos and Wright (2005), in any equilibrium, the solution of (13) involves $d = z$ and $q = g^{-1}(\rho z)$, where

$$g(q) \equiv \frac{\theta c(q)\upsilon'(q) + (1-\theta)\upsilon(q)c'(q)}{\theta\upsilon'(q) + (1-\theta)c'(q)}. \tag{14}$$

Notice $\partial q/\partial z = \rho/g'(q) > 0$, so bringing more money gets h more KW goods, but non-linearly (unless $\theta = 1$ and c is linear).

[10] In the working paper Berentsen, Menzio and Wright (2008), we consider alternative pricing mechanisms for both MP and KW, including price taking and price posting. Here, we focus on bargaining because it is easy, and it is standard in the literatures on search unemployment and money.

Given the bargaining outcome $d = z$ and $q = g^{-1}(\rho z)$, we can rewrite the choice of $\hat{z}$ by h in AD as

$$\max_{\hat{z} \geq 0} \left\{ -\hat{z} + \beta \alpha_h \upsilon \left[g^{-1}(\rho \hat{z}) \right] + \beta(1 - \alpha_h) \rho \hat{z} \right\}, \tag{15}$$

using the fact that ρ is constant in the steady state. The solution satisfies

$$\frac{1}{\beta \rho} = \alpha_h \frac{\upsilon'(q)}{g'(q)} + 1 - \alpha_h. \tag{16}$$

Using $1/\beta\rho = 1 + i$ and $\alpha_h = \mathcal{M}(1, 1 - u)$, we get

$$\frac{i}{\mathcal{M}(1, 1 - u)} = \frac{\upsilon'(q)}{g'(\hat{q})} - 1. \tag{17}$$

This is the LW curve, determining q as in Lagos and Wright (2005), except there α_h was fixed and now $\alpha_h = \mathcal{M}(1, 1 - u)$. Its properties follow from well-known results. For instance, simple conditions guarantee that $\upsilon'(q)/g'(q)$ is monotone, so there is a unique $q > 0$ solving (17), with $\partial q / \partial u < 0$.[11] Intuitively, the higher is u, the lower is the probability that h matches in KW, which lowers the demand for money and hence reduces its value q. Also, given u, (17) implies q is decreasing in i. These and other properties of the LW curve are summarized below.

Proposition 1 *Let q^* solve $\upsilon'(q^*) = c'(q^*)$. For all $i > 0$ the LW curve slopes downward in (u, q) space, with $u = 0$ implying $q \in (0, q^*)$ and $u = 1$ implying $q = 0$. The curve shifts down with i and up with θ. As $i \to 0$, $q \to q_0$ for all $u < 1$, where q_0 is independent of u, and $q_0 = q^*$ iff $\theta = 1$.*

[11] Sufficient conditions for $\upsilon'(q)/g'(q)$ monotonicity are either: decreasing absolute risk aversion; or $\theta \approx 1$. Alternatively, the analysis in Wright (forthcoming) implies there is generically a unique solution for q given any u, with $\partial q / \partial u < 0$, even if $\upsilon'(q)/g'(q)$ is not monotone.

2.2 Labor Market Equilibrium

In MP, we use Nash bargaining over w with threat points given by continuation values and η the bargaining power of f. It is routine to solve for

$$w=\frac{\eta\left[1-\beta\left(1-\delta\right)\right]\left(b+\ell\right)+\left(1-\eta\right)\left[1-\beta\left(1-\delta-\lambda_h\right)\right]R}{1-\beta\left(1-\delta\right)+\left(1-\eta\right)\beta\lambda_h}, \tag{18}$$

exactly as in Mortensen and Pissarides (1994). Substituting this and $R=y+\alpha_f[\rho d-c(q)]$ into (12), the free entry condition becomes

$$k=\frac{\lambda_f\eta\left[y-b-\ell+\alpha_f\left(\rho d-q\right)\right]}{r+\delta+(1-\eta)\lambda_h}. \tag{19}$$

To simplify (19), use the steady-state condition $(1-u)\delta=N(u,v)$ to implicitly define $v=v(u)$ and write $\alpha_f=\mathcal{M}(1,1-u)/(1-u)$, $\lambda_f=\mathcal{N}[u,v(u)]/v(u)$ and $\lambda_h=\mathcal{N}[u,v(u)]/u$. Using these plus $\rho d=g(q)$, (19) becomes

$$k=\frac{\eta\frac{\mathcal{N}[u,v(u)]}{v(u)}\left\{y-b-\ell+\frac{\mathcal{M}(1,1-u)}{1-u}[g(q)-c(q)]\right\}}{r+\delta+(1-\eta)\frac{\mathcal{N}[u,v(u)]}{u}}. \tag{20}$$

This is the MP curve, determining u as in Mortensen and Pissarides (1994), except the total surplus (the term in braces) includes not just $y-b-\ell$ but also the expected gain from trade in KW. Routine calculations show the MP curve is downward sloping. Intuitively, when q is higher, profit and hence the benefit from opening a vacancy are higher, so ultimately unemployment is lower. Also, given q, u is increasing in b, ℓ and k and decreasing in y. These and other properties of the MP curve are summarized below, under a maintained assumption $k(r+\delta)<\eta\left[y-b-\ell+g(q^*)-c(q^*)\right]$, since without this condition the market simply shuts down.

Proposition 2 *The MP curve slopes downward in (u,q) space and passes through $(\underline{u},q^*)$, where $\underline{u}\in(0,1)$. If $k(r+\delta)\geq\eta(y-b-\ell)$, it passes through $(1,\underline{q})$, where*

$\underline{q} > 0$, *and if* $k(r+\delta) < \eta(y-b-\ell)$, *it passes through* $(\overline{u}, 0)$, *where* $\overline{u} > 0$. *It shifts to the right with* b, ℓ *and* k, *and to the left with* y.

2.3 General Equilibrium

The LW and MP curves both slope downward in the box $\mathcal{B} = [0,1] \times [0,q^*]$ in (u,q) space, as shown in Figure 4 (this is a stylized representation; curves for actual calibrated parameter values are shown below). Notice that LW enters $\mathcal{B}$ from the left at $(0,q_0)$ and exits from the right at $(1,0)$. If $k(r+\delta) \geq \eta(y-b-\ell)$, MP enters $\mathcal{B}$ from the top at $(\underline{u}, q^*)$ and exits from the right at $(1,q_1)$. In this case, there exists a non-monetary equilibrium at $(1,0)$ and, depending on parameter values, monetary equilibria may also exist (see the curves labelled MP_2 and MP_3). If $k(r+\delta) < \eta(y-b-\ell)$, MP enters $\mathcal{B}$ from the top at $(\underline{u}, q^*)$ and exits from the bottom at $(\overline{u}, 0)$ (see the curve labelled MP_1). In this case, a non-monetary equilibrium exists at $(\overline{u}, 0)$, as well as at least one monetary equilibrium.

Generally, equilibrium exists but need not be unique, as shown in Figure 4 for different parameter configurations implying different MP curves but the same LW curve. For instance, given parameters leading to the curve labeled MP_2, one non-monetary and two monetary equilibria will exist. If monetary equilibrium is not unique, for quantitative work we focus on the one with the lowest u. In any case, once we have (u,q), we easily recover v, α_j, λ_j, z, etc.[12] Also note that changes in i shift only the LW curve, while changes in y, η, r, k, δ, b or ℓ shift only the MP curve, making it very easy to study the effects of parameter changes.

In particular, in monetary equilibrium, an increase in i shifts the LW curve toward

[12] In particular, given the AD price $p = M/g(q)$, the budget equation yields x for every h as a function of z and I_e. In the case with many AD goods and utility $x + \Upsilon_e(\mathbf{x})$, standard consumer theory yields individual demand $\mathbf{x} = \mathbf{D}_e(\mathbf{p})$, market demand is $\mathbf{D}(\mathbf{p}) = u\mathbf{D}_0(\mathbf{p}) + (1-u)\mathbf{D}_1(\mathbf{p})$, and equating this to supply yields a system of equations that solve for $\mathbf{p}$.

the origin, decreasing q and increasing u if the equilibrium is unique (or, if there are multiple equilibria, in the one with the lowest u). The result $\partial q/\partial i < 0$ holds in standard LW models, with fixed α_h, but here there is a general equilibrium (multiplier) effect: Once q falls, u goes down and this reduces α_h, which further reduces q. The result $\partial u/\partial i > 0$ is novel, since the nominal rate has no role in standard MP models, and there is no unemployment in standard LW models. This effect captures the idea suggested by Friedman (1977) and discussed in the Introduction. Intuitively, a higher i increases the cost of holding money, leading h to economize on real balances; this hurts retail trade and profit; and ultimately this reduces employment. Other experiments can be analyzed similarly, and are left as exercises.[13]

Proposition 3 *Steady-state equilibrium exists. If $k(r + \delta) \geq \eta(y - b - \ell)$, there is a non-monetary steady state at $(1, 0)$ and monetary steady states may also exist. If $k(r+\delta) < \eta(y-b-\ell)$, there is a non-monetary steady state at $(\overline{u}, 0)$ and at least one monetary steady state. If the monetary steady state is unique, a rise in i decreases q and increases u, while a rise in y, or a fall in k, b or ℓ, increases q and decreases u.*

3 Quantitative Analysis

We have developed a consistent framework to analyze labor and goods markets with frictions. The model is very tractable, and many results can be established by shifting curves, including the result that increasing i raises u through the qualitative channel suggested by Friedman (1977). We now show that the theory is amenable to quantitative analysis. Precisely, we study how well it can account for the low-frequency

[13] Consider an increase in b. This shifts the MP curve out, increasing u and reducing q if the equilibrium is unique (or in the one with the lowest u). The result $\partial u/\partial b > 0$ holds in standard MP models, but now we have the novel effect $\partial q/\partial b < 0$, plus a multiplier effect.

behavior of u from 1955-2005, assuming (counterfactually) the only driving force is monetary policy. Although above we only considered steady states, here we use the generalized model described in the Appendix, with a stochastic process for productivity y, and a policy rule that gives the next period's nominal rate by $\widehat{i} = \bar{i} + \rho_i (i - \bar{i}) + \epsilon_i$, $\epsilon_i \sim N(0, \sigma_i)$.

3.1 Parameters and Targets

We choose a model period as one quarter. In terms of parameters, preferences are described by the discount factor β, the value of leisure ℓ, and $\upsilon(q) = Aq^{1-a}/(1-a)$. Technology is described by the vacancy cost k, the job-destruction rate δ, and $c(q) = q^{\gamma}$. Matching is described by $\mathcal{N}(u, v) = Zu^{1-\sigma}v^{\sigma}$ (truncated to keep probabilities below 1), as in much of the macro-labor literature, and $\mathcal{M}(B, S) = BS/(B + S)$, following Kiyotaki and Wright (1993). Policy is described by a UI benefit b and a stochastic process for i summarized by $(\bar{i}, \rho_i, \sigma_i)$. Finally, we have bargaining power in MP and KW, η and θ.

We set β so the real interest rate in the model matches the data, measured as the difference between the rate on Aaa bonds and realized inflation. We set $(\bar{i}, \rho_i, \sigma_i)$ to match the average, autocorrelation, and standard deviation of the nominal rate. The parameters k, δ, Z, σ, η and b are fixed using the standard approach in the macro-labor literature (e.g., Robert Shimer 2005 or Menzio and Shi 2009). Thus, k and δ match the average unemployment rate and UE (unemployment-to-employment) transition rate; Z is normalized so that the vacancy rate is 1; σ is to set match the regression coefficient of v/u on the UE transition rate; η is equated to σ, by the Arthur J. Hosios (1990) rule; and b is set so that UI benefits are half of average w.

We then set A, a, γ and θ as in the relevant monetary economics literature. First, set A and a so the relationship between money demand M/pY and i is the same in the model and data. In the model,

$$\frac{M}{pY} = \frac{M/p}{Y} = \frac{g(q)}{(1-u)\left\{\alpha_f[g(q)-c(q)]+y\right\}}, \tag{21}$$

which depends on i via q and u, and on A and a via the function $g(q)$. Although there are alternative ways to fit this relation, we set A to match average M/pY and a to match the empirical elasticity, using $M1$ as our measure of money.[14] Notice that (21) also involves γ in $c(q)$ and θ in $g(q)$. For now, we set $\gamma = 1$, so that the MRT between x and q is 1, as is often assumed in related models (but see below). Finally, we set θ so the markup in KW matches the retail data summarized by Miguel Faig and Belen Jerez (2005), which gives a target markup of 30 percent.[15]

The targets discussed above and summarized in Table 1 are sufficient to pin down all but one parameter, the value of leisure ℓ. As is well known, the literature has not reached a consensus on how to set this value. For instance, Shimer (2005) assumes $\ell = 0$; Marcus Hagedorn and Iourii Manovskii (2008) calibrate it using the cost of hiring and find that $(b+\ell)/y = 0.95$; and Robert E. Hall and Paul R. Milgrom (2008) calibrate it using consumption data and find that $(b+\ell)/y = 0.71$. Here we follow a different strategy, and set ℓ so that the model implies that, at the business cycle frequency, measured fluctuations in productivity y (holding monetary policy fixed) account for 2/3 of the observed fluctuations in u. While the exact target is somewhat arbitrary, this method reflects a common view, articulated in Mortensen

[14] We use $M1$ mainly to facilitate comparison with the literature. Although at first sight it may seem that $M0$ better suits the theory, one can reformulate this kind of model so that demand deposits circulate in the goods market, either instead of or along with currency (see Berentsen, Camera and Christopher J. Waller 2007; Ping He, Lixin Huang and Wright 2008; or Jonathan Chiu and Cesaire Meh 2008).

[15] S. Boragan Aruoba, Waller and Wright (2009) for more on calibrating LW-type models, including matching the markup data.

and Eva Nagypal (2007), that productivity is a major but not the only cause of cyclical fluctuations in labor markets.[16]

Table 1: Calibration Targets

Description	Value
average unemployment u	.006
average vacancies v (normalization)	1
average UE rate λ_h (monthly)	.450
elasticity of $\lambda_h$wrt v/u	.280
firm's bargaining power in MP η	.280
average UI replacement rate b/w	.500
average money demand M/pY(annual)	.179
elasticity of M/pYwrt i (negative)	.556
elasticity γ of cost function	1
retail sector markup	.300
average nominal interest rate i (annual)	.074
autocorrelation of i (quarterly)	.989
standard deviation of i	.006
average real interest rate r (annual)	.033

Table 2: Key Parameter Values

	Description	Baseline	Markup	Leisure	Elasticity
β	discount factor	.992	.992	.992	.992
ℓ	value of leisure	.504	.517	.514	.491
A	KW utility weight	1.08	1.10	1.07	1.10
a	KW utility elasticity	.179	.211	.179	.105
δ	job destruction rate	.050	.050	.050	.050
k	vacancy posting cost (10^{-4})	8.44	8.68	6.47	8.25
Z	MP matching efficiency	.364	.364	.364	.364
σ	MP matching $v$elasticity	.280	.280	.280	.280
η	MP firm bargaining share	.280	.280	.280	.280
θ	KW firm bargaining share	.275	.225	.275	.275

Table 2 summarizes parameter values. The first column is for the baseline calibration described above. For robustness, we also present three alternative calibrations

[16]We also tried several alternative calibration strategies: Berentsen, Menzio and Wright (2008) report results when ℓ is set as in Hagedorn and Manovskii (2008), and when it is set to minimize deviations between predicted and actual u. While details differ, the overall message is similar.

in the other columns. In the first alternative, labeled Markup, we set θ so that the KW markup is 40 percent rather than 30 percent. In the second, labeled Leisure, we set ℓ so that at the business cycle frequency the model accounts for all, rather than 2/3, of unemployment volatility in response to fluctuations in y. In the third, labeled Elasticity, we set a so that the elasticity of money demand is -1 rather than -0.556 as in the base case. Although these alternatives are somewhat arbitrary, they suffice to illustrate how the results depend on the parameters. Notice that given these parameters, the share of the KW market in total output is pinned down by $\mathcal{M}(1, 1-u)M/pY$. For the record, with our baseline calibration, KW accounts for 42 percent and AD for 58 percent of consumption.

3.2 Results

Using the calibrated parameters, we compute equilibrium for the model when i and y follow stochastic processes, as described in the Appendix. Then we input the actual time series for i, holding y constant, and compute the implied path of u. To focus on longer-run behavior, we pass u through an HP filter to eliminate business-cycle fluctuations. The resulting series is our prediction of what trend unemployment would have been if monetary policy had been the only driving force over the period.

TABLE 3: 1972-1992

	u 1972(1)	u 1982(1)	u 1992(1)	Δ1972-1982	Δ1982-1992
Data	5.33	8.16	6.48	2.83	-1.68
Baseline	5.83	7.02	5.96	1.19	-1.06
Markup	5.83	7.97	6.02	2.14	-1.95
Leisure	5.83	7.91	6.01	2.08	-1.90
Elasticity	5.83	7.55	6.02	1.72	-1.53

All data is passed through a 1600 HP-filter

For the baseline parameters, Figure 5 plots the time-series of the actual and counterfactual trend u, as well as the unfiltered series. While, obviously, the u predicted by the model does not match all of the movement in the data, there is a very similar basic pattern. Changes in i alone account for around 40 percent of the 2.83 increase in u between 1972 and 1982, and around 60 percent of the 1.68 decline between 1982 and 1992 (Table 3). The model also generates the overall decline in u between 1992 and 2005, if not all the ups and downs. The 1960s are the only extended episode where the actual and counterfactual u move in opposite directions.[17] Figure 6 shows the scatter plot of actual (grey) and counterfactual (black) i versus u; and Figure 7 repeats this with inflation replacing interest rates. The relationships generated by the model are very similar to the regression lines implied by the data in Figures 1 and 2. We conclude that we can account for the overall pattern in u solely by monetary policy, even if there is plenty left in the data to be explained by other factors.

Table 3 also summarizes results from the other calibrations. As one can see, money accounts for more if we target a higher markup, assuming y shocks generate a larger fraction of business-cycle fluctuations, or make money demand more elastic. Figure 8 shows how the model is closer to the data when ℓ is higher. To understand what happens when ℓ is higher, note that y and i have different effects on R, but given the effect on R they have the same effect on u. If u responds more to y, as it does when ℓ is higher, u also responds more to i. One can also interpret this using the MP and LW curves (even if formally these curves describe only steady states). Increasing ℓ flattens the MP curve, as shown in Figure 9 for the actual calibrated parameters, so

[17] Clearly we cannot explain u in the 1960s as a function i alone, since theory predicts $\partial u/\partial i < 0$. We could, however, say this decline in u was due to other factors, say increased productivity, and slack monetary policy actually prevented u from falling by more. Quantitatively, we need to increase y only from 1 to 1.0275 in order to explain lower u despite higher i during the 1960s.

a shift in LW from changing i has a larger impact on u.[18]

We conclude that monetary policy may have been responsible for a sizable part of movements in trend u over the last half century. Moreover, we conclude that monetary policy is more important for labor market performance when the markup is higher, money demand is more elastic, or the contribution of productivity shocks to unemployment over the business cycle is greater. Although we focus on low-frequency implications – again, because we abstract from several factors that may be important for business cycles – the model does make predictions about high-frequencies, too, as shown by dotted lines in Figures 5 and 8. Especially in Figure 8, we actually not only match the trend well but also the peak in the *unfiltered* u series in the 1980s. Of course, we do not claim that money can account for all of the high- or low-frequency behavior of u, which is good, in the sense that it leaves plenty of room for other factors to play a role. What we claim is that money may be more important than previously understood.

3.3 Financial Innovation

The baseline model generates a relationship between nominal interest rates and money demand that closely resembles its empirical counterpart prior to the 1990s. Since the 1990s, however, M/pY is systematically lower for all i – that is, the money demand curve has shifted down – and the baseline parameters do not match the data well. See Figure 10 below. This is a concern, since we just saw that the shape of money demand plays an important role in determining the effect of i on u. We now carry out a counterfactual analysis like the one above in a generalized version of the model that is better able to replicate observed money demand, similar in spirit to an exercise in

[18] Similar economic intuition can be used to understand how the markup or money demand elasticity affect results.

Veronica Guerrieri and Guido Lorenzoni (forthcoming).

Recall that the probability that a KW meeting is anonymous, and hence that money is essential, can be any $\omega \in [0, 1]$. Here, we allow ω to differ before and after 1990. This is meant to capture the idea that the downward shift in money demand was due to innovations in payments, such as the proliferation of credit cards, and perhaps also ATMs, and sweep accounts, for instance, that permit households to economize on real balances. We keep $\omega = 1$ from 1955-1990, and set $\omega = 0.62$ after 1990 to match average M/pY in the latter period, with other parameters set as in the baseline calibration.[19] Given these parameter values, we compute equilibrium under the assumption that a one-time unexpected change in ω occurred in 1990, which is crude but still illustrative. We then feed in the actual path for i and compute the predicted path for u.

Figure 10 depicts the money demand relationship generated by the model with financial innovation (solid black) and the baseline model (dashed black), as well as the actual data (gray). In all cases, the series have been filtered, so the chart shows the scatters of the HP trends. The model with financial innovation generates a money demand curve that has a higher mean and elasticity before 1990, and a much lower mean after 1990. Generally, with financial innovation, the money demand relationship in the model is much closer to the data. Figure 11 shows actual u (grey), the path generated by the model with financial innovation (solid black), and the path generated by the baseline model (dotted black). As one can see, the model with financial innovation implies money accounts for more of the movement in u. In particular, the

[19] By comparison, Aruoba, Waller and Wright (2009) argue for $\omega = 0.88$ to match Elizabeth C. Klee's (2008) finding that shoppers use credit cards (as opposed to cash, checks and debit cards) for 12% of supermarket transactions in the scanner data (which is close to the 16% reported by Cooley and Hansen (1991) from earlier consumer survey data). While future work on better matching micro payments data is desirable, calibrating ω as we do here suffices for the basic point.

model with financial innovation generates more of a run-up in u during stagflation, because during the 1970s we were in the regime with the higher ω.

This experiment provides another robustness check on the baseline model. Additionally, this extension implies that the observed shift in money demand is likely to reduce the impact of monetary policy on labor markets in the future. In Figure 12, the black line shows the path for u assuming $\omega = 0.62$ over the entire period. In this case, the inflation of the 1970s would have had a much smaller effect on u. Hence, we predict that in the future, assuming money demand does not shift back, inflation will not lead to as large an increase in u as we observed during stagflation.

4 Comparison with CIA

Questions that often comes up in monetary economics are: Why do we need microfoundations? Do they matter for any substantive results? At one level, we obviously do not *need* a search-and-bargaining model to study the effect of money on unemployment, since some of the papers mentioned in the Introduction use cash-in-advance, henceforth CIA, models. One does not actually need a model in the modern sense at all – one could use the IS-LM paradigm combined with Okun's Law. The interesting issue is not one of *need*, but whether it *matters* for the results whether one uses a search-and-bargaining or a reduced-form approach. To discuss this issue, here, we consider a version of our model with a frictionless competitive goods market – no search or bargaining – except that we impose a CIA constraint.[20]

We compare the two models in two ways: We examine the mechanisms analytically; and we also use calibrated versions to contrast results numerically. For the first

[20] This is similar to Andofatto, Hendry and Moran (2004) and Cooley and Quadrini (2004), who impose CIA in MP models, but to give the reduced-form approach a chance, we really need both cash and credit goods: simple CIA models simply do not match empirical money demand at all well.

comparison, without going through the rudimentary details, the setup with CIA but otherwise no frictions in KW implies a demand for q given by

$$\upsilon'(q) = (1+i)c'\left(\frac{q}{1-u}\right). \tag{22}$$

The left side is the MRS between q and x, and the right is the opportunity cost of q in terms of x, including the interest rate $1+i$ and the marginal cost c' evaluated at the equilibrium quantity produced by an active firm (i.e., one matched with a worker). An increase in i raises the cost of q due to CIA, while an increase in u raises marginal cost for each matched f, since there are fewer of them. Hence, an increase in either i or u reduces demand for KW goods.

By comparison, in our search-and-bargaining model, the demand for q satisfies

$$\upsilon'(q) = \left(1+\frac{i}{\alpha_h}\right)g'(q). \tag{23}$$

There are two differences between (22) and (23). First, because of search frictions, h only gets to trade in the KW market with probability α_h, making the effective interest rate i/α_h, instead of i. Second, because we use Nash bargaining rather than Walrasian pricing, the effective price is $g'(q)$ rather than $c'(q)$, where $g(q)$ is given in (14). In our model, an increase in i reduces the demand for q, as in the CIA model, but the effect is larger given that $\alpha_h < 1$ and given that $g(q)$ is typically less convex than $c(q)$. Moreover, in our model an increase in u affects q by lowering the probability of trade, which is different from the CIA model, where an increase in u merely raises the price since each active f has to produce more.

Additionally, in both models the entry (vacancy posting) decision of f is based on expected revenue R, but in the CIA model,

$$R = c'\left(\frac{q}{1-u}\right)\frac{q}{1-u} - c\left(\frac{q}{1-u}\right) + y. \tag{24}$$

An increase in the demand for q increases R in the CIA model by increasing the difference between the revenue and cost associated with the KW good. And an increase in u increases R because it increases the equilibrium price of the KW good.

By comparison, in our search-and-bargaining model,

$$R = \alpha_f \left[g(q) - c(q) \right] + y. \tag{25}$$

An increase in q here increases R by raising the surplus that f gets from KW sales, $g(q) - c(q)$. This is similar to the effect of q on R in the CIA model, except with bargaining the magnitude depends not only on the shape of the cost function but also on the utility function and bargaining power via the equilibrium object $g(q)$. Additionally, an increase in u raises R in our model by increasing the probability of KW trade α_f, an effect that is totally missing in the CIA model. We conclude that the channels via which q affects u, the channels via which u affects q, and the impact of a change in i, are qualitatively different in the two models. The CIA model simply does not capture the same underlying economics underlying our model.

We now turn to the quantitative comparison. First, for simplicity here, suppose $c(q) = q^\gamma$ is linear, $\gamma = 1$, which is a standard case in the literature. In the CIA model, an increase in i increases the opportunity cost of money, which reduces the demand for q, but with linear cost and a competitive market the price of q and hence R are completely unaffected. Therefore, in the CIA model with linear cost, an increase in i has no effect on the incentive for f to open vacancies and hence has no effect on u. By contrast, in our model f has market power, and price exceeds cost in KW. Thus, in our model, R falls with a decline in demand and an increase in i reduces vacancies and employment. In our baseline calibration, increasing inflation from 0 to 10 percent raises u from 5.2 to 7.4 across steady states, while in the CIA model this same policy has literally no effect on u.

Suppose now that cost is convex: $\gamma > 1$. Then the price of KW goods exceeds average cost, and so R depends on demand, even in the CIA model. Thus, with $\gamma > 1$, a fall in demand for q reduces employment even in that model. But when we calibrate the two models, as shown in Table 4, we find that the magnitudes of the effect are very different. In the CIA model, increasing inflation from 0 to 10 percent raises u from 5.4 to 6.6 when $\gamma = 1.05$, and from 5.4 to 6.8 when $\gamma = 1.1$. By comparison, in our search-and-bargaining model, the same policy increases u from 5.2 to 7.9 when $\gamma = 1.05$, and from 5.1 to 8.7 when $\gamma = 1.10$. Thus our model generates much bigger effects, mainly because the share of the surplus accruing to f in KW is determined differently, and ends up both larger and more sensitive to changes in demand.

Table 4: Calibrated Parameters

	$\gamma = 1$		$\gamma = 1.05$		$\gamma = 1.1$	
	BMW	CIA	BMW	CIA	BMW	CIA
β	.992	.992	.992	.992	.992	.992
ℓ	.504	.480	.511	.497	.517	.513
A	1.08	1.01	1.13	1.04	1.21	1.08
a	.179	.030	.156	.001	1.13	.001
δ	.050	.050	.050	.050	.050	.050
k	8.44	4.11	8.56	4.25	8.68	4.40
Z	.364	.364	.364	.364	.364	.364
σ	.720	.720	.720	.720	.720	.720
η	.280	.280	.280	.280	.280	.280
θ	.275	–	.250	–	.275	–

In both models, increasing γ magnifies the response of u to i, but it also dampens the response of M/PY to i, and thus makes it harder to match empirical money demand. Intuitively, the higher is γ, the smaller the effect of an increase in i on q and hence on M/PY. Quantitatively, the CIA model can match the elasticity of M/PY when $\gamma = 1$, but fails for $\gamma = 1.05$ or higher: for $\gamma \geq 1.05$, there are no parameters for which the CIA model looks like the actual money demand curve. In contrast, our model can match the empirical money demand curve for $\gamma = 1$, 1.05 or 1.1. This is because, in our model, h faces an effective interest rate of i/α_h, rather than i, which means that an increase in i has a larger impact on q and M/PY. So, to the extent that one is disciplined by money demand, and not free to pick γ arbitrarily, our model generates a bigger quantitative impact of monetary policy on labor markets.

Figures 13-16 summarize the results. Figure 13 shows how calibrated versions of both models match money demand at $\gamma = 1$, but as seen in Figure 14 the CIA model predicts a smaller effect of i on u. Figures 15 and 16 show that at $\gamma = 1.1$ the CIA model can generate a bigger effect of i on u, although still not as big an effect as the search-and-bargaining model, but the CIA model with $\gamma = 1.1$ cannot match money demand while the search-and-bargaining model can. These findings show that using search-and-bargaining theory in monetary economics can matter a lot, qualitatively as well as quantitatively. We conclude that while one may not *need* microfoundations for monetary economics, microfoundations certainly do *matter* for the results.

5 Conclusion

This paper studied the long-run relation between unemployment and monetary policy. We first documented that unemployment is positively related to inflation and interest rates in the low-frequency data. We then developed a theory in which both labor markets and goods markets are modeled using the search-and-bargaining approach. The framework is tractable and many results, at least for steady states, can be derived simply by shifting curves. The framework is also amenable to quantitative analysis, and to illustrate this, we asked how much we can account for in unemployment behavior when the sole driving force is monetary policy. We found that we can account for quite a lot.

Of course, there is still much in unemployment left to be explained by other factors, potentially including demography, productivity, fiscal policy, and energy prices. In the current economic environment, it may well be that problems in banking, housing, and asset markets generally are contributing significantly to high unemployment despite low inflation; a serious analysis of this idea is well beyond the scope of the current project. We also showed how the results depend on certain key parameters, including a parameter representing financial innovation. Finally, we asked if it matters, qualitatively and quantitatively, whether one uses monetary theory based on search-and-bargaining microfoundations or based on an ad hoc cash-in-advance specification. The answer is yes.

Appendix: The Dynamic-Stochastic Model

At the beginning of a period, the state is $s = (u, i, y)$, where u is unemployment, i the nominal interest rate and y productivity. The state s was known in the previous AD market, including the return on nominal bonds maturing this period. Although these bonds are not traded in equilibrium, i matters because it pins down the expected return on real balances $\hat{\rho}(s) = \mathbb{E}[\rho(\hat{s})|s]$ via the no-arbitrage condition $1 = \beta(1 + i)\hat{\rho}(s)$. The nominal interest rate and productivity follow exogenous (independent) processes:

$$
\begin{aligned}
\hat{\imath} &= \bar{\imath} + \rho_i(i - \bar{\imath}) + \epsilon_i,\ \epsilon_i \sim N(0, \sigma_i) \\
\hat{y} &= \overline{y} + \rho_y(y - \overline{y}) + \epsilon_y,\ \epsilon_y \sim N(0, \sigma_y)
\end{aligned}
$$

Unemployment behaves as follows. In MP, each unemployed h finds a job with probability $\lambda_h[\tau(s)]$ and each f with a vacancy fills it with probability $\lambda_f[\tau(s)]$, where $\tau(s) = v/u$ and $v = v(s)$ were set in the previous AD market. Therefore, at the beginning of KW,

$$\hat{u}(s) = u - u\lambda_h[\tau(s)] + (1 - u)\delta.$$

When h and f meet in MP, $w(s)$ is determined by generalized Nash bargaining, but is paid (in units of x) in AD; $w(s)$ can be renegotiated in MP each period.

In the KW market, h meets f with probability $\alpha_h[Q(s)]$ and f meets h with probability $\alpha_f\left[Q(s)\right]$, where $Q(s) = 1/\left[1-\hat{u}(s)\right]$, whence $q(z,s)$ and $d(z,s)$ are determined according to generalized Nash bargaining, where z denotes real balances held by h. After KW, in the AD market, the realization of $\hat{s}$ becomes known, f liquidates inventories, pays wages and dividends, and posts $v(\hat{s})$ vacancies for the next MP. Also, h chooses $z(\hat{s})$, and the government collects $T(\hat{s})$, pays b, and announces $\hat{\imath}$.

In MP, taking as given the equilibrium wage function $w(s)$, the value functions for h are

$$
\begin{aligned}
U_0^h(z;s) &= V_0^h(z;s) + \lambda_h[\tau(s)]\left\{V_1^h[z,w(s);s] - V_0^h(z;s)\right\} \\
U_1^h(z;s) &= V_1^h[z,w(s);s] - \delta\left\{V_1^h[z,w(s);s] - V_0^h(z;s)\right\}.
\end{aligned}
$$

In KW, taking as given the equilibrium terms of trade $q(z;s)$ and $d(z;s)$,

$$
\begin{aligned}
V_0^h(z;s) &= \alpha_h\left[\tfrac{1}{1-\hat{u}(s)}\right]\left\{\upsilon\left[q(z;s)\right] - \hat{\rho}(s)d(z;s)\right\} + \hat{\rho}(s)\left[z - d(z;s)\right] + \mathbb{E}W_0^h(0;\hat{s}) \\
V_1^h(z,w;s) &= \alpha_h\left[\tfrac{1}{1-\hat{u}(s)}\right]\left\{\upsilon\left[q(z;s)\right] - \hat{\rho}(s)d(z;s)\right\} + \hat{\rho}(s)\left[z - d(z;s)\right] + \mathbb{E}W_1^h(0,w;\hat{s})
\end{aligned}
$$

using the linearity of $W_e^h(\cdot;\hat{s})$. Finally, in AD,

$$
\begin{aligned}
W_0^h(z;\hat{s}) &= z + b + \ell + \Delta(\hat{s}) - T(\hat{s}) + \max_{\hat{z}\geq 0}\left\{-\hat{z} + \beta U_0^h(\hat{z};\hat{s})\right\} \\
W_1^h(z,w;\hat{s}) &= z + w + \Delta(\hat{s}) - T(\hat{s}) + \max_{\hat{z}\geq 0}\left\{-\hat{z} + \beta U_1^h(\hat{z};\hat{s})\right\}.
\end{aligned}
$$

Let $z(\hat{s})$ solve the above maximization, $d(s) = d\left[z(s);s\right]$ and $q(s) = q\left[z(s);s\right]$.

For f, in MP, taking as given $w(s)$, the value functions are

$$\begin{aligned} U_0^f(s) &= \lambda_f[\tau(s)]V_1^f[w(s);s] \\ U_1^f(s) &= (1-\delta)V_1^f[w(s);s]. \end{aligned}$$

In KW, taking as given $q(z;s)$, $d(z;s)$ and $z(s)$,

$$V_1^f(w;s) = \alpha_f\left[\tfrac{1}{1-\hat{u}(s)}\right]\{\hat{\rho}(s)d(s) - c[q(s)]\} + \beta\mathbb{E}W_1^f(0,y,w;\hat{s}).$$

And in AD,

$$\begin{aligned} W_0^f(\hat{s}) &= \max\{0, -k + U_0^f(\hat{s})\} \\ W_1^f(z,y,w;\hat{s}) &= y + z - w + \beta U_1^f(\hat{s}). \end{aligned}$$

In MP the surplus of a match is

$$S(s) = V_1^h[z,w;s] + V_1^f[w;s] - V_0^h(z;s),$$

where we note that both z and w vanish on the right hand side. The bargaining solution implies $w(s)$ is such that

$$\begin{aligned} V_1^h[z,w(s);s] - V_0^h(z;s) &= (1-\eta)S(s) \\ V_1^f[w(s);s] &= \eta S(s). \end{aligned}$$

In KW, the bargaining solution implies that $d(z;s) = z$ and $q(z;s)$ is such that $\hat{\rho}(s)z = g\left[q(z;s)\right]$, with $g(q)$ as defined in the text.

The transition probability function $\mathcal{P}(\hat{s}; s)$ is constructed from the laws of motion for i, y, and u in the obvious way. Then a Recursive Equilibrium is a list of functions $S(s)$, $q(s)$, $\tau(s)$, and $\mathcal{P}(\hat{s}; s)$ such that:

$$\begin{aligned}
S(s) &= y + b - \ell + \alpha_f \left[\tfrac{1}{1-\hat{u}(s)}\right] \{g[q(s)] - c[q(s)]\} + \beta \mathbb{E}\{1 - \delta - (1-\eta)\lambda_h[\tau(\hat{s})]\} S(\hat{s}) \\
1 &= \frac{\upsilon'\left[q(s)\right]}{g'\left[q(s)\right]} - \frac{i}{\alpha_h \left[\tfrac{1}{1-\hat{u}(s)}\right]} \\
k &= \beta \lambda_f[\tau(s)] \eta S(s)
\end{aligned}$$

and $\mathcal{P}$ is consistent with the law of motion for (i, u, y). Now standard methods in quantitative macroeconomics allow us to solve for the equilibrium functions numerically. Details, including programs for calibration and simulation, are available by request.

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Figure 1: Inflation and Unemployment

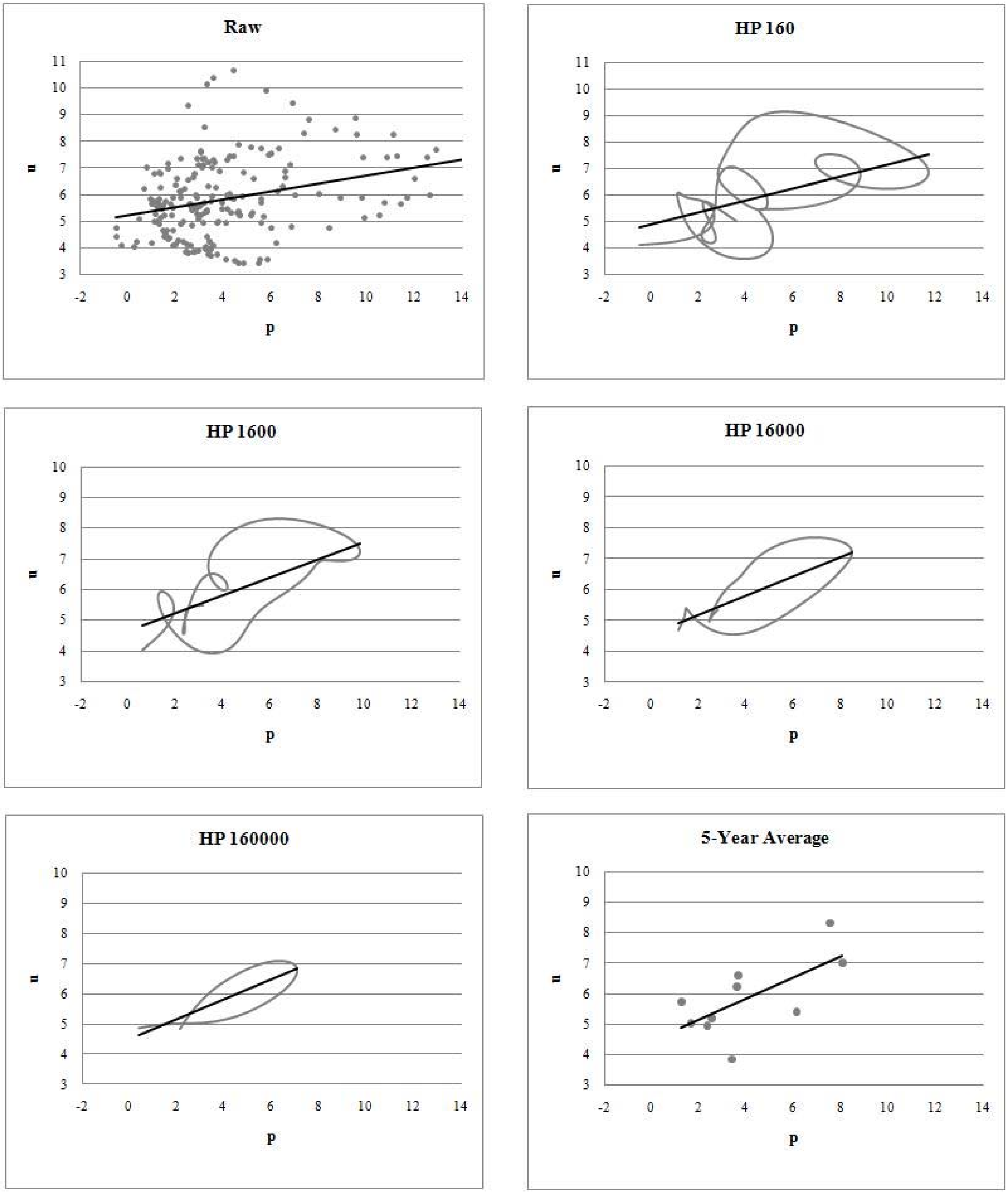

Figure 2: Interest Rate and Unemployment

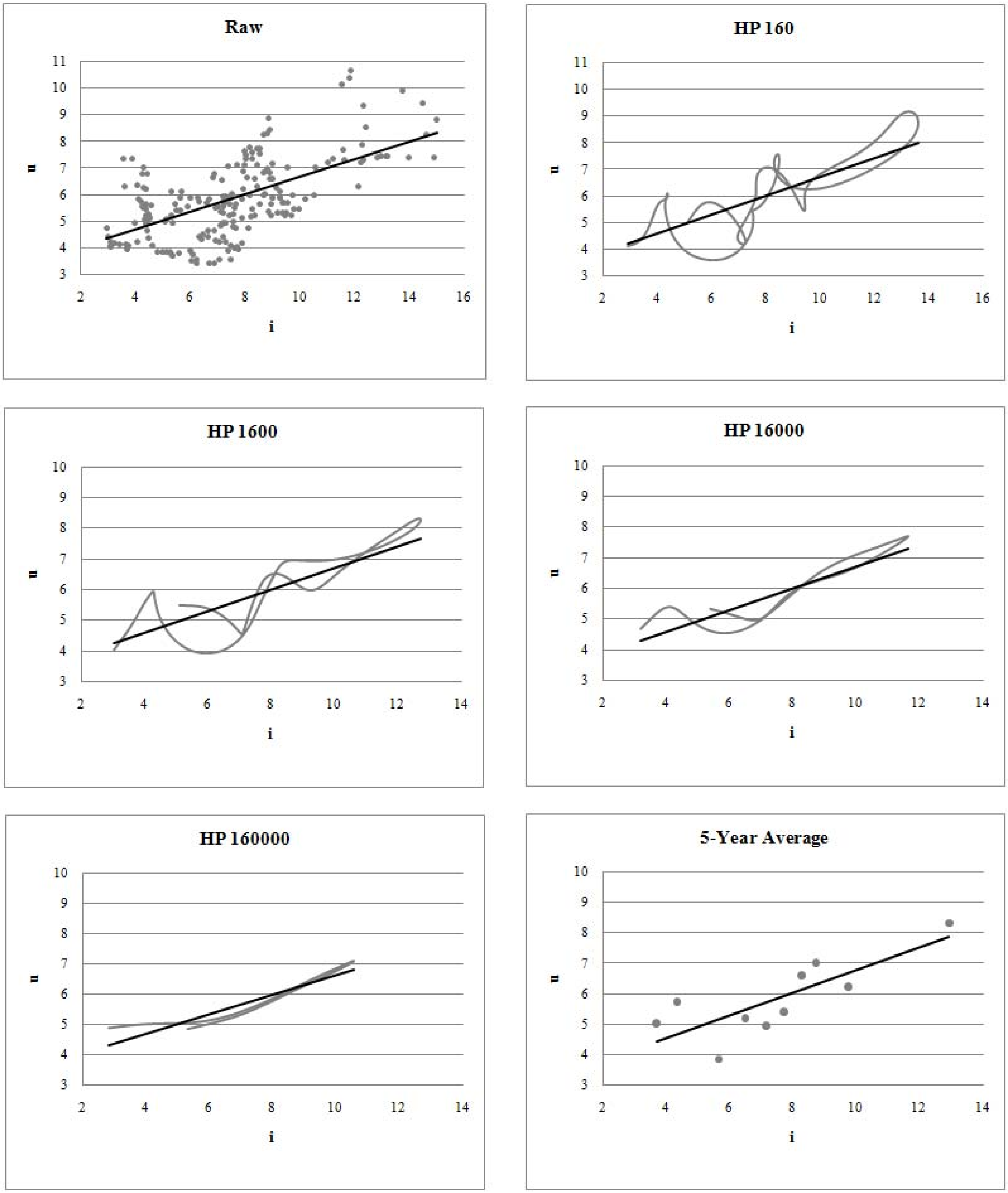

Figure 3: Inflation and Unemployment

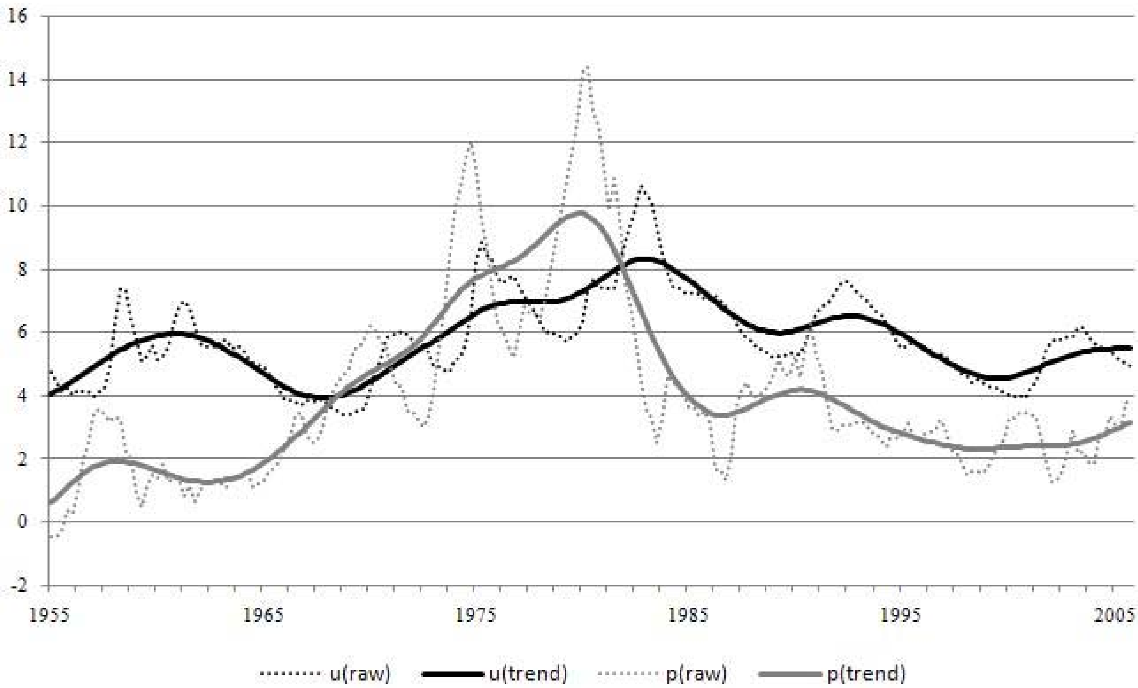

Figure 4: Steady State Equilibrium

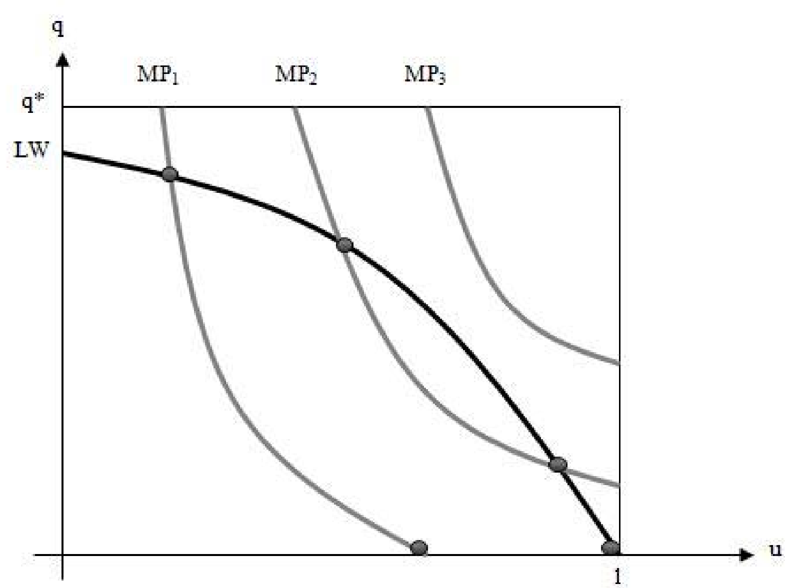

Figure 5: Counterfactual Unemployment

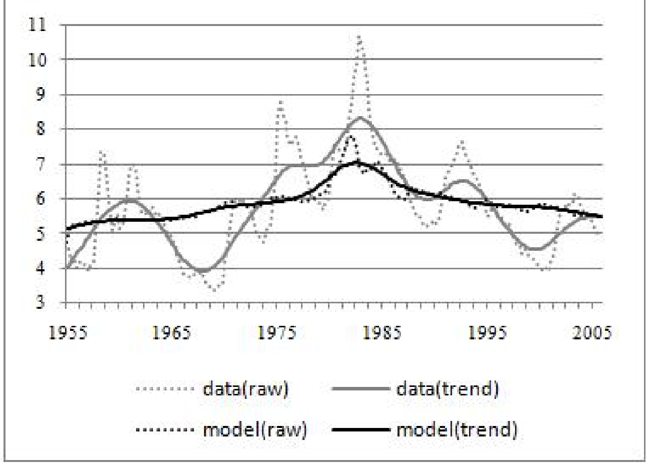

Figure 6: Interest Rate and Unemployment

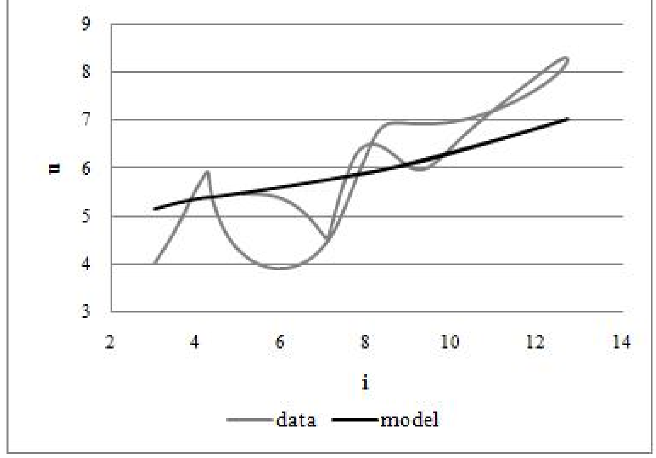

Figure 7: Inflation and Unemployment

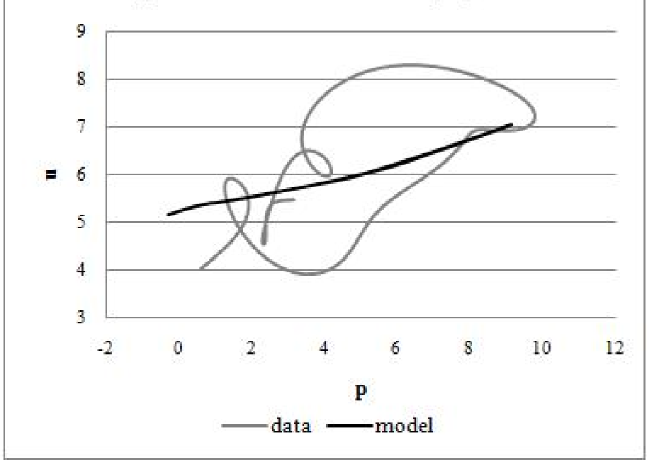

Figure 8: High Value of Leisure

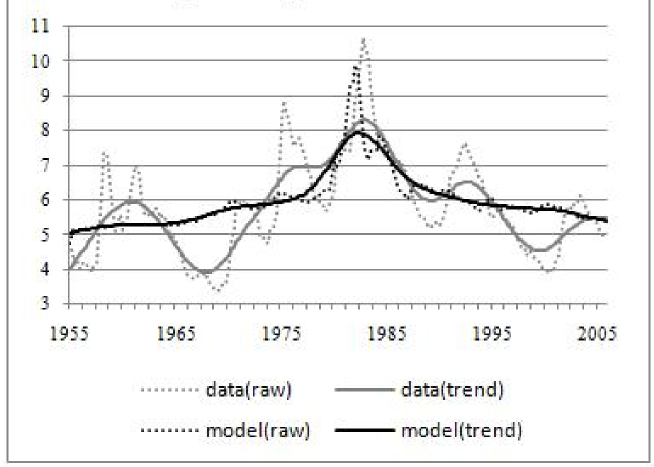

Figure 9: High Value of Leisure

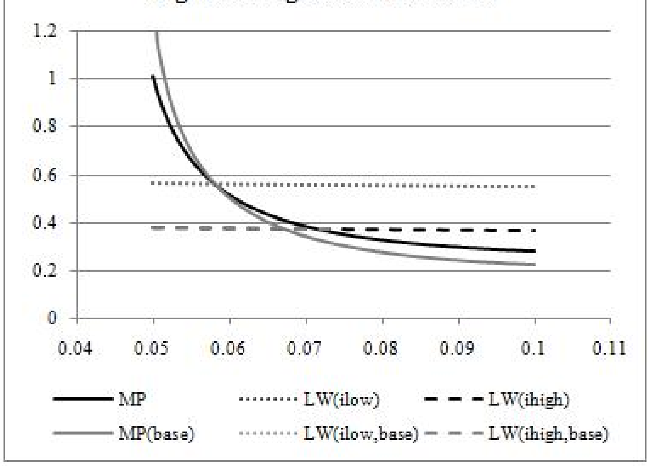

Figure 10: Money Demand

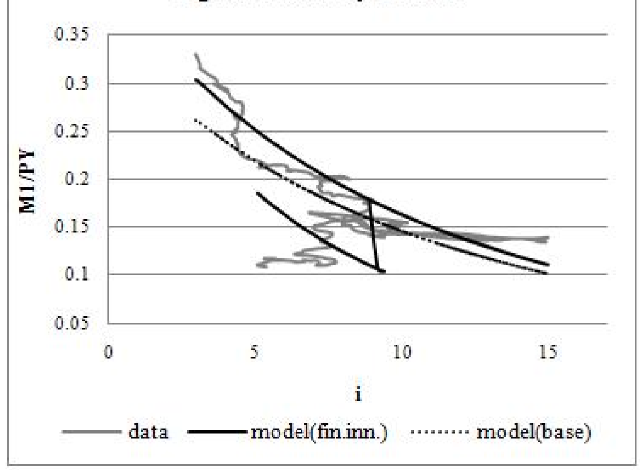

Figure 11: Financial Innovation in 1990

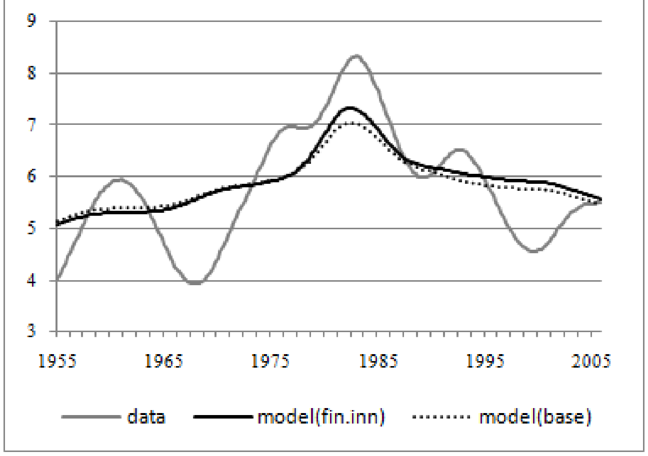

Figure 12: Financial Innovation in 1955

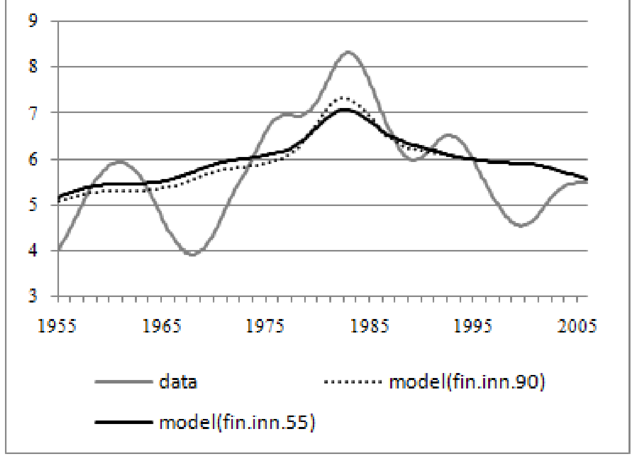

Figure 13

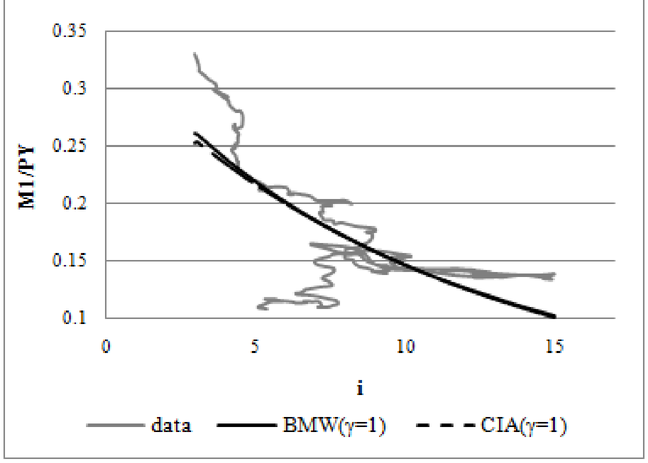

Figure 14

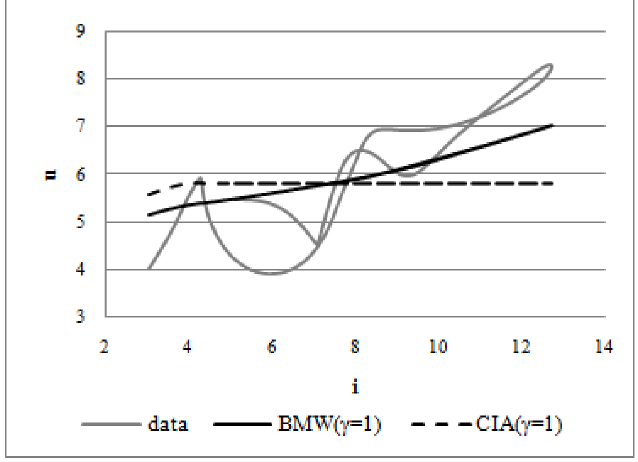

Figure 15

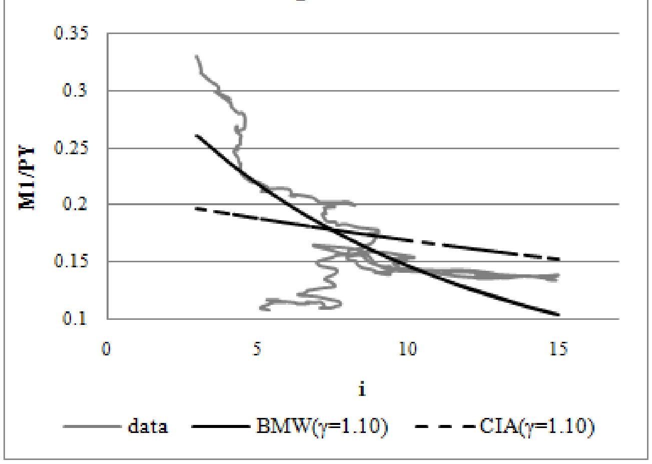

Figure 16